ACCELERATING UNIVERSE – THE SURPRISING DISCOVERY DUE TO AN UNDISCOVERED ASPECT

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ABSTRACT

The comparison of redshift-distance relationship for high and low-redshift supernovae has revealed the surprising transition of Universe's expansion from deceleration to acceleration. As compared to local supernovae, remote supernovae are further away than expected. The expansion rate obtained for local supernovae is higher with low redshifts as compared to the expansion rate obtained for remote supernovae with high redshifts. Since observed redshifts provide an estimate of recession velocities in order to determine the expansion rate (km s^{-1} Mpc^{-1}) of the local and the remote Universe, therefore, it is very disturbing to find that low recession velocities indicate acceleration (faster rate of expansion), whereas high recession velocities indicate deceleration (slower rate of expansion). In this paper I unravel an undiscovered aspect that perfectly mimics cosmic acceleration. I show in this paper that remote structures began expanding into the Universe before the expansion got initiated for the local structures, for this reason, remote structures are not only further away than expected, but they also happen to yield a slower rate of expansion as compared to the expansion rate obtained for the local structures. The analysis is based on the redshift-distance relationship plotted for 580 type Ia supernovae from the Supernova Cosmology Project, 7 additional high-redshift type Ia supernovae discovered through the Advanced Camera for Surveys on the Hubble Space Telescope from the Great Observatories Origins Deep Survey Treasury program, and 1 additional very high-redshift type Ia supernova discovered with Wide Field and Planetary Camera 2 on the Hubble Space Telescope. The results obtained by the High-Z Supernova Search Team through observations of type Ia supernovae have also been analysed.

Key words: cosmology: observations – dark energy.

1 INTRODUCTION

The research conducted by the High-Z Supernova Search Team (Riess et al. 1998) and by the Supernova Cosmology Project team (Perlmutter et al. 1999) by using type Ia supernovae as standard candles resulted into a very surprising discovery that made the teams win the 2011 Nobel Prize in Physics. By comparing the brightness of the very distant supernovae with the brightness of the nearby ones, distant supernovae were found to be 10% to 25% dimmer than the nearby supernovae; this indicated that the distances to those remote supernovae were larger than expected. A surprising feat was found being displayed by the Universe, a feat that was so extraordinary that the remarkable results obtained were not even expected. It was the remarkable discovery of Universe expanding at an accelerating rate. A research that was aimed at observing the expected deceleration of the Universe was welcomed by something completely unexpected.

"By establishing the distance to the supernovae and the speed at which they are moving away from us, scientists hoped to reveal our cosmic fate. They expected to find signs that the expansion of the Universe was slowing down, which would lead to equilibrium between fire and ice. What they found was the opposite – the expansion was accelerating" (an excerpt from "Written in the stars" by The Nobel Committee for Physics – The Royal Swedish Academy of Sciences).

A mysterious energy of unknown origin rightfully coined as dark energy is considered responsible for accelerating the Universe's expansion. According to Durrer (2011), "our single indication for the existence of dark energy comes from distance measurements and their relation to redshift. Supernovae, cosmic microwave background anisotropies and observations of baryon acoustic oscillations simply tell us that the observed distance to a given redshift is larger than the one expected from a locally measured Hubble parameter".

The expansion history of the Universe is depicted by the Hubble diagram as shown in Figure 1 (plotted by using the Supernova Cosmology Project data for 580 type Ia supernovae from Union 2 (Amanullah et al. 2010) and Union 2.1 (Suzuki et al. 2012), 7 additional high-redshift type Ia supernovae discovered through the ACS (Advanced Camera for Surveys) on the Hubble Space Telescope from the GOODS (Great Observatories Origins Deep Survey) Treasury program (joint work conducted by Giavalisco et al. 2004 and Riess et al. 2004), and 1 additional very high-redshift type Ia supernova discovered with WFPC2 (Wide Field and Planetary Camera 2) on the Hubble Space Telescope (Gilliland et al. 1999)).

The observed deviation from redshift-distance linearity in Figure 1 indicates an accelerating Universe since the distances to the remote supernovae are larger than expected with respect to the nearby ones. The value of slope (or the expansion rate measured in km s^{-1} Mpc^{-1}) is higher for the local structures and lower for the remote structures, suggesting that the Universe is accelerating now (locally) and was decelerating in the past (remotely). "A purely kinematic interpretation of the SN Ia sample provides evidence at the greater than 99% confidence level for a transition from deceleration to acceleration or, similarly, strong evidence for a cosmic jerk" (Riess et al. 2004).

By comparing the slope and thus the expansion rate of remote and local supernovae, cosmologists have come to an important, ground-breaking conclusion that the very local Universe is accelerating, whereas the remote Universe is decelerating. "Observations of Type Ia supernovae (SNe Ia) at redshift $z < 1$ provide startling and puzzling evidence that the expansion of the universe at the present time appears to be *accelerating*" (Riess et al. 2004). It is believed that the Universe was decelerating in the past due to the gravitational attraction of matter (Riess et al. 2001, Riess 2012). "A single SN Ia at $z \approx 1.7$, SN 1997ff, discovered with WFPC2 on the *Hubble Space Telescope* (*HST*) (Gilliland et al. 1999), provided a hint of past deceleration" (Riess et al. 2004).

Why does it appear that the Universe was expanding slowly in the past (decelerating remotely) even with high recession velocities and is expanding faster now (accelerating locally) even with low recession velocities? Why are the distances to the remote supernovae larger than expected, thereby making them appear 10% to 25% dimmer than the nearby local supernovae? Could the distant supernovae appear dim due to intervening dust? Or could it be that those distant supernovae have different properties as compared to the nearby supernovae? These possibilities have already been taken into account. Dust is not a factor. Similarly, the brightness of local and remote supernovae differing due to property mismatch brought about by evolution is also not a factor.

2 THE SURPRISING TRANSITION OF UNIVERSE'S EXPANSION FROM DECELERATION TO ACCELERATION: ANALYSING THE 588 TYPE Ia SUPERNOVAE

In an expanding Universe the observed redshifts provide an estimate of recession velocities. For instance, a redshift (z) of 0.1 corresponds to a recession velocity of 30,000 km s^{-1}. Once the redshifts and the distances are known (distances of type Ia supernovae estimated from their standard luminosities), the relation between redshift and distance is then used to determine the expansion rate (km s^{-1} Mpc^{-1}) of the Universe.

In Figure 1, the redshift of the most distant remote supernova at 41.6119 Gly is 1.7, this yields a slope of 1.2949×10^{-18} m s^{-1} m^{-1} (≈ 40 km s^{-1} Mpc^{-1}) – a lower value of slope (or a slower rate of expansion) even with high recession velocity – does this imply deceleration?

On the other hand, the redshift of a very nearby local supernova that happens to fall within the linear regime of the Hubble diagram in Figure 1 at 0.2148 Gly is 0.015166, this yields a slope of 2.2379×10^{-18} m s^{-1} m^{-1} (≈ 70 km s^{-1} Mpc^{-1}) – a higher value of slope (or a faster rate of expansion) even with low recession velocity – does this imply acceleration?

The paper was submitted to one of the most prestigious astronomy journals. **Reviewer's Report:** "The author has presented a novel interpretation of the redshift-distance relationship of observed supernovae".

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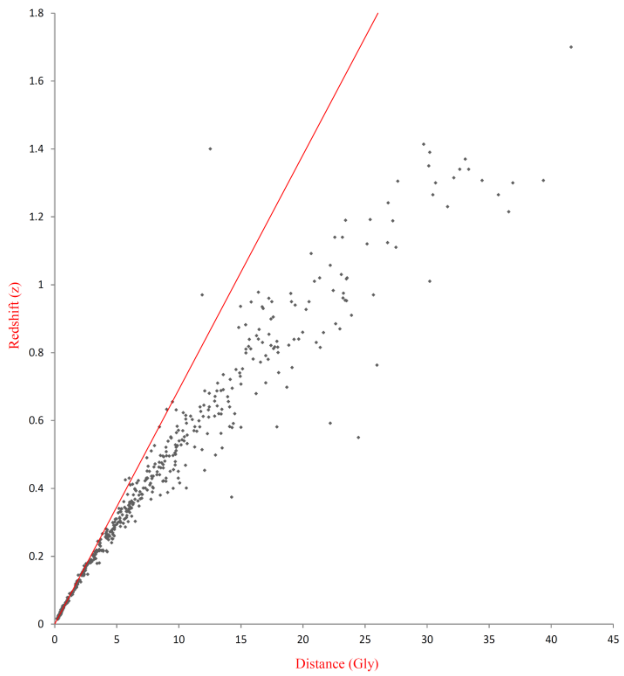

Figure 1. Redshift-distance relationship for 588 type Ia supernovae (580 type Ia supernovae plotted by using the data (Union 2 and Union 2.1) from the Supernova Cosmology Project, 7 additional high-redshift type Ia supernovae discovered through the ACS (Advanced Camera for Surveys) on the Hubble Space Telescope from the GOODS (Great Observatories Origins Deep Survey) Treasury program, and 1 additional very high-redshift type Ia supernova discovered with WFPC2 (Wide Field and Planetary Camera 2) on the Hubble Space Telescope). The red line indicates the linear redshift-distance relationship exhibited by the local structures. The deviation from linearity indicates an accelerating Universe since the distances to the remote supernovae are larger than expected with respect to the local supernovae. The slope is steeper for the local structures suggesting a faster rate of expansion (acceleration) and shallower for the remote structures, suggesting a slower rate of expansion (deceleration).

