Brief analysis of Einstein Fallacies - Special Relativity

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Abstract

This is a brief analysis of some fallacies as they appear in standard *Special Relativity* as taught in Universities. Specifically regarding the concept of *simultaneity* and *light clock* thought experiments. We also discuss a consequence of Lorentz transformation which causes material objects in relative motion to *dematerialise*.

I. CLASSICAL PARTICLE EMISSION

Assume the mass of Atom is M and mass of photon is m. Let them have an excess wound up energy of $\frac{1}{2}mc'^2$, like energy stored in a compressed spring. Say initially their bulk velocity was v_B according an inertial observer. What will be their velocities after emission?. Suppose the final velocities of m and M are v_1 and v_2.

Momentum equation according to moving inertial observer,

$$mv_1 + Mv_2 = (m+M)v_B = p \tag{1}$$

Note, $\frac{p^2}{2(m+M)} = \frac{1}{2}(m+M)v_B^2$.

Energy equation according to moving inertial observer,

$$\frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 = \frac{p^2}{2(m+M)} + \frac{1}{2}mc'^2 = E \tag{2}$$

From momentum(Eqn(1)) eqn,

$$v_2 = \frac{p - mv_1}{M} \tag{3}$$

Using v_2 in energy(Eqn(2)) eqn

$$\frac{1}{2}mv_1^2 + \frac{1}{2}M\left(\frac{p-mv_1}{M}\right)^2 = E$$
$$\left(1+\frac{m}{M}\right)v_1^2 - \frac{2pv_1}{M} + \frac{p^2}{mM} - \frac{2E}{m} = 0 \tag{4}$$

Solving the quadratic eqn in Eqn(4) we get,

$$v_1 = v_B \pm c'\sqrt{\frac{M}{m+M}}$$

Therefore,

$$v_2 = \frac{p-mv_1}{M} = \frac{p - m\left(v_B \pm c'\sqrt{\frac{M}{m+M}}\right)}{M}$$
$$v_2 = v_B \mp c'\frac{m}{M}\sqrt{\frac{M}{m+M}}$$

That is v_1 and v_2 are given by,

$$v_1 = v_B \pm c'\sqrt{\frac{M}{m+M}}$$
$$v_2 = v_B \mp c'\frac{m}{M}\sqrt{\frac{M}{m+M}} \tag{5}$$

Relative velocity between m and M would be,

$$v_2 - v_1 = \mp c'\left(1+\frac{m}{M}\right)\sqrt{\frac{M}{m+M}}$$
$$v_2 - v_1 = \mp c'\sqrt{1+\frac{m}{M}} \tag{6}$$

Thus wrt mass M the emitted particle is going away at $c'\sqrt{1+\frac{m}{M}}$ m/s in either +ve or -ve X-direction. If we suppose that $v_2 - v_1 = c$, that is the speed is always c wrt emission source then,

$$v_2 - v_1 = c = \mp c'\sqrt{1+\frac{m}{M}}$$
$$c' = \mp c\sqrt{\frac{M}{M+m}}$$
$$So, \frac{1}{2}mc'^2 = \frac{1}{2}\frac{mM}{m+M}c^2 \tag{7}$$

Thus if the initial stored(wound up) energy is $\frac{1}{2}\frac{mM}{m+M}c^2$ then the relative velocity between the emitter(M) and the emitted(m) particle is always c m/s. And wrt an observer moving at v_B m/s wrt initial bound state, the final unbound state velocities are,

$$v_1 = v_B \pm c\frac{M}{m+M}$$
$$v_2 = v_B \mp c\frac{m}{m+M} \tag{8}$$

In this scheme, regardless of the value of relative velocity v_B all frames note the same amount of wound up energy in the system, $\frac{1}{2}mc'^2$.

The pair v_1, v_2 also represent the velocity outcomes of a collision between masses m and M with total kinetic energy E and total momentum p when there is no absorption/emission process involved.

In the relativistic framework there is even no pretense of conserving the momentum(or energy) of the system emitting light. That is the system at rest(with zero net momentum) can suddenly emit light and still remain at rest after creating some momentum out of nothing in the universe. To the external observers this also appears as excess energy creation out of nothing. We can formally prove the breakdown of the relativistic energy/momentum equations for a pair of colliding particles. But we can illustrate the same more easily with the same example as used by Einstein in his 1905 paper.

II. NOTE ON EINSTEIN'S SIMULTANEITY

Consider a stationary frame S with a box AB and a source of light located stationary at the center(x_0) of the box. At time $t = 0$ photons are emitted in either direction. Photon 1 travels a distance ct in time t and is described by $x_1 = x_0 - ct$ similarly Photon 2 is described by $x_2 = x_0 + ct$ in frame S at time t. Frame S' is moving with velocity v m/s along +ve X-axis.

