Brief analysis of Einstein Fallacies - General Relativity

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Abstract

According to *General Relativity*, i)A frame of reference experiencing free fall due to Gravity is equivalent to an inertial frame of reference in free space and ii)A frame at rest on the surface of Earth experiencing Gravity is equivalent to a frame in free space being accelerated with constant acceleration. Here we describe two simple experiments which prove the above statements wrong.

I. FREE FALL VS INERTIAL FRAMES

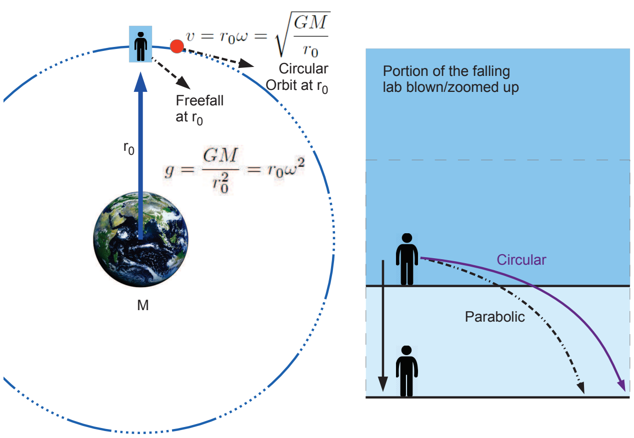

FIG. 1. Illustrating the difference between i) A free falling frame under gravity and ii) An inertial frame of reference.

We know from the work of Kepler, Newton and from all the space programs run by various countries/organizations that if an object is situated at a distance from the center of the Earth as shown in Fig.1, it can have various possible trajectories. That is, i) It can have a straight line/parabolic free fall trajectory, ii) It can have circular/elliptical orbits and iii) In certain cases it can even have an hyperbolic orbit depending on its energy and angular momentum values. When the total energy of the object is too low to hold it in any circular or elliptic orbit of its own, then we see the straight line or parabolic free fall trajectory.

Suppose we are situated in a lab which is under free fall and is located at some distance r_0 from Earth(Fig.1) and we do the experiments of dropping an object or firing it sideways or in various other angles within the lab. Suppose the time scale and length scales of the experiment is too small to detect the higher order tidal forcing components of gravity. So when we throw our projectiles at various angles and various trajectories. There is a definite chance that we produce some trajectories which have (i)parabolic free fall and (ii)circular orbit trajectories.

At the distance r_0 over a short time/length scale the lab frame can be thought of as falling with uniform acceleration(g) given by,

$$g = \frac{GM}{r_0^2} \tag{1}$$

Here G = Universal Gravitational Constant and M = Mass of Earth.

Thus as viewed from Earth the free falling lab frame can be described by,

$$\begin{aligned} x_0 &= 0 \\ z_0 &= r_0 - \frac{1}{2}gt^2 \end{aligned} \tag{2}$$

Now suppose in the lab frame we throw some low energy projectiles with vertical velocity u and horizontal velocity v. Then the parabolic trajectory viewed from Earth appears as,

$$\begin{aligned} x_1 &= vt \\ z_1 &= r_0 + ut - \frac{1}{2}gt^2 \end{aligned} \tag{3}$$

If in the lab frame we throw some high energy projectiles in an horizontal orientation then at an appropriate value of velocity(v_c) we get circular orbit trajectory. The angular frequency(ω) of the circular orbit is given by Kepler's 3^{rd} law or Newton's law of gravity. That is,

$$\begin{aligned} r_0^3\omega^2 &= GM \\ \omega &= \pm\sqrt{\frac{GM}{r_0^3}} \\ v_c &= r_0\omega = \pm\sqrt{\frac{GM}{r_0}} \end{aligned} \tag{4}$$

Comparing Eqn(4) witn Eqn(1) we can also note that,

$$g = \frac{GM}{r_0^2} = r_0\omega^2$$

So using ω from Eqn(4) we can write the circular orbit trajectory as,

$$\begin{aligned} x_2 &= r_0 Sin(\omega t) \\ z_2 &= r_0 Cos(\omega t) \end{aligned} \tag{5}$$