The redshift of the remote supernova is 112 times higher than the redshift of this very nearby local supernova. Since observed redshifts provide an estimate of recession velocities, therefore, confidently, those recession velocities corresponding to those observed high redshifts exhibited by the remote supernovae are undoubtedly much higher.

The unit of expansion rate (km s^{-1} Mpc^{-1}) makes it clear enough that there is a velocity and a distance component associated with the measurement of Universe's rate of expansion; it is this unit of measurement that helps us to compare the expansion rate of the remote and the local Universe in order to determine if the Universe is expanding at a slower rate, or at a faster rate. According to Riess et al. (2004), "It is valuable to consider the distance-redshift relation of SNe Ia as a purely *kinematic* record of the expansion history of the universe".

Such high redshift of the remote supernova does not indicate in any way a low recession velocity, or a slower rate of expansion, or deceleration due to the gravitational attraction of matter! One should therefore explain why does this remote supernova with such high recession velocity yield a lower value of slope (or a slower rate of expansion, thereby suggesting deceleration) as compared to the value of slope for the local supernova with low recession velocity and then be further away than expected?

3 ANALYSING THE SUPERNOVA SN 1995K

SN 1995K was the first and the most distant type Ia supernova discovered in 1995 by the High-Z Supernova Search Team. As compared to the nearby type Ia supernovae that happen to fall within the linear regime of the Hubble diagram as shown in Figure 2, SN 1995K happens to deviate from linearity as it is further away than expected – SN 1995K was already indicating that the Universe is accelerating. However, additional supernovae were required by the team to confirm if the Universe was accelerating or decelerating, and, it was only through further observations of additional type Ia supernovae at even larger distances that confirmed an accelerating Universe (Figure 3).

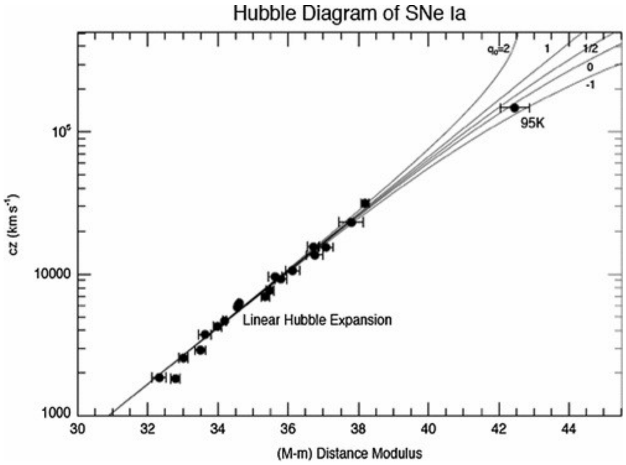

Figure 2. Velocity-distance relationship (Hubble Diagram of SNe Ia) showing SN 1995K at a redshift (z) of 0.479 from the proposal put forward by the High-Z Supernova Search Team. Credit: Schmidt B. P., Reviews of Modern Physics, vol. 84, 1151, page 1158, year 2012, reprinted with permission, Copyright (2012) American Physical Society. https://doi.org/10.1103/RevModPhys.84.1151

In Figure 2, the redshift of SN 1995K, the most distant supernova at 9.7211 Gly is 0.479, this yields a slope of 1.5617 x 10^{-18} m s^{-1} m^{-1} (≈ 50 km s^{-1} Mpc^{-1}). On the other hand, the redshift of a nearby supernova falling within the linear regime of the Hubble diagram in Figure 2 at 0.4604 Gly is 0.0333, this yields a slope of 2.2925 x 10^{-18} m s^{-1} m^{-1} (≈ 70 km s^{-1} Mpc^{-1}).

The comparison of expansion rate (km s^{-1} Mpc^{-1}) for these supernovae shows that SN 1995K is expanding at a slower rate (decelerating) as compared to the nearby supernova obeying the linear Hubble expansion.

Since observed redshifts provide an estimate of recession velocities, therefore, in Figure 2, Figure 3, and Figure 4, the observed redshifts have been interpreted as recession velocities by the corresponding research teams. The recession velocity of SN 1995K is 14.38 times higher than the recession velocity of the nearby supernova that falls within the linear regime of the Hubble diagram. Does this imply that SN 1995K even with high recession velocity is expanding at a slower rate (decelerating) as compared to a local supernova with low recession velocity?

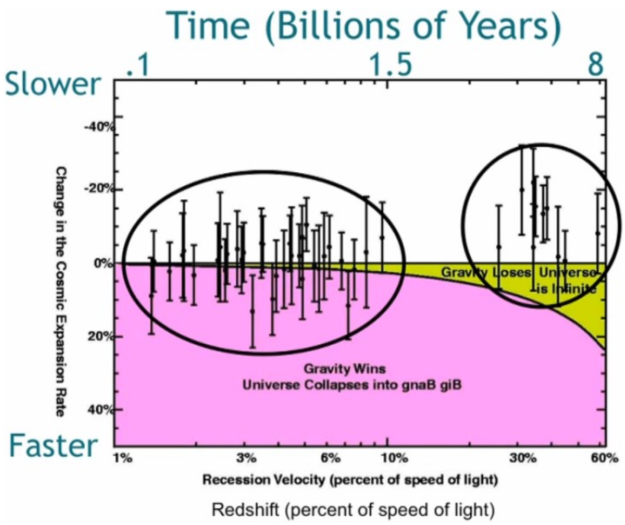

Figure 3. Observations of additional type Ia supernovae by the High-Z Supernova Search Team. The plot confirmed the result that the Universe is accelerating – remote supernovae are expanding at a slower rate (decelerating), whereas local supernovae are expanding at a faster rate (accelerating). Credit: High-Z Supernova Search Team.

SN 1995K, the first and the most distant type Ia supernova discovered by the High-Z Team already indicated that the Universe is accelerating, however, to confirm if the Universe was accelerating or decelerating, additional supernovae were required by the team.

Figure 3 depicts the result of additional type Ia supernovae observations at even larger distances carried out by the High-Z Team that confirmed Universe's acceleration. Distant supernovae were dimmer than expected (as they were further away than expected) and the expansion rate for them was found to be lower than the expansion rate for the nearby supernovae.

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Figure 3 clearly shows the transition of Universe's expansion from deceleration to acceleration – Universe was expanding slowly in the past (decelerating remotely) and is expanding faster now (accelerating locally).

However, if we look at the observed redshifts that provide an estimate of recession velocities in Figure 3, then there seems to be a conundrum, it is very disturbing to find that recession velocities ranging from 1% to 10% of speed of light indicate a faster rate of expansion (acceleration), whereas recession velocities ranging from 30% to 60% of speed of light indicate a slower rate of expansion (deceleration).

Why is it that an object with high recession velocity is not only further away than expected, but is also yielding a lower value of slope (or a slower rate of expansion, thereby suggesting deceleration) as compared to an object with low recession velocity?