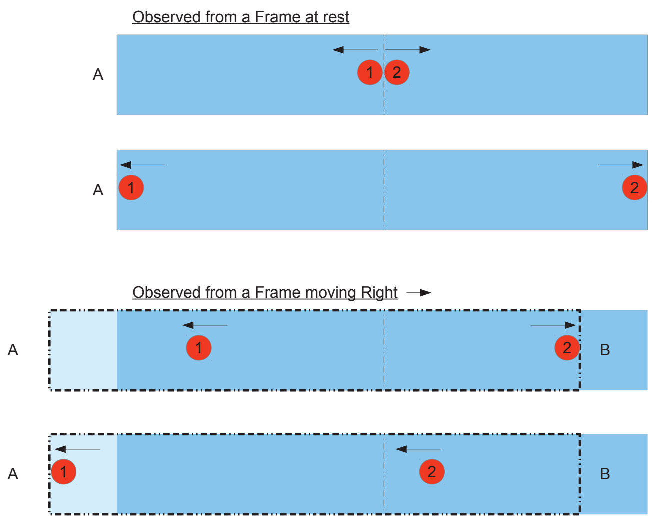

FIG. 1. Top 2 panels show the process as viewed from a rest frame. Here the two photons hit the ends simultaneously. The bottom 2 panels show the same process as viewed from a moving frame(According to Einstein). Note that Photon 1 travels a larger distance compared to Photon 2 according to Relativistic perception.

Einstein used semi-Newtonian analysis, adding up velocities like in Galilean transformation on his way to derive Lorentz transformation. We shall not resort to such wooly arguments. We shall rely entirely on Lorentz transformation for the analysis of the thought experiment. Because that is the formal tool of relativity.

At time $t = 0$, origin of S' coincides with the origin of S and time is synchronized ($t' = 0$). Instead of c, for generality let us assume the two objects move at $\pm u$ that is, $x_1 = x_0 - ut$ and $x_2 = x_0 + ut$.

The time and location of first object coming towards the origin of S as noted by S' frame are,

$$x'_1 = \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}}, t'_1 = \frac{t - \frac{vx_1}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$x'_1 = \frac{x_0 - (u+v)t}{\sqrt{1-\frac{v^2}{c^2}}}, t'_1 = \frac{t + \frac{vut}{c^2} - \frac{vx_0}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$x'_1 = \frac{x_0\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{vu}{c^2}} - \frac{(u+v)t'_1}{1+\frac{vu}{c^2}} \quad (9)$$

The time and location of second object moving away from the origin of S as noted by S' frame are,

$$x'_2 = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}}, t'_2 = \frac{t - \frac{vx_2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$x'_2 = \frac{x_0 + (u-v)t}{\sqrt{1-\frac{v^2}{c^2}}}, t'_2 = \frac{t - \frac{vut}{c^2} - \frac{vx_0}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$x'_2 = \frac{x_0\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{vu}{c^2}} + \frac{(u-v)t'_2}{1-\frac{vu}{c^2}} \quad (10)$$

At $t = 0$ Eqns(9,10) give,

$$x'_1 = \frac{x_0}{\sqrt{1-\frac{v^2}{c^2}}}, t'_1 = -\frac{vx_0}{c^2\sqrt{1-\frac{v^2}{c^2}}}$$
$$x'_2 = \frac{x_0}{\sqrt{1-\frac{v^2}{c^2}}}, t'_2 = -\frac{vx_0}{c^2\sqrt{1-\frac{v^2}{c^2}}}$$
$$Note, x'_1 = x'_2, And, t'_1 = t'_2 \quad (11)$$

At $t = \frac{L}{u}$ Eqns(9,10) give,

$$x'_1 = \frac{x_0 - (u+v)\frac{L}{u}}{\sqrt{1-\frac{v^2}{c^2}}}, t'_1 = \frac{\frac{L}{u} + \frac{vL}{c^2} - \frac{vx_0}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$x'_2 = \frac{x_0 + (u-v)\frac{L}{u}}{\sqrt{1-\frac{v^2}{c^2}}}, t'_2 = \frac{\frac{L}{u} - \frac{vL}{c^2} - \frac{vx_0}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\Delta x' = x'_2 - x'_1 = \frac{2L}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\Delta t' = t'_2 - t'_1 = -\frac{2vL}{c^2\sqrt{1-\frac{v^2}{c^2}}} \quad (12)$$

$\Delta x'$ and $\Delta t'$ in Eqn(12) show the path difference and time difference between the 2 paths as seen from S' frame.