When the time scale is short $t \approx 0$, so $\omega t \approx 0$. Hence we can approximate Eqn(5) using the limits $Sin(\omega t) \approx \omega t$ and $Cos(\omega t) \approx 1$.

$$\begin{aligned} x_2 &\approx r_0\omega t = v_c t \\ z_2 &= r_0 \end{aligned} \tag{6}$$

Let us now consider the relative motion between lab trajectory(x_0, y_0) and parabolic trajectory(x_1, y_1)

$$
\begin{aligned}
x_1 - x_0 &= vt - 0 = vt \\
z_1 - z_0 &= \left(r_0 + ut - \frac{1}{2}gt^2\right) - \left(r_0 - \frac{1}{2}gt^2\right) = ut
\end{aligned}
\tag{7}
$$

Thus viewed from the free falling lab frame, the low energy parabolic trajectories appear as uniform velocity trajectories. Hence the lab frame may be thought of as an inertial frame under limited conditions. Let us now consider the relative motion between lab trajectory (x_0, y_0) and high energy circular trajectory (x_2, y_2)

$$
\begin{aligned}
x_2 - x_0 &\approx v_c t - 0 = v_c t \\
z_2 - z_0 &= r_0 - \left(r_0 - \frac{1}{2}gt^2\right) = \frac{1}{2}gt^2
\end{aligned}
\tag{8}
$$

Thus viewed from the free falling lab frame, the high energy circular trajectories appear to be rising with **uniform acceleration** along Z-axis. Hence the free falling lab frame is not equivalent to an inertial frame because some of the trajectories produced from within it can be used to measure the value of gravitational acceleration.

As long as we ignore the tidal forcing effects by considering only experiments of short length/time scales. Or by considering experiments with very low energy projectiles then we can argue that a free falling frame is equivalent to an inertial frame. But with careful and sensitive experiments with wide ranging parameters we can detect they are not really equivalent.

II. RESTING UNDER GRAVITY VS UNIFORM ACCELERATION

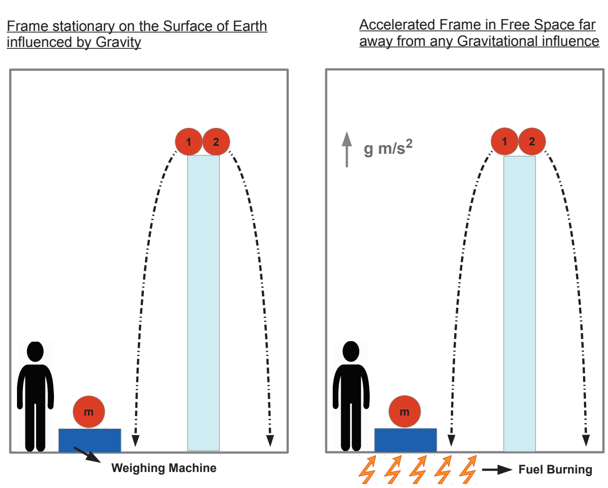

FIG. 2. Illustrating the difference between i) A frame resting under gravity on the surface of Earth and ii) An uniformly accelerating frame in free space.

If we throw away(or disengage) some mass from an accelerating frame like a ship/aircraft/etc, the acceleration of the frame/vehicle increases. Because the same fuel burn rate will be accelerating a lesser mass in the altered condition. This change in acceleration can be measured from within the frame by using a weighing machine and measuring the weight of a standard test mass kept in the frame for that purpose. If we throw away some mass from the frame then the weight of the test mass increases because acceleration increases.

However, in case of gravity throwing away some mass from the frame at rest on the surface of Earth does not alter the state of acceleration of the frame. Hence the two cases are not equivalent.

Consider two identical frames of reference as shown in Fig.2. One frame is at rest on the surface of Earth and the other frame is being accelerated uniformly in free space. In both the frames we have a weighing machine(shown as a blue box) on top of which we have placed our standard test mass (marked m). Within each frame is a tall tower oriented along the direction of acceleration. On top of the tower we have placed two masses (marked 1 and 2). Masses 1 and 2 are held together by a compressed spring(or magnets, or glue, etc), they can be released whenever needed. Once released the two masses experience free fall along the direction of acceleration as indicated in Fig.2.