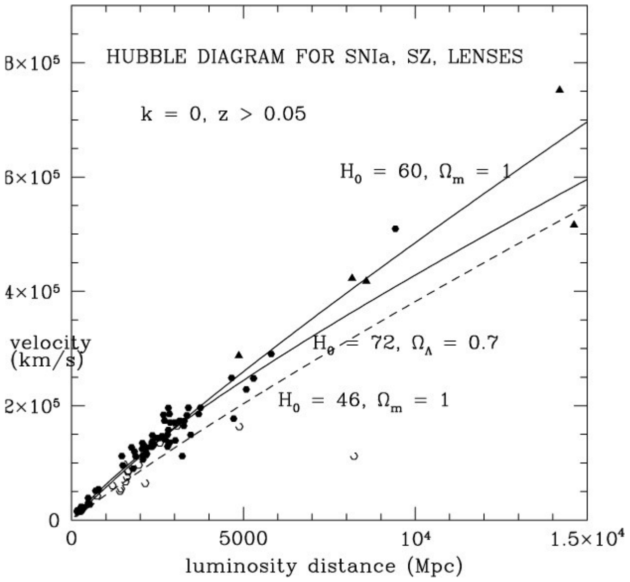

Figure 4. "Velocity versus luminosity-distance for type Ia supernovae (filled circles), S–Z clusters (open circles) and gravitational lens time-delay systems (filled triangles), with z > 0.05". Credit: Blanchard A., et al., A&A, vol. 412, 35, page 39, year 2003, reproduced with permission © ESO. https://doi.org/10.1051/0004-6361:20031425

Remote measurement yields an expansion rate of 46 km s^{-1} Mpc^{-1} which is significantly lower than the local measurement of 72 km s^{-1} Mpc^{-1} obtained from the Hubble Key Project determination (Freedman et al. 2001). The expansion rate measured for the local objects is significantly greater than the expansion rate measured for the remote objects.

"It has been noted by Zehavi et al. (1998) that the SNe Ia out to 7000 km s^{-1} exhibit an expansion rate that is 6% greater than that measured for the more distant objects" (Riess et al. 1998). One might say that local void is expanding faster than the remote expansion rate. According to Riess et al. (1998), "In principle, a local void would increase the expansion rate measured for our low-redshift sample relative to the true, global expansion rate. Mistaking this inflated rate for the global value would give the false impression of an increase in the low-redshift expansion rate relative to the high-redshift expansion rate".

However, according to Riess et al. (1998), "only a small fraction of our nearby sample is within this local void, reducing its effect on the determination of the low-redshift expansion rate". Furthermore, the reanalysis carried out (Riess et al. 1998) by discarding the seven SNe Ia within 7000 km s^{-1} (108 Mpc for 65 km s^{-1} Mpc^{-1}) ruled out the possibility of local void and confirmed cosmic acceleration.

Anyways, the recession velocities of local structures are not high enough as compared to the recession velocities of remote structures. In other words, the recession velocities of remote structures are not low to yield a lower rate of expansion (deceleration), similarly, the recession velocities of local structures are not high to yield a higher rate of expansion (acceleration).

The key point is, remote structures are not only further away than expected, but they also happen to yield a lower rate of expansion even with high recession velocities as compared to the higher rate of expansion for the local structures even with low recession velocities.

Here the observed redshift has clearly been interpreted as recession velocity of 7000 km s^{-1} by the researchers to determine the expansion rate of the local structures (65 km s^{-1} Mpc^{-1} (7000 km s^{-1}/108 Mpc)); the researchers then compare this local expansion rate with the expansion rate of more distant objects to find that the expansion rate for local objects is 6% greater than the expansion rate measured for the more distant objects.

Since quantities that have the same units can only be compared together – this clearly indicates that redshifts of the more distant objects have also been interpreted as recession velocities by the researchers to compare the expansion rate of local and remote structures. Therefore, why is it that an object with high recession velocity is not only further away than expected, but is also yielding a lower value of slope (or a slower rate of expansion, thereby suggesting deceleration) as compared to an object with low recession velocity?

4 AN UNDISCOVERED ASPECT

It remains undiscovered that an object that begins expanding before will not only be further away than expected, but it will also yield a lower value of slope (or a slower rate of expansion) even with high recession velocity as compared to an object with low recession velocity that begins expanding comparatively later.

Logically, an object that begins expanding before has an utmost probability of being further away than expected; the observational fact, that such object, which happens to be further away than expected, yields a lower value of slope (or a slower rate of expansion) even with high recession velocity as compared to an object with low recession velocity is the most compelling evidence in favour of this undiscovered aspect.

There is absolutely no other reason for an object with high recession velocity to yield a lower value of slope (or a slower rate of expansion) and then be further away than expected, unless it began expanding before. Plotting together the high-recession-velocity remote structures that began expanding before and the low-recession-velocity local structures that began expanding comparatively later into the Universe causes the Hubble diagram to deviate from linearity.

Comparing the slope and thus the expansion rate of high-recession-velocity remote structures that began expanding before into the Universe with the slope and thus the expansion rate of low-recession-velocity local structures that began expanding comparatively later into the Universe causes the high-recession-velocity remote structures to appear as if they are receding slower than expected as compared to the low-recession-velocity local structures.

It is important to note that even with high recession velocity, an object that begins expanding before will never yield a value of slope, or the expansion rate that is higher than the value of slope, or the expansion rate for an object with low recession velocity that begins expanding comparatively later. Comparing the slope and thus the expansion rate of such objects results into the apparent transition of Universe's expansion from deceleration to acceleration – an object with high recession velocity that began expanding before will be further away than expected and will appear to be decelerating, whereas an object with low recession velocity that began expanding comparatively later will appear to be accelerating.

It is this comparison that makes it appear that the Universe is accelerating now (locally) even with low recession velocities and was decelerating in the past (remotely) even with high recession velocities.

Requiring mysterious dark energy of unknown origin to explain this apparent transition of Universe's expansion from deceleration to acceleration has complicated things to an unimaginable extent.

5 A SIMPLE NUMERICAL PROOF USING HIGH AND LOW VELOCITY TEST PARTICLES

Let us consider two test particles – particle A and particle B. Particle A has an extreme recession velocity of 10^6 m s^{-1}, whereas particle B has a recession velocity of just 0.4 m s^{-1}.

Initially, particle A begins expanding into the Universe. After 4 seconds, particle B begins expanding and is observed for 1 second. By the time particle B is observed for 1 second, particle A has already been expanding for 5 seconds.

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Since particle A began expanding before, therefore, logically, as compared to particle B, particle A will be further away than expected.

The distance covered by particle A in 5 seconds with a recession velocity of 10^6 m s^{-1} is 5 x 10^6 m, whereas the distance covered by particle B in 1 second with a recession velocity of 0.4 m s^{-1} is 0.4 m.

The slope or the expansion rate for these particles is obtained by using the relation,

$$H = \frac{v}{D} \tag{1}$$

where H is the slope or the expansion rate (m s^{-1} m^{-1}), v is the recession velocity of the particles (m s^{-1}), and D is the distance covered by them (m). The inverse of slope or the expansion rate ($1/H$ or H^{-1}) gives back the time (t_H) in seconds.

The value of slope or the expansion rate for particle A with a whopping recession velocity of 10^6 m s^{-1} turns out to be 0.2 m s^{-1} m^{-1}. On the other hand, for particle B, the value of slope or the expansion rate with a mere recession velocity of just 0.4 m s^{-1} turns out to be 1 m s^{-1} m^{-1}.

The value of slope or the expansion rate for particle A even with an extreme recession velocity of 10^6 m s^{-1} is much lower (5 times lower) than the value of slope or the expansion rate for particle B even with a mere recession velocity of just 0.4 m s^{-1}.

Does this imply that particle A with high recession velocity of 10^6 m s^{-1} is decelerating, whereas particle B with low recession velocity of 0.4 m s^{-1} is accelerating?

10^6 m s^{-1} – recession velocity of particle A is 2.5 x 10^6 times higher than the recession velocity of particle B! Such high recession velocity of particle A does not indicate in any way a low recession velocity, or a slower rate of expansion, or deceleration due to the gravitational attraction of matter!

Then why is particle A with a whopping recession velocity of 10^6 m s^{-1} yielding a lower value of slope or a slower rate of expansion, thereby suggesting deceleration as compared to particle B with a minuscule recession velocity of just 0.4 m s^{-1}?

There is absolutely no other reason for an object with such high recession velocity to yield a lower value of slope (or a slower rate of expansion) and then be further away than expected, unless it began expanding before.

As already stated, even with high recession velocity (no matter how high), an object that begins expanding before will never yield a value of slope, or the expansion rate that is higher than the value of slope, or the expansion rate for an object with low recession velocity (no matter how low) that begins expanding comparatively later.