Thus regardless of photons or other material objects released from x_0 in frame S, according to Lorentz transformation all produce the same non-simultaneous behaviour wrt a moving frame.

The path/time differences found in Eqn(12) depend only on the spatial distance L and relative velocity v and is a consequence of using the Lorentz transformation. From Eqns(9,10),

$$x'_1 = \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}}, t'_1 = \frac{t - \frac{vx_1}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$x'_2 = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}}, t'_2 = \frac{t - \frac{vx_2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\Delta x' = x'_2 - x'_1 = \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}}, Note, x_2 - x_1 = 2L$$
$$\Delta t' = t'_2 - t'_1 = -\frac{v(x_2 - x_1)}{c^2\sqrt{1-\frac{v^2}{c^2}}} = -\frac{2Lv}{c^2\sqrt{1-\frac{v^2}{c^2}}} \quad (13)$$

The stationary point $x = x_0$ in S appears in S' as,

$$x'_0 = \frac{x_0 - vt}{\sqrt{1-\frac{v^2}{c^2}}}, t'_0 = \frac{t - \frac{vx_0}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x'_0 = x_0\sqrt{1-\frac{v^2}{c^2}} - vt'_0 \tag{14}$$

Using Eqn(14) at $t = 0$ we get, $x'_0 = x'_1 = x'_2$ and $t'_0 = t'_1 = t'_2$.

III. MOMENTUM BALANCE OF THE BOX

Since we can use any material object, let us assume that we send two particles of mass m kg with velocity u m/s from x_0. For simplicity of analysis, let us also assume the mass of the box is m kg. Let us do some classical physics analysis of this case when the objects do not hit the walls simultaneously (as we imagine seeing from S' frame).

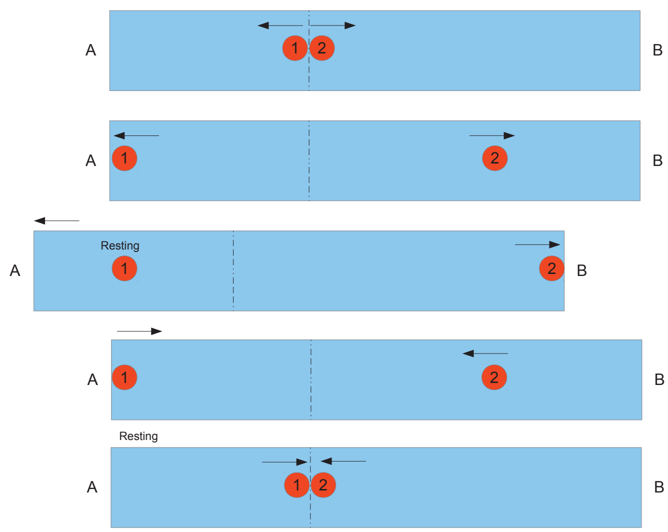

FIG. 2. If the objects 1 and 2 hit the walls of the box simultaneously, then net momentum added to the box is zero. But when the objects hit the ends of the box separately, then the box acquires some non-zero momentum during the process.

Suppose the smaller side be $L - d$ and larger side be $L + d$,

Step-1: Objects 1 and 2 are given equal velocities in opposite directions. Note the two objects are not at the center of the box.

Step-2: Object 1 hits the side A transfers momentum and comes to rest. The box picks up the momentum and moves left. Time when 1 hits A, $t_1 = \frac{L-d}{u}$.

Step-3: Objects 2 moving right hits the box moving left, after collision Object 2 moves left and box moves right. By this time Object 2 has travelled $L - d$ m. The box travels d m towards 2 and 2 travels d m towards B. Thus the excess time needed is $\frac{d}{u}$. Thus 2 hits B at $t_2 = t_1 + \frac{d}{u} = \frac{L}{u}$.

Step-4: The box moving right hits Object 1 at rest and transfers all the momentum. Object 2 continues moving left. Side A is now d m away from 1. Thus it will take another $\frac{d}{u}$ seconds for the collision, $t_3 = t_2 + \frac{d}{u} = \frac{L+d}{u}$.

Step-5: Objects 1 and 2 collide and repeat Step-1. The 2 objects in Step-4 are separated by $2(L - d)$ m and it takes another $\frac{L-d}{u}$ s for the objects to collide. That is total time $= t_3 + \frac{L-d}{u} = \frac{2L}{u}$.

Thus from S' frame the observer must also see these momentum transfers with the box. That is S' observer must see the box also move back and forth due to collisions with the objects. This is an altered reality process not seen in rest frame. Hence this is an inconsistent method.