Suppose we weigh the test mass m before we release the masses 1 and 2 from top of the tower. In both the frames we measure the same weight $W_i = mg = \frac{GMm}{r_0^2}$, ($W_i$ means initial weight). Then we release the masses 1 and 2 and again weigh the test mass m(W_f, final weight) while masses 1 and 2 are in free fall in their respective frames.

While the masses 1 and 2 are in free fall, they will be disengaged from rest of the lab-frame/vehicle to which they belong. That is they will not be pressing onto any surface of the frame and exerting some *action*. Hence there will be no *reaction* from the frame. Under this(disengaged) condition the two frames in Fig.2 behave quiet differently; (i)If the acceleration is due to Earth's mass and the sum of masses 1 and 2 is too negligible to exert any gravitational influence on the frame of reference then the weight measured before and after masses 1 and 2 are released will be same, $W_i = W_f = mg$. (ii)If the acceleration is due to fuel burn and the sum of masses 1 and 2 is too negligible to exert any gravitational influence on the frame of reference then the weight measured before and after masses 1 and 2 are released can be estimated as below.

Suppose the mass of the whole frame initially be M_f and it is being accelerated with $g\ ms^{-2}$. Therefore its(frames) velocity at any time instant would be $v = v_0 + gt$. If we are conducting the experiment from within the frame then assume $v_0 = 0$ because we are at rest wrt ourselves to begin with. Then the power delivered by the engine at any moment is given by $\frac{dE}{dt}$, rate of change of kinetic energy, that is

$$E = \frac{1}{2} M_f v^2$$
$$\frac{dE}{dt} = M_f v \frac{dv}{dt} = M_f g^2 t \tag{9}$$

Assume power delivery rate($\frac{dE}{dt}$) rate is maintained even when the masses 1 and 2 are diengaged. That is $M_f \to (M_f - m_1 - m_2)$. Let us call the new mass(when m_1 and m_2 are disengaged) M_f' and the new acceleration as g'. Then,

$$\frac{dE}{dt} = M_f g^2 t = M_f' g'^2 t$$
$$g' = \sqrt{\frac{M_f}{M_f'}} g \tag{10}$$

In the frame accelerated by fuel burn, weight of test mass changes from an initial value $W_i = mg$ to a new weight $W_f = mg\sqrt{\frac{M_f}{M_f'}}$ during the time of flight of masses 1 and 2. Larger the mass we disengage from our frame(i.e. lower M_f') greater the weight of test mass measured in the frame. Thus by doing experiment within the frame of reference we can differentiate between a frame at rest under gravity from an uniformly accelerating frame maintained by an engine burning some fuel. All we need to do is disengage some masses(1 and 2) give them some time of flight and measure the weight of test mass(m) beforem during and after the time of flight of masses 1 and 2.

From this experiment we can also conclude that the two(gravity vs fuel burn) mechanisms of producing acceleration must be quiet different. In the case of fuel burn the energy transfer is occuring at the *contact surfaces* between objects(masses 1, 2 and m). So once we diengage a mass from contact with the engine, the engine feels a different load and responds by producing an altered acceleration.

In case of gravity, each body(masses 1,2 or m) might be accelerated by some form of *bulk mechanism*(not mere surface contact) which accelerates each and every individual mass point(protons, neutrons, etc) within the body by similar extents. The mechanism of gravity permeates into the core of the objects probably pointing at a revival of *Fatio-Le Sage* type dynamic gravity but acting at the level of protons and neutrons.

III. CONCLUSION

We suggest experiments that can be done staying within ones own frame of reference to differentiate between,

i) A frame of reference experiencing free fall due to Gravity versus an inertial frame of reference in free space, and

ii) A frame at rest on the surface of Earth experiencing Gravity versus a frame in free space being accelerated with constant acceleration.

Einstein's *General Relativity* is based on not being able to differentiate between the frames as listed in i) and ii) hence *General Relativity* is wrong.