Therefore, we should never compare the slope and thus the expansion rate of such objects, doing so, without any doubt, will result into the apparent transition of Universe's expansion from deceleration to acceleration – an object with high recession velocity that began expanding before will be further away than expected and will appear to be decelerating, whereas an object with low recession velocity that began expanding comparatively later will appear to be accelerating. Requiring mysterious dark energy of unknown origin to explain this apparent transition would only complicate things to an unimaginable extent.

It is only the result of this comparison that particle A even with an extreme recession velocity of 10^6 m s^{-1} appears to be expanding at a slower rate (decelerating) as compared to particle B with a mere recession velocity of just 0.4 m s^{-1}.

Comparing the slope and thus the expansion rate of high-recession-velocity object that began expanding before into the Universe with the slope and thus the expansion rate of low-recession-velocity object that began expanding comparatively later into the Universe causes the high-recession-velocity object to appear as if it is receding slower than expected as compared to the low-recession-velocity object.

6 GRAPHICAL CONFIRMATION

To further confirm the credibility of this undiscovered aspect, it is necessary to plot some graphical relationships for such scenario where an object with high recession velocity (high redshift) begins expanding before, and an object with low recession velocity (low redshift) begins expanding comparatively later. Therefore, we will consider 11 test particles that have been assigned random velocities. These test particles expand consecutively (one particle after another) into the Universe. Based on calculations, we will plot some graphical relationships to verify if this undiscovered aspect perfectly mimics cosmic acceleration.

6.1 Velocity-distance relationship

Initially, particle A (3517.60 m s^{-1}) begins expanding into the Universe, 0.1 second later, particle B (2983.93 m s^{-1}) begins expanding, the expansion of particle B is followed by the expansion of particle C (2648.64 m s^{-1}) after another 0.1 second. Expansion of particles continues in the same way for particle D (2496.43 m s^{-1}), particle E (2223.52 m s^{-1}), particle F (1676.20 m s^{-1}), particle G (1219.96 m s^{-1}), particle H (917.97 m s^{-1}), and particle I (768.62 m s^{-1}). Particle J (530.48 m s^{-1}) and particle K (257.85 m s^{-1}) are the last particles to expand, and they expand at the same time into the Universe and are observed for 1 second. By the time these last two particles expand and are observed for 1 second, particle A has already been expanding for 1.9 second, and particle B for 1.8 second, this becomes their respective observation time.

The velocity-distance relationship for these 11 test particles has been plotted in Figure 5. The plot is remarkably similar to the redshift-distance relationship for 588 type Ia supernovae plotted in Figure 1. The deviation from linearity in Figure 5 clearly indicates that remote particles are not only further away than expected, but they also happen to yield a lower value of slope, or a slower rate of expansion (deceleration) even with high recession velocities as compared to the local particles that yield a higher value of slope, or a faster rate of expansion (acceleration) even with low recession velocities.

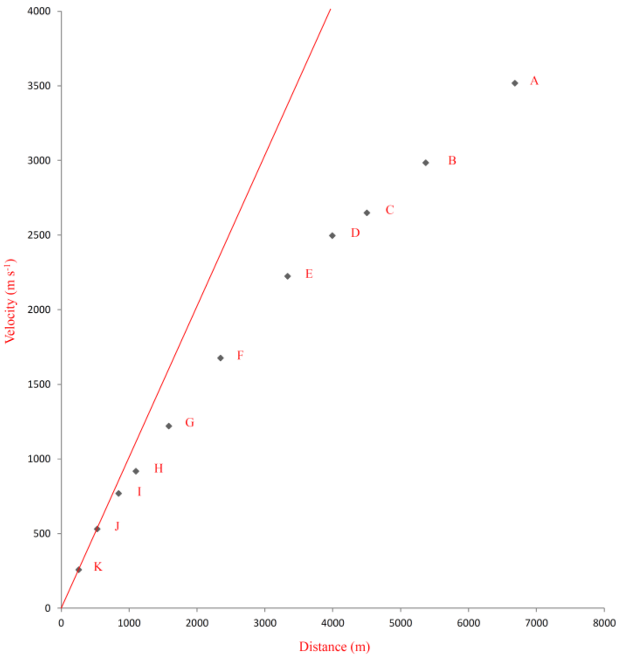

Figure 5. Velocity-distance relationship for 11 test particles (local and remote particles) expanding consecutively (one particle after another) into the Universe. Distances to remote particles are larger than expected with respect to local particles without acceleration. In other words, expansion initiated for remote particles before it did for local particles (see Figure 15 for comparison).

The value of slope for the most distant remote particle in Figure 5, that is, particle A, is 0.5263 m s^{-1} m^{-1} (a lower value of slope, or a slower rate of expansion even with high recession velocity of 3517.60 m s^{-1} – does this imply deceleration?), the inverse of this gives us the original observation/expansion time of 1.9 second.

For local particles, particle J and particle K, the value of slope (slope of the red line) turns out to be 1 m s^{-1} m^{-1} (a higher value of slope, or a faster rate of expansion even with low recession velocities of 530.48 m s^{-1} and 257.85 m s^{-1} respectively – does this imply acceleration?), the inverse of this gives the original observation/expansion time of 1 second.

The recession velocity of particle A is 6.63 times higher than the recession velocity of particle J, and 13.64 times higher than the recession velocity of particle K. Particle A still happens to yield a lower value of slope, thereby suggesting a slower rate of expansion or deceleration as compared to these two particles (not to mention again that particle A is further away than expected as compared to these two particles).

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Could there be any other reason why an object with high recession velocity would be yielding a lower value of slope, thereby suggesting a slower rate of expansion or deceleration and then be further away than expected as compared to an object with low recession velocity?

There is absolutely no other reason for an object with high recession velocity to yield a lower value of slope (or a slower rate of expansion, thereby suggesting deceleration) and then be further away than expected, unless it began expanding before.

High recession velocities of remote objects yielding a lower value of slope do not indicate their deceleration. Similarly, low recession velocities of local objects yielding a higher value of slope do not indicate their acceleration. Requiring mysterious dark energy of unknown origin to explain such transition would only complicate things to an unimaginable extent.

Since expansion began for remote particles before it did for local particles, therefore, remote particles are not only further away than expected, but they also yield a lower value of slope (or a slower rate of expansion) even with high recession velocities as compared to the higher value of slope (or a faster rate of expansion) for local particles even with low recession velocities. It therefore appears that local particles are expanding at a faster rate as compared to remote particles. One would therefore be forced into believing that local particles, as compared to remote particles, are accelerating.

6.2 Expansion rate versus time relationship

Although the observational fact that a remote object which happens to be further away than expected yields a slower rate of expansion even with high recession velocity as compared to a local object that yields a faster rate of expansion even with low recession velocity is the most compelling evidence to suggest that remote structures began expanding into the Universe before the expansion got initiated for local structures, however, to further confirm upon this aspect, we will plot expansion rate versus time relationship for such scenario where remote particles with high recession velocities (high redshifts) began expanding into the Universe before the expansion got initiated for local particles with low recession velocities (low redshifts).

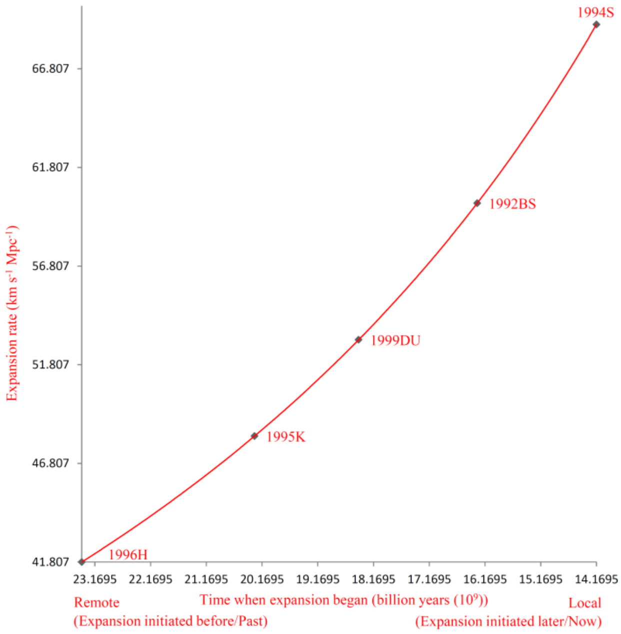

Figure 6. Plot of expansion rate versus time when expansion began (measured from past to present) for 5 type Ia supernovae (remote and local supernovae from Figure 1) shows an accelerating expansion (expansion rate increasing with time). Expansion rate for remote supernovae that are further away than expected (see Figure 1) is lower even with high recession velocities as compared to the expansion rate for nearby local supernovae even with low recession velocities.