<u>How will S' see?</u> Eqns(9,10) shows how objects 1 and 2 appear to S' observer. Let us see how do the sides A and B appear to S'.

For S frame $x_A = x_0 - L$ and $x_B = x_0 + L$. Thus from S',

$$x'_A = \frac{x_0 - L - vt}{\sqrt{1-\frac{v^2}{c^2}}}, t'_A = \frac{t - \frac{v(x_0-L)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x'_B = \frac{x_0 + L - vt}{\sqrt{1-\frac{v^2}{c^2}}}, t'_B = \frac{t - \frac{v(x_0+L)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{15}$$

The trajectories are,

$$x'_A = (x_0 - L)\sqrt{1-\frac{v^2}{c^2}} - vt'_A$$

$$x'_B = (x_0 + L)\sqrt{1-\frac{v^2}{c^2}} - vt'_B \tag{16}$$

At first, Object 2 and Side B collide. Let us assume they collide at x', t' as seen from S'. This implies the x'_2 trajectory from Eqn(10) and x'_B trajectory from Eqn(16) meet at x', t'.

$$x'_2 = \frac{x_0\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{vu}{c^2}} + \frac{(u-v)t'_2}{1-\frac{vu}{c^2}} = x'$$

$$x'_B = (x_0 + L)\sqrt{1-\frac{v^2}{c^2}} - vt'_B = x'$$

$$And, t'_2 = t'_B = t'$$

Therefore we get the meeting point of Object 2 and Side B, x', t' as,

$$t' = \frac{\frac{L}{u}\left(1-\frac{vu}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\frac{vx_0}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x' = \frac{x_0 + L - \frac{vL}{u}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{17}$$

Which corresponds to $x = x_0 + L$ and $t = \frac{L}{u}$ in frame S. This is what is observed in Frame S as the impact between object 2 and side B.

At this point we expect the box(of mass m) moving with $-v$ m/s to exchange momentum with object 2(of mass m) moving with velocity $\frac{u-v}{1-\frac{vu}{c^2}}$ wrt S' because of elastic collision. That is object 2 should start moving with $-v$ m/s and the box should start moving right with $\frac{u-v}{1-\frac{vu}{c^2}}$ m/s after collision. But that is not what we see from S'. Breaking the elastic collision process, the box keeps nochalantly moving with its original $-v$ and object 2 reverses its direction of motion and acquires a different magnitude of velocity if we simply follow the Lorentz tranformation of variables from x' frame. If we forget what is happening in S frame and focus on conserving the momentum in S' frame then we get a different time gap between the events.

If momentum of the objects and the box is conserved in S' frame then the time gap is $\frac{Lv}{c^2}\frac{\left(1-\frac{vu}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$. This not only depends on L and v but also depends on the velocity of the objects and is incompatible with the expectations of Lorentz transformation.

Another way of looking at it is by hitting a box of mass m externally with objects 1 and 2 simultaneously in rest frame and watch the similar breakdown of momentum conservation from an S' frame.

IV. COLLISION TEST OF RELATIVITY

$$\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = p = momentum$$
$$\frac{m_1 c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 c^2}{\sqrt{1-\frac{v_2^2}{c^2}}} = E = energy \tag{18}$$

Consider the energy equation,

$$\frac{m_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{E}{c^2}$$

Mutiply the above Eqn with v_2 on LHS and RHS and then use the momentum Eqn with it,

$$\frac{m_1 v_2}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{E v_2}{c^2}$$
$$\frac{m_1 v_2}{\sqrt{1-\frac{v_1^2}{c^2}}} + p - \frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{E v_2}{c^2} \tag{19}$$

Thus we can find v_2 as,

$$p - \frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \left[\frac{E}{c^2} - \frac{m_1}{\sqrt{1-\frac{v_1^2}{c^2}}}\right] v_2$$
$$Let, y = \sqrt{1-\frac{v_1^2}{c^2}}$$
$$So, v_2 = \frac{p - \frac{m_1 v_1}{y}}{\frac{E}{c^2} - \frac{m_1}{y}} = \frac{(py - m_1 v_1)c^2}{(Ey - m_1 c^2)} \tag{20}$$

Therefore,

$$\sqrt{1-\frac{v_2^2}{c^2}} = \frac{\sqrt{(Ey - m_1 c^2)^2 - (py - m_1 v_1)^2 c^2}}{(Ey - m_1 c^2)}$$