Here in Figure 6, we see that expansion rate is increasing with time; expansion rate for remote supernovae is lower than the expansion rate for local supernovae – Universe is expanding slower in the past and is expanding faster now.

Now we need to plot the expansion rate versus time relationship when particles with high recession velocities (high redshifts) began expanding into the Universe before the expansion got initiated for particles with low recession velocities (low redshifts).

As discussed previously in Section 6.1, initially, the high-recession-velocity (high-redshift) particle A began expanding into the Universe, 0.1 second later, particle B began expanding, expansion of particle B was followed by the expansion of particle C after another 0.1 second. Expansion of particles continued in the same way for remaining particles. Low-recession-velocity (low-redshift) particles – particle J and particle K were the last particles to expand, and, they expanded at the same time into the Universe and were observed for 1 second. By the time these last two particles expanded and were observed for 1 second, particle A had already been expanding for 1.9 second, particle B for 1.8 second, particle C for 1.7 second, and so on.

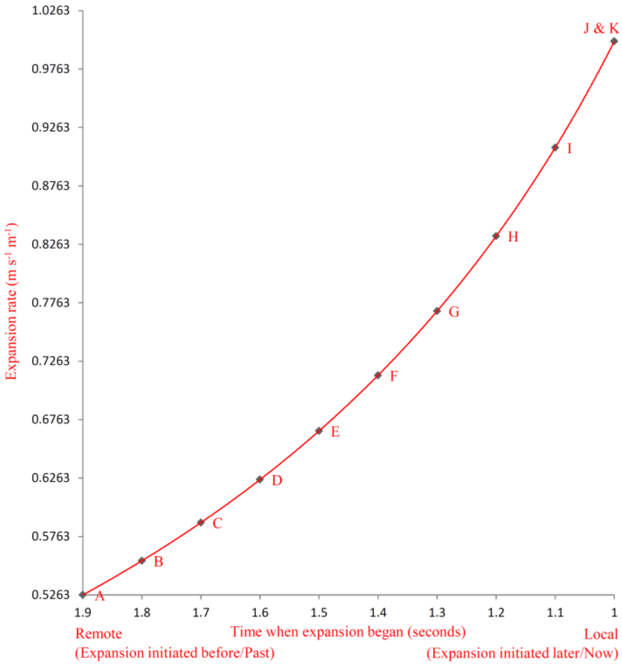

Figure 7. Plot of expansion rate versus time when expansion began (measured from past to present) for 11 test particles (remote and local particles from Figure 5) mimics an accelerating expansion (expansion rate appears to be increasing with time) when remote particles with high recession velocities began expanding into the Universe before the expansion got initiated for local particles with low recession velocities. Expansion rate for remote particles that are further away than expected (see Figure 5) is lower even with high recession velocities as compared to the expansion rate for nearby local particles even with low recession velocities, similar to what we observe for supernovae in Figure 6.

Here in Figure 6 and Figure 7, the time when expansion began has been obtained by using the relation,

$$t_H = \frac{1}{H}$$

See H from equation (1). The similarities incurred while plotting Figure 6 and Figure 7 (expansion rate increasing with time) are strong enough to indicate that remote structures with high recession velocities (high redshifts) began expanding into the Universe before the expansion got initiated for local structures with low recession velocities (low redshifts).

6.3 Expansion factor versus time relationship

If redshift also happens to indicate the size of the Universe now as compared to its size when the light was emitted, then the study conducted here based on redshifts of 10 test particles should also help us confirm that remote structures with high redshifts began expanding into the Universe before the expansion got initiated for local structures with low redshifts.

Velocity of particles (v) in m s^{-1} has been converted to redshift (z) by using the relation,

$$z = \frac{v}{c}$$

where c is the velocity of light in m s^{-1}. Similarly, the light-travel-time (t_c) in seconds corresponding to the distance (D) to particles in meters has been calculated by using the relation,

$$t_c = \frac{D}{c}$$

Just like a high-redshift remote supernova that we observe to be further away than expected (Figure 1 and Figure 2), we are

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observing the high-redshift remote particle A which is also further away than expected, as it began expanding before (Figure 5), at a distance (D) of 2.227813333 x 10^{-5} light seconds (6683.44 m) with a redshift (z) of 1.172533333 x 10^{-5} (3517.60 m s^{-1}).

The percentage of shift in the spectral lines towards the red-end of the electromagnetic spectrum for particle A is 1.172533333 x 10^{-3}% (this also corresponds to the percentage of expansion that has occurred while the light from particle A has been in transit before reaching us), in other words, the Universe is 1.172533333 x 10^{-3}% larger now than it was when the light was emitted.

To get a factor (expansion factor) which would help us calculate the time when the Universe was 100% smaller than now, we need to divide 100% by the percentage of shift in the spectral lines towards the red-end of the electromagnetic spectrum for a particular particle (this percentage of shift in the spectral lines also corresponds to the percentage of expansion that has occurred while the light from that particular particle has been in transit before reaching us), therefore, the expansion factor for the remote particle A is,

$$\frac{100}{1.172533333 \times 10^{-3}} = 85285.4219$$

(The expansion factor can also be obtained directly by taking an inverse of the redshift (z)). This factor suggests that we will have to reverse the expansion 85285.4219 times back into the past when the scale factor was zero and everything was at the same place – the Big Bang. Therefore, multiplying the expansion factor obtained for particle A (85285.4219) with its light-travel-time in seconds (2.227813333 x 10^{-5}) gives back the original expansion time for particle A (1.9 second), that is, the time in the past when particle A began expanding (expansion of particles has occurred at a steady rate; we have not subjected any particle to acceleration or deceleration).

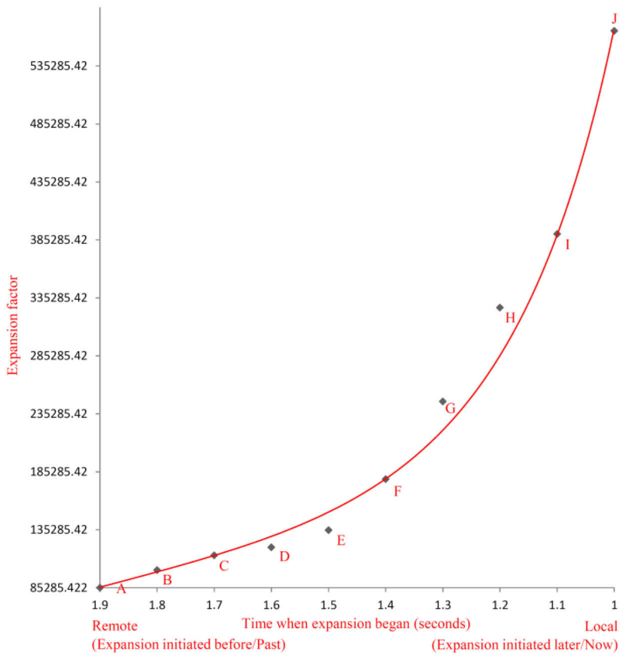

Figure 8. Plot of expansion factor versus time when expansion began (measured from past to present) for 10 test particles (remote and local particles from Figure 5) mimics an accelerating expansion (expansion factor increasing exponentially with time) when remote particles with high redshifts began expanding into the Universe before the expansion got initiated for local particles with low redshifts. Figure is consistent with Universe that is accelerating as expansion factor is increasing exponentially with time.

Here in Figure 8, the time when expansion began for test particles has been obtained by multiplying their expansion factor with their light-travel-time in seconds, and, this is consistent with time when expansion began for these test particles obtained in Figure 7 by using the relation,

$$t_H = \frac{1}{H}$$

See H from equation (1). It can be seen that high-redshift remote particles that began expanding into the Universe before yield a smaller expansion factor (expansion factor for remote particle A (85285.4219)) as compared to the expansion factor that increases exponentially with time for low-redshift local particles that began expanding comparatively later (expansion factor for local particle J (565525.5618)).

We will now follow the same method for type Ia supernovae (remote and local supernovae from Figure 1) to see if they also exhibit a similar expansion factor versus time relationship. Such similarity, if incurred, will further help us confirm that remote structures began expanding into the Universe before the expansion got initiated for local structures.

We have remote supernova 1996H in a distant galaxy with a redshift (z) of 0.62 at a distance (D) of 14.5043 Gly; this remote supernova is further away than expected as compared to local supernovae. The percentage of shift in the spectral lines towards the red-end of the electromagnetic spectrum for this supernova is 62% (this also corresponds to the percentage of expansion that has occurred while the light from this remote supernova has been in transit before reaching us), in other words, the Universe is 62% larger now than it was when the light was emitted.

To get the expansion factor which would help us calculate the time when the Universe was 100% smaller than now, we need to divide 100% by the percentage of shift in the spectral lines towards the red-end of the electromagnetic spectrum for a particular supernova (this percentage of shift in the spectral lines also corresponds to the percentage of expansion that has occurred while the light from that particular supernova has been in transit before reaching us), therefore, the expansion factor for the remote supernova 1996H is,

$$\frac{100}{62} = 1.612903226$$

This factor suggests that we will have to reverse the expansion 1.612903226 times back into the past when the scale factor was zero and everything was at the same place – the Big Bang. Therefore, multiplying the expansion factor obtained for supernova 1996H (1.612903226) with its light-travel-time in years (14.510739 x 10^9 years) gives back the original expansion time for supernova 1996H (23.4044 x 10^9 years), that is, the time in the past when supernova 1996H began expanding.

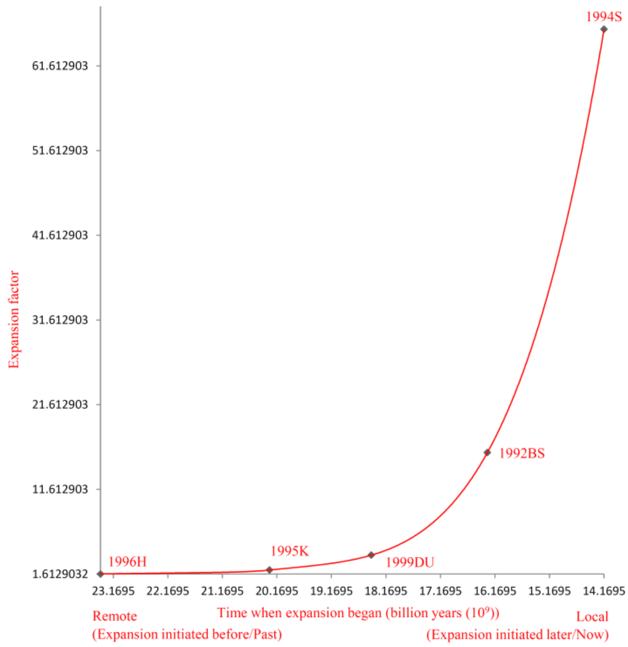

Figure 9. Plot of expansion factor versus time when expansion began (measured from past to present) for 5 type Ia supernovae (remote and local supernovae from Figure 1) shows an accelerating expansion (expansion factor increasing exponentially with time).

Here in Figure 9, the time when expansion began for supernovae has been obtained by multiplying their expansion factor with their light-travel-time in years, and, this is consistent with time when expansion began for these supernovae obtained in Figure 6 by using the relation,

$$t_H = \frac{1}{H}$$

See H from equation (1). It can be seen that high-redshift remote supernovae that began expanding into the Universe before yield a smaller expansion factor (expansion factor for

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remote supernova 1996H (1.612903226)) as compared to the expansion factor that increases exponentially with time for low-redshift local supernovae that began expanding comparatively later (expansion factor for local supernova 1994S (65.93696426)).

In Figure 8, we see that high-redshift remote particles that are further away than expected are yielding a smaller expansion factor as compared to the expansion factor that increases exponentially with time for low-redshift local particles, and, such exponential increase in expansion factor has occurred when remote particles with high redshifts began expanding into the Universe before the expansion got initiated for local particles with low redshifts, in other words, such exponential increase in expansion factor has occurred without subjecting any test particle to acceleration or deceleration.

In Figure 9, high-redshift remote supernovae that are further away than expected are also yielding a smaller expansion factor as compared to the expansion factor that increases exponentially with time for low-redshift local supernovae – similar to what we observe in Figure 8 using test particles.

Such similarity further confirms that remote structures with high redshifts began expanding into the Universe before the expansion got initiated for local structures with low redshifts, for this reason, remote supernovae are further away than expected as compared to the nearby local supernovae – acceleration cannot be the reason why remote supernovae are further away than expected.

6.4 Expansion factor versus light-travel-time relationship

In the previous section we obtained the expansion factor, the light-travel-time, and the time when expansion began. We plotted expansion factor versus time (time when expansion began) relationship and found expansion factor increasing exponentially with time (measured from past to present) when objects with high redshifts began expanding into the Universe before the expansion got initiated for objects with low redshifts. The time when expansion began was obtained by multiplying the expansion factor with the light-travel-time.

Here we will consider plotting expansion factor versus light-travel-time relationship for such scenario when remote particles with high redshifts began expanding into the Universe before the expansion got initiated for local particles with low redshifts, we will then plot expansion factor versus light-travel-time relationship for 5 type Ia supernovae to see if they also exhibit a similar relationship. A similar relationship, if incurred, will further help us confirm that remote structures with high redshifts began expanding into the Universe before the expansion got initiated for local structures with low redshifts.

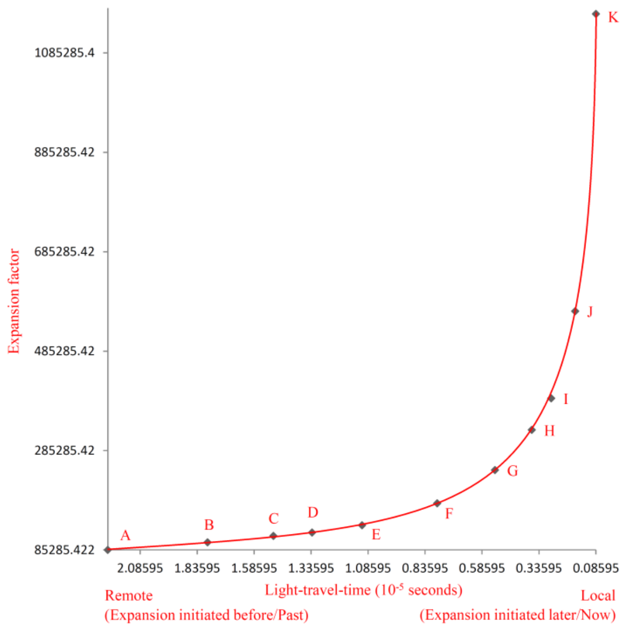

Figure 10. Plot of expansion factor versus light-travel-time (measured from past to present) for 11 test particles (remote and local particles from Figure 5) mimics an accelerating expansion (expansion factor increasing exponentially with time) when remote particles with high redshifts began expanding into the Universe before the expansion got initiated for local particles with low redshifts. Figure is consistent with Universe that is accelerating as expansion factor is increasing exponentially with time.

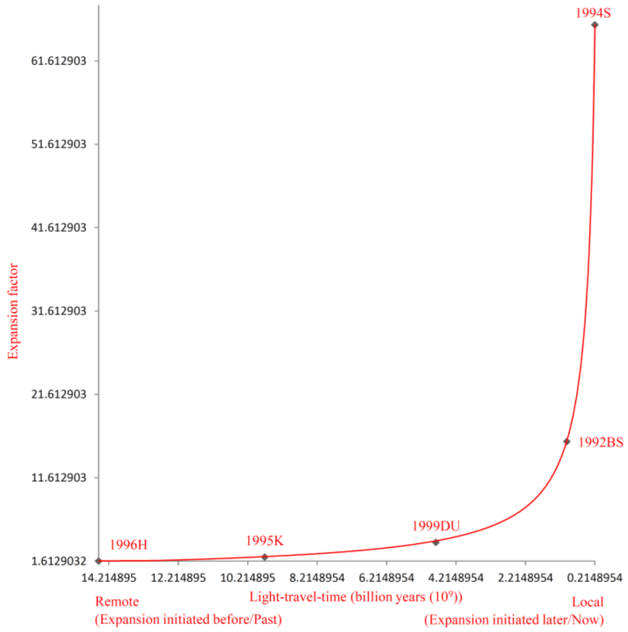

Figure 11. Plot of expansion factor versus light-travel-time (measured from past to present) for 5 type Ia supernovae (remote and local supernovae from Figure 1) shows an accelerating expansion (expansion factor increasing exponentially with time).

6.5 Scale factor versus light-travel-time relationship

If redshift also happens to indicate the size of the Universe when the light was emitted, then the study conducted here based on redshifts of 11 test particles should also help us confirm that remote structures with high redshifts began expanding into the Universe before the expansion got initiated for local structures with low redshifts.