Use these v_2 expressions back in momentum eqn

$$\frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 v_2 (Ey - m_1 c^2)}{\sqrt{(Ey - m_1 c^2)^2 - (py - m_1 v_1)^2 c^2}} = p$$
$$\frac{m_2 (py - m_1 v_1) c^2}{\sqrt{(Ey - m_1 c^2)^2 - (py - m_1 v_1)^2 c^2}} = p - \frac{m_1 v_1}{y}$$
$$\frac{m_2 c^2}{\sqrt{(Ey - m_1 c^2)^2 - (py - m_1 v_1)^2 c^2}} = \frac{1}{y} \tag{21}$$

We have completely eliminated v_2 from our original set. Squaring the LHS and RHS in the above eqn we get,

$$(Ey - m_1 c^2)^2 - (py - m_1 v_1)^2 c^2 = m_2^2 c^4 y^2$$

Note, since $y = \sqrt{1-\frac{v_1^2}{c^2}}$ we get $v_1 = \pm c\sqrt{1-y^2}$

$$(Ey - m_1 c^2)^2 - (py \mp m_1 c\sqrt{1-y^2})^2 c^2 = m_2^2 c^4 y^2$$
$$(Ey - m_1 c^2)^2 - m_2^2 c^4 y^2 = (py \mp m_1 c\sqrt{1-y^2})^2 c^2$$

The eqn reduces to,

$$E^2 y - 2E m_1 c^2 - m_2^2 c^4 y =$$
$$p^2 y c^2 - m_1^2 c^4 y \mp 2p m_1 c^3 \sqrt{1-y^2}$$

Or in other words,

$$[E^2 - m_2^2 c^4 - p^2 c^2 + m_1^2 c^4] y - 2E m_1 c^2 =$$
$$\mp 2p m_1 c^3 \sqrt{1-y^2} \tag{22}$$

Note at this point if $p = 0$ we get,

$$[E^2 - m_2^2 c^4 + m_1^2 c^4] y - 2E m_1 c^2 = 0$$
$$Or, y = \frac{2E m_1 c^2}{E^2 - m_2^2 c^4 + m_1^2 c^4} \tag{23}$$

which gives a unique solution for y and hence only 2 possible solutions for $v_1 = \pm c\sqrt{1-y^2}$.

But in general $p \neq 0$. Let $D_0 = E^2 - m_2^2 c^4 - p^2 c^2 + m_1^2 c^4$ so,

$$D_0 y - 2E m_1 c^2 = \mp 2p m_1 c^3 \sqrt{1-y^2} \tag{24}$$

Note if Eqn(24) is real then $|y| \leq 1$

Squaring both sides and rearranging,

$$[D_0^2 + 4p^2m_1^2c^6]y^2 - 4Em_1c^2D_0y + 4E^2m_1^2c^4 - 4p^2m_1^2c^6 = 0 \quad (25)$$

Equivalent to,...$\alpha y^2 + \beta y + \gamma = 0$

Let us evaluate $\beta^2 - 4\alpha\gamma$ term,

$$\beta^2 - 4\alpha\gamma =$$
$$(-4Em_1c^2D_0)^2 - 4(D_0^2 + 4p^2m_1^2c^6)(4E^2m_1^2c^4 - 4p^2m_1^2c^6)$$
$$16m_1^2c^4[(ED_0)^2 - (D_0^2 + 4p^2m_1^2c^6)(E^2 - p^2c^2)]$$
$$16p^2m_1^2c^6[D_0^2 - 4m_1^2c^4(E^2 - p^2c^2)] \quad (26)$$

$$D_0 = (E^2 - p^2c^2) + c^4(m_1^2 - m_2^2)$$
$$16p^2m_1^2c^6[D_0^2 - 4m_1^2c^4D_0 + 4m_1^4c^8 - 4m_1^2c^8m_2^2]$$
$$\beta^2 - 4\alpha\gamma = 16p^2m_1^2c^6[(D_0 - 2m_1^2c^4)^2 - 4m_1^2c^8m_2^2] \quad (27)$$

Note that,

$$E^2 - p^2c^2 = m_1^2c^4 + m_2^2c^4 + \frac{2m_1m_2c^2(c^2 - v_1v_2)}{\sqrt{\left(1 - \frac{v_1^2}{c^2}\right)\left(1 - \frac{v_2^2}{c^2}\right)}} \quad (28)$$

$$D_0 - 2m_1^2c^4 = (E^2 - p^2c^2) - c^4(m_1^2 + m_2^2) = \frac{2m_1m_2c^2(c^2 - v_1v_2)}{\sqrt{\left(1 - \frac{v_1^2}{c^2}\right)\left(1 - \frac{v_2^2}{c^2}\right)}} \quad (29)$$

$$\beta^2 - 4\alpha\gamma = 16p^2m_1^2c^6[(D_0 - 2m_1^2c^4)^2 - (2m_1m_2c^4)^2]$$
$$= 16p^2m_1^2c^6(2m_1m_2c^4)^2\left[\frac{(c^2 - v_1v_2)^2}{(c^2 - v_1^2)(c^2 - v_2^2)} - 1\right]$$
$$= 64p^2m_1^4m_2^2c^{14}\frac{c^2(v_1 - v_2)^2}{(c^2 - v_1^2)(c^2 - v_2^2)} \quad (30)$$

From Eqn(30) we get $\beta^2 - 4\alpha\gamma = 0$ when $v_1 = v_2$.

Thus the condition $\beta^2 = 4\alpha\gamma$ is the same as the condition $v_1 = v_2$ i.e. relative velocity is zero which means there will be no collision. That is, when $v_1 = v_2$, we get a single unique solution for y.

$$y = \frac{2Em_1c^2D_0}{D_0^2 + 4p^2m_1^2c^6} \quad (31)$$

In all other cases we get a some non-zero positive value for it, i.e $\beta^2 - 4\alpha\gamma > 0$, hence we get 2 unique solutions for y. We can also note that $0 \le \sqrt{\beta^2 - 4\alpha\gamma} \le -\beta$ because $\beta = -4Em_1c^2D_0$ and $\alpha > 0, \gamma > 0$. We can further prove that both the distinct solutions of y lie in the range [0,1]. That is $0 \le y \le 1$. Let,

$$y_1 = \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}$$
$$y_2 = \frac{-\beta - \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \quad (32)$$

So, Eqn(32) implies $0 \le y_2 < y_1$. Suppose $y_1 \le 1$

$$y_1 = \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \le 1$$
$$\frac{\beta^2 - 4\alpha\gamma}{4\alpha^2} \le \left(1 + \frac{\beta}{2\alpha}\right)^2$$
$$1 + \frac{\beta}{\alpha} + \frac{\gamma}{\alpha} \ge 0 \quad (33)$$

That means if the 2 solutions y_1, y_2 are distinct i.e. $\beta^2 \ne 4\alpha\gamma$ such that $y_2 < y_1$, then $\alpha + \beta + \gamma \ge 0$ implies $y_2 < y_1 \le 1$.

$$\alpha = D_0^2 + 4p^2m_1^2c^6 > 0$$
$$\beta = -4Em_1c^2D_0 < 0$$
$$\gamma = 4m_1^2c^4[E^2 - p^2c^2] > 0$$
$$\alpha + \beta + \gamma = (D_0 - 2Em_1c^2)^2 \ge 0$$

Therefore when $\beta^2 - 4\alpha\gamma > 0$ the 2 distinct solutions for y satisfy $0 \le y \le 1$.

So we get a single unique solution for y when i) Net momentum of the two objects is zero, i.e. $p = 0$ or ii)when the velocities of the 2 objects are the same(i.e. $v_1 = v_2$). In all other cases there will be 2 solutions for y given by the quadratic equation(Eqn(25)). This implies that the momentum-energy set of equations have 4 possible unique solutions for v_1 and v_2. This means the collision process is ill-defined under relativistic transformations. Because it has more than 2 possible unique solutions for v_1 and v_2 we can not be sure what will be the outcome of a particular collision because we have 3 outcome options to chose from. This is a mathematical inconsistency with Relativistic transformations.

However in the special case when $m_1 = m_2 = m$ the 4 different possible solutions reduce to 2 unique possible solutions for v_1 and v_2. It is easy to conserve both relativistic momentum and energy just by exchanging the respective velocities.

Suppose $m_1 = m_2 = m$ as in the box momentum examples then, $D_0 = E^2 - p^2c^2$

$$[D_0^2 + 4p^2m^2c^6]y^2 - 4Emc^2D_0y + 4E^2m^2c^4 - 4p^2m^2c^6 = 0 \quad (34)$$

Equivalent to...$\alpha y^2 + \beta y + \gamma = 0$

Therefore,

$$y = 2mc^2\left[\frac{ED_0 \pm pc\sqrt{D_0^2 - 4m^2c^4D_0}}{D_0^2 + 4p^2m^2c^6}\right]$$

Suppose the 2 solutions be,

$$y_1 = 2mc^2\left[\frac{ED_0 + pc\sqrt{D_0^2 - 4m^2c^4D_0}}{D_0^2 + 4p^2m^2c^6}\right]$$
$$y_2 = 2mc^2\left[\frac{ED_0 - pc\sqrt{D_0^2 - 4m^2c^4D_0}}{D_0^2 + 4p^2m^2c^6}\right] \quad (35)$$