We have remote supernova 1996H in a distant galaxy with a redshift (z) of 0.62 at a distance (D) of 14.5043 Gly; this remote supernova is further away than expected as compared to local supernovae.

The scale factor ($a(t)$) which denotes the size of the Universe when the light was emitted is obtained by using the relation,

$$a(t) = \frac{1}{1+z}$$

where z is the redshift. The light-travel-time (t_c) in years corresponding to the distance (D) to the supernovae in meters has been calculated by using the relation,

$$t_c = \frac{D}{c}$$

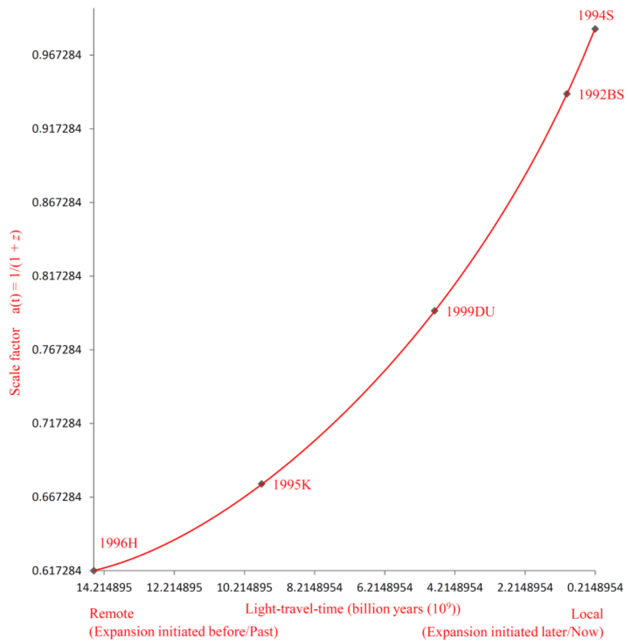

Figure 12. Plot of scale factor versus light-travel-time (measured from past to present) for 5 type Ia supernovae (remote and local supernovae from Figure 1) shows an accelerating expansion (scale factor increasing with time).

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Here in Figure 12, high-redshift remote supernovae that are further away than expected are yielding a smaller scale factor (scale factor for remote supernova 1996H (0.61728395)) as compared to the scale factor that increases with time for low-redshift local supernovae (scale factor for local supernova 1994S (0.985060571)).

Now we need to plot the scale factor versus light-travel-time relationship when particles with high redshifts began expanding into the Universe before the expansion got initiated for particles with low redshifts.

Velocity of particles (v) in m s^{-1} has been converted to redshift (z) by using the relation,

$$z = \frac{v}{c}$$

where c is the velocity of light in m s^{-1}. Similarly, the light-travel-time (t_c) in seconds corresponding to the distance (D) to particles in meters has been calculated by using the relation,

$$t_c = \frac{D}{c}$$

Just like a high-redshift remote supernova that we observe to be further away than expected (Figure 1 and Figure 2), we are observing the high-redshift remote particle A which is also further away than expected, as it began expanding before (Figure 5), at a distance (D) of 2.227813333 x 10^{-5} light seconds (6683.44 m) with a redshift (z) of 1.172533333 x 10^{-5} (3517.60 m s^{-1}).

The scale factor ($a(t)$) which denotes the size of the Universe when the light was emitted is obtained by using the relation,

$$a(t) = \frac{1}{1+z}$$

where z is the redshift. The light-travel-time (t_c) in seconds corresponding to the distance (D) to the particles in meters has been calculated by using the relation,

$$t_c = \frac{D}{c}$$

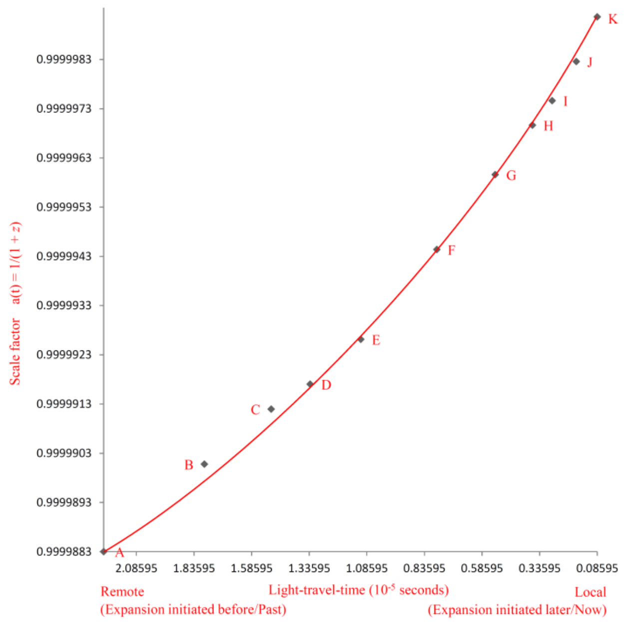

Figure 13. Plot of scale factor versus light-travel-time (measured from past to present) for 11 test particles (remote and local particles from Figure 5) mimics an accelerating expansion (scale factor increasing with time) when remote particles with high redshifts began expanding into the Universe before the expansion got initiated for local particles with low redshifts. Figure is consistent with Universe that is accelerating as scale factor is increasing with time.

Here in Figure 13, we see that high-redshift remote particles that are further away than expected are yielding a smaller scale factor (scale factor for remote particle A (0.999988274)) as compared to the scale factor that increases with time for low-redshift local particles (scale factor for local particle K (0.99999914)), and, such increase in scale factor has occurred when remote particles with high redshifts began expanding into the Universe before the expansion got initiated for local particles with low redshifts, in other words, such increase in scale factor has occurred without subjecting any test particle to acceleration or deceleration.

In Figure 12, high-redshift remote supernovae that are further away than expected are also yielding a smaller scale factor as compared to the scale factor that increases with time for low-redshift local supernovae – similar to what we observe here in Figure 13 using test particles.

Such similarity further confirms that remote structures began expanding into the Universe before the expansion got initiated for local structures, for this reason, remote supernovae are further away than expected as compared to the nearby local supernovae – again, acceleration cannot be the reason why remote supernovae are further away than expected.

6.6 Redshift-distance and velocity-distance relationships for 5 type Ia supernovae from Figure 1

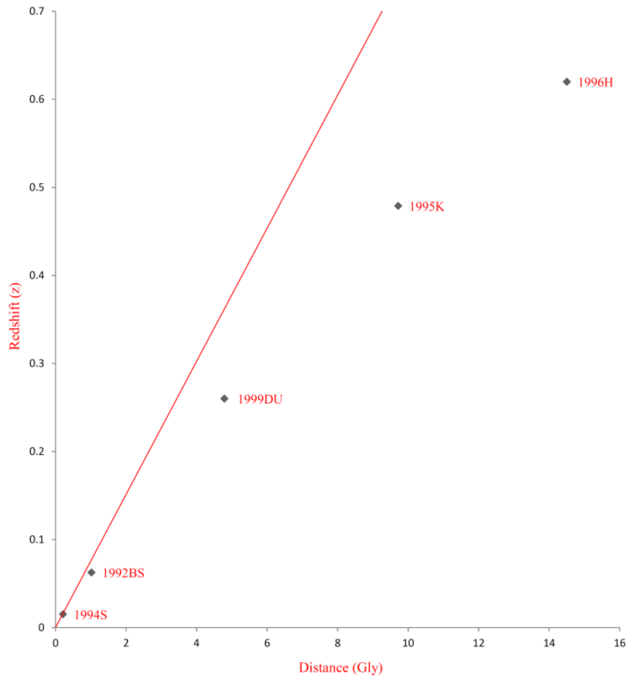

Figure 14. Redshift-distance relationship for 5 type Ia supernovae (local and remote supernovae from Figure 1). Remote supernovae are further away than expected as compared to the nearby local supernovae.