When solution is y_1, $v_1 = \pm c\sqrt{1-y_1^2}$ so from Eqn(20),

$$v_2 = \frac{(py - m_1 v_1)c^2}{(Ey - m_1 c^2)} = \frac{(py_1 \mp m_1 c\sqrt{1-y_1^2})c^2}{(Ey_1 - m_1 c^2)} \quad (36)$$

When solution is y_2, $v_1 = \pm c\sqrt{1-y_2^2}$ so from Eqn(20),

$$v_2 = \frac{(py_2 \mp m_1 c\sqrt{1-y_2^2})c^2}{(Ey_2 - m_1 c^2)} \quad (37)$$

Also from Eqn(35) we get,

$$y_1 + y_2 = \frac{4mc^2 E D_0}{D_0^2 + 4p^2 m^2 c^6}$$
$$y_1 . y_2 = \frac{4m^2 c^4 D_0}{D_0^2 + 4p^2 m^2 c^6} \quad (38)$$

Therefore,

$$\frac{1}{y_1} + \frac{1}{y_2} = \frac{y_1 + y_2}{y_1 y_2} = \frac{E}{mc^2}$$
$$\frac{1}{y_1} + \frac{1}{y_2} = \frac{1}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{1}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{E}{mc^2} \quad (39)$$

If $y_1 = \pm\sqrt{1-\frac{v_1^2}{c^2}}$, Eqn(39) implies $y_2 = \mp\sqrt{1-\frac{v_2^2}{c^2}}$. Thus we get $v_2 = -v_1\sqrt{\frac{1-y_2^2}{1-y_1^2}}$.

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Thus the 2 possible sets of solutions converge to 1 set of unique solution when $m_1 = m_2$. Otherwise theoretically the relativistic collision is illdefined.

V. CO-MOVING LIGHT SOURCE - MIRROR SYSTEM

The example of light clock in a moving train may not be due to Einstein but it is pedagogically used to build an intuition for the time dilation concept in special relativity. One of the major fallacies in that is that the assumption that we can **simply see** a ray of light, or a photon in flight and measure its speed from a distance!. In that example they do not specify any realistic mechanism as to how the two frames measure the speed of the same ray as seen from different frames. Nevertheless we will carry on with the same thought experiment example, just change the angles of the mirror a bit and show that the ray diagrams or photon trajectories do not work that way as imagined in that thought experiment.

Suppose there is a light source at the origin which lights up at time $t = 0$ when S and S' frames coincide and synchronize their clocks S'. It sends a ray up along Z-axis (Fig.3 Panel A). There is a slanted mirror which meets the Z-axis at an height x_0. In the rest frame, S sees that the ray goes straight up along Z-axis, gets reflected and the emerging ray is parallel to X-axis.

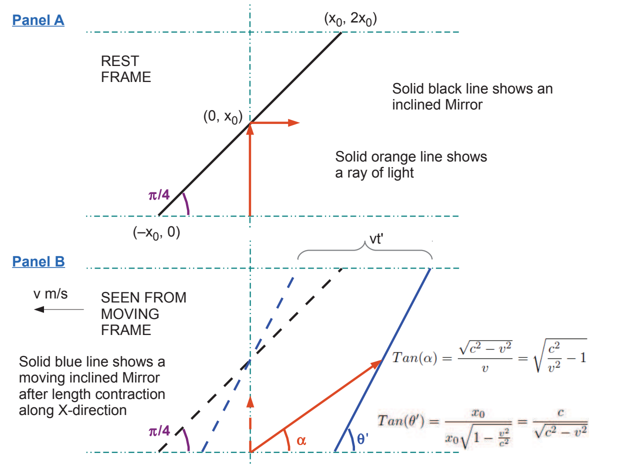

FIG. 3. Panel A) Reflection from a slanted mirror as seen in rest frame. Panel B) Reflection from a slanted mirror as seen from a relativistic inertial frame.

In much of the cases where the mirror is placed between 0 and 90 degrees wrt X-axis (Fig. 3), the light ray has to tunnel through from behind the mirror if it has to follow the Lorentz transformations blindly.