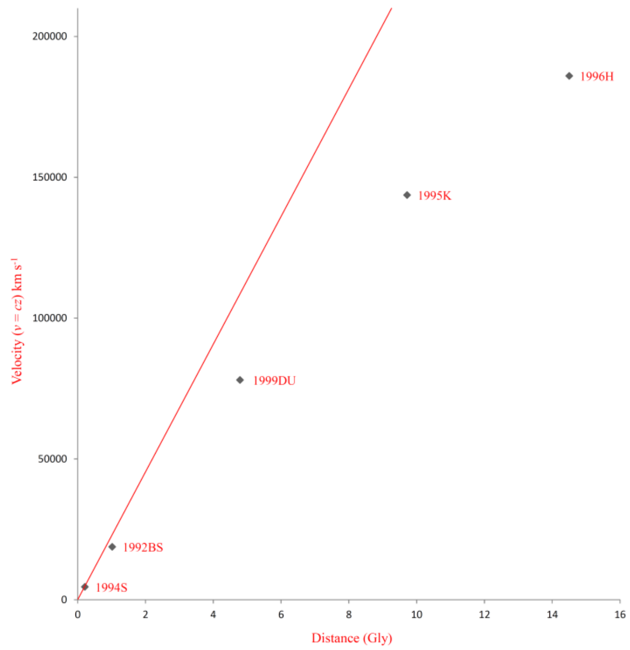

Figure 15. Velocity-distance relationship for 5 type Ia supernovae (local and remote supernovae from Figure 1). Remote supernovae are further away than expected as compared to the nearby local supernovae. These are the same supernovae plotted in Figure 6, Figure 9, Figure 11, and Figure 12. This plot is similar to the velocity-distance relationship obtained for test particles in Figure 5 when remote particles with high recession velocities began expanding into the Universe before the expansion got initiated for local particles with low recession velocities – expansion rate for remote supernovae that are further away than expected is lower even with high recession velocities as compared to the expansion rate for nearby local supernovae even with low recession velocities, similar to what we observe for test particles in Figure 5.

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7 CONCLUSIONS

(1) Direct evidence for an accelerating Universe came from the comparison of redshift-distance relationship for high and low redshift supernovae. As compared to local supernovae, remote supernovae are further away than expected. The expansion rate obtained for local supernovae is higher with low redshifts as compared to the expansion rate obtained for remote supernovae with high redshifts. Since observed redshifts provide an estimate of recession velocities in order to determine the expansion rate (km s^{-1} Mpc^{-1}) of the local and the remote Universe, therefore, it is very disturbing (a conundrum) to find that low recession velocities indicate acceleration (faster rate of expansion), whereas high recession velocities indicate deceleration (slower rate of expansion).

(2) The redshift of a remote supernova in Figure 1 (z = 1.7) is 112 times higher than the redshift of a local supernova (z = 0.015166), similarly, the redshift of the most distant supernova in Figure 2, SN 1995K (z = 0.479) is 14.38 times higher than the redshift of a local supernova (z = 0.0333). Since observed redshifts provide an estimate of recession velocities, therefore, confidently, those recession velocities corresponding to those observed high redshifts exhibited by the remote/distant supernovae are undoubtedly much higher.

(3) The unit of expansion rate (km s^{-1} Mpc^{-1}) makes it clear enough that there is a velocity and a distance component associated with the measurement of Universe's rate of expansion in order to determine if the Universe is expanding at a slower rate, or at a faster rate. According to Riess et al. (2004), "It is valuable to consider the distance-redshift relation of SNe Ia as a purely *kinematic* record of the expansion history of the universe". Also, according to Riess et al. (2004), "A purely kinematic interpretation of the SN Ia sample provides evidence at the greater than 99% confidence level for a transition from deceleration to acceleration".

(4) The evidence for accelerating Universe came from measuring how the expansion rate (km s^{-1} Mpc^{-1}) has changed over time. Since expansion rate for local Universe is found to be higher than the expansion rate for remote Universe, therefore, we say that the Universe is expanding faster now and had a slower expansion in the past. This apparent transition of the Universe's expansion from deceleration to acceleration (past to present) is explained by invoking dark energy – a mysterious and hypothetical energy of unknown origin. As pointed out by Durer (2011), "our single indication for the existence of dark energy comes from distance measurements and their relation to redshift. Supernovae, cosmic microwave background anisotropies and observations of baryon acoustic oscillations simply tell us that the observed distance to a given redshift is larger than the one expected from a locally measured Hubble parameter".

(5) Theoretical calculation for the value of dark energy believed to be the intrinsic energy associated with empty space or the vacuum energy according to the quantum field theory results into a huge 120 orders of magnitude (10^{120}) discrepancy. This suggests that dark energy is only introduced to account for the apparent transition of the Universe's expansion from deceleration to acceleration.

(6) "Expansion of gas molecules into the vacuum by the virtue of dark energy" has never been heard off; "such claim" if considered to be true would only suggest that gas molecules do not possess any energy.

(7) It is worth noting that an experiment conducted by Sabulsky et al. (2019) by using atom interferometry to detect dark energy acting on a single atom inside an ultra-high vacuum chamber showed no trace of any mysterious energy. Dark energy believed to be stronger in high vacuum environments should have easily been detected acting on a minuscule mass – a single atom.

(8) The surprising discovery of accelerating Universe is the result of an undiscovered aspect that has been unravelled in this paper. With 100% confidence level this undiscovered aspect perfectly mimics cosmic acceleration.

(9) It remains undiscovered that an object that begins expanding before will not only be further away than expected, but it will also yield a lower value of slope (or a slower rate of expansion) even with high recession velocity as compared to an object with low recession velocity that begins expanding comparatively later. Logically, an object that begins expanding before has an utmost probability of being further away than expected; the observational fact, that such object, which happens to be further away than expected, yields a lower value of slope (or a slower rate of expansion) even with high recession velocity as compared to an object with low recession velocity is the most compelling evidence in favour of this undiscovered aspect.

(10) There is absolutely no other reason for an object with high recession velocity to yield a lower value of slope (or a slower rate of expansion, thereby suggesting deceleration) and then be further away than expected as compared to an object with low recession velocity, unless it began expanding before.

(11) Plotting together the high-recession-velocity remote structures that began expanding before and the low-recession-velocity local structures that began expanding comparatively later into the Universe causes the Hubble diagram to deviate from linearity.

(12) Comparing the slope and thus the expansion rate of high-recession-velocity remote structures that began expanding before into the Universe with the slope and thus the expansion rate of low-recession-velocity local structures that began expanding comparatively later into the Universe causes the high-recession-velocity remote structures to appear as if they are receding slower than expected as compared to the low-recession-velocity local structures. For this reason, the velocity of a remote structure appears to be lower than the velocity predicted by the Hubble's law for a local structure.

(13) An object with high recession velocity that began expanding before will be further away than expected and will appear to be decelerating, whereas an object with low recession velocity that began expanding comparatively later will appear to be accelerating.

(14) Expansion rate versus time (time when expansion began) relationship for objects (supernovae and particles, Figure 6 and Figure 7 respectively) shows expansion rate increasing with time (past to present) – possible only when objects with high recession velocities began expanding into the Universe before the expansion got initiated for objects with low recession velocities.

(15) Expansion factor versus time (time when expansion began) relationship for objects (particles and supernovae, Figure 8 and Figure 9 respectively) shows expansion factor increasing exponentially with time (past to present) – possible only when objects with high redshifts began expanding into the Universe before the expansion got initiated for objects with low redshifts.

(16) Expansion factor versus light-travel-time relationship for objects (particles and supernovae, Figure 10 and Figure 11 respectively) shows expansion factor increasing exponentially with time (past to present) – possible only when objects with high redshifts began expanding into the Universe before the expansion got initiated for objects with low redshifts.

(17) Scale factor versus light-travel-time relationship for objects (supernovae and particles, Figure 12 and Figure 13 respectively) shows scale factor increasing with time (past to present) – possible only when objects with high redshifts began expanding into the Universe before the expansion got initiated for objects with low redshifts.

(18) Such similar plots obtained confirm the study conducted in this paper that remote structures began expanding into the Universe before the expansion got initiated for local structures, for this reason, remote supernovae are further away than expected as compared to the nearby local supernovae; acceleration cannot be the reason why remote supernovae are further away than expected.

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supernovae have been plotted in Figure 1). I am grateful to Gilliland et al. (1999) for the very high-redshift type Ia supernova discovered by them with WFPC2 (Wide Field and Planetary Camera 2) on the Hubble Space Telescope (1 high-redshift type Ia supernova has been plotted in Figure 1). I am grateful to American Physical Society for allowing me to illustrate the Hubble Diagram of SNe Ia in my manuscript (Figure 2; illustrated from Schmidt (2012)). I am also grateful to European Southern Observatory (ESO) for allowing me to illustrate the Velocity versus luminosity-distance for type Ia supernovae, S–Z clusters and gravitational lens time-delay systems in my manuscript (Figure 4; illustrated from Blanchard et al. (2003)).

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