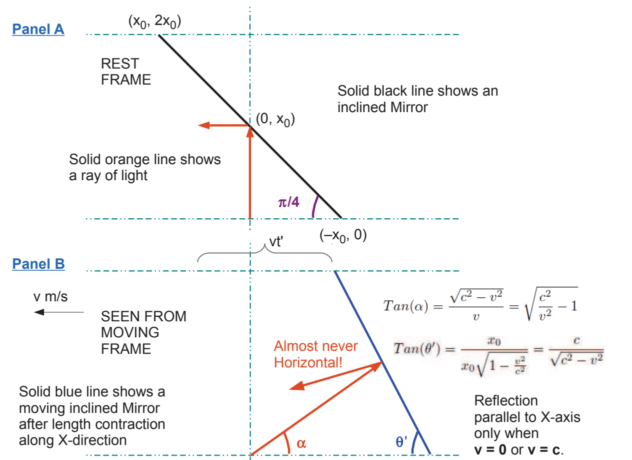

FIG. 4. Panel A) Reflection from a slanted mirror as seen in rest frame. Panel B) Reflection from a slanted mirror as seen from a relativistic inertial frame.

And in most cases where the mirror is placed between 90 and 180 degrees wrt X-axis (Fig. 4), the light ray must break the laws of reflection(angle of incidence = angle of reflection) inorder to satisfy Lorentz transformation. This seems to work well only in 1 configuration when the mirror placed at angle 0 wrt X-axis, or parallel to X-axis in all other cases it eiter breaks the laws of common sense or atleast the laws of reflection.

This essentially means that the reflection problem should be treated in its proper frame of occurence. i.e. the frame where the source and the mirrors where at rest.

VI. DEMATERIALISATION OF OBJECTS

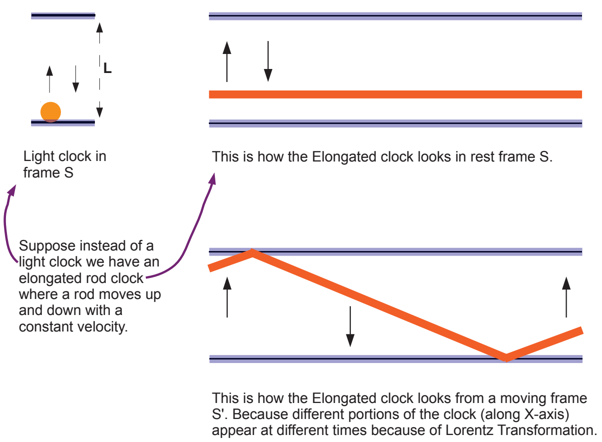

FIG. 5. An extended object executing an up and down motion at constant velocity appears to dematerialise as seen from a frame moving with some non-zero relative velocity. Because the moving frame looks at different parts of the objects from different time instances of their rest frame values, as one single instance of S' frame.

From Lorentz Transformation we know,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, z' = z$$
$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{40}$$

Suppose for simplicity we consider $z = ut$. In rest frame each part of the rod, located at any x, is moving with the same rate vertically, defined by $z = ut$. So at an instant t, they would all be located at the same height z.

$x = x_0 =$ some stationary point on the rod at distance x_0. Vertical motion $z = ut$. So, from Eqn(40) we get,

$$x' = \frac{x_0 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, z' = z = ut$$
$$t' = \frac{t - \frac{vx_0}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{41}$$

Inverting the time eqn from Eqn(41) we get the time at location x_0 in S frame,

$$t_{x_0} = t'\sqrt{1 - \frac{v^2}{c^2}} + \frac{vx_0}{c^2} \tag{42}$$

Let us assume that S and S' had synchronized their clocks at $x = x' = 0$ and $t = t' = 0$ therefore whenever it is time t on clock S it will be time $t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$ in S' clocks. Using this in Eqn(42) we get,

$$t_{x_0} = t'\sqrt{1 - \frac{v^2}{c^2}} + \frac{vx_0}{c^2} = t + \frac{vx_0}{c^2} \tag{43}$$

Thus at time instance t' in S' frame a point at x_0 in S(rest) frame appears to be from a time instant $t + \frac{vx_0}{c^2}$(Eqn(43)). That is if $x_0 > 0$ then S' will be looking at a future time instant of that place and if $x_0 < 0$ then S' will be looking at a past time instant. In fact by placing mirrors and cameras in S frame and controlling them, S' can gaze at its own indefinite past and infinite future and perhaps even manipulate it too.

A consequence of seeing multiple time instances at once is that the material object will dematerialise, its different parts seem to execute their own independent motion, lose all its rigidity, appear as a much elongated(Fig.5) saw tooth like wave. All this due to a few m/s of relative velocity of the observer.

VII. CONCLUSION

From the above 3 considerations, i)Einstein Simultaneity concept not conserving relativistic energy and momentum in all intertial frames(there is one preferred frame where they are both satisfied) during collisions, ii)Impossibility of realistic reflection in much of the light clock configurations and iii)Dematerialisation of objects in relative motion, we should declare special relativity as unfit to be a realistic theory of physics. And we should forget all the fantastical predictions of being able to gaze at and manipulate past and future alike, they are just fiction.