Action–reaction symmetry breaking by induced internal forces

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Abstract. Newton’s third law of motion would be the cornerstone of physics were it not for certain experimental results that demonstrate a conflict with conservation of momentum under certain special circumstances. Such conceptual conflicts appear in systems in which the interacting parts of the system are mediated by a nonequilibrium environment that gives rise to nonreciprocal forces. The present theoretical study of a new mechanism of motion that utilizes induced internal forces in an isolated system (internally powered), addresses the probable cause behind the breaking of action–reaction symmetry and explores the potential implications of these findings for Einstein’s special relativity and Lorentz transformations.

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I. INTRODUCTION

In classical mechanics, any realization of the action–reaction principle presupposes two bodies exerting equal and opposite forces on each other. The notion of an internally powered isolated system moving in the absence of external forces, touches upon the extraordinary idea that object A is attracted to object B while object B is simultaneously repelled by object A. Nonreciprocal forces associated with the breaking of action–reaction symmetry is a subject being addressed by various disciplines in physics as in statistical mechanics [1–3], condensed matter [4–9], quantum mechanics [10–12], high energy physics [13], relativistic mechanics [14–16] and optics [17–21]. The present paper proposes and reveals the potentially missing part of Newton’s third law of motion that enables the realization of the above idea by simultaneously establishing the breaking of Newton’s action–reaction symmetry as a fundamental law of nature. The acceleration of an isolated system that utilizes induced internal forces leads inevitably to a new phenomenon, namely the reduction of the effective inertia, with profound implications for Einstein’s special relativity.

II. METHODS

The general form of Newton’s 2nd law that includes varying mass is developed by applying the derivative product rule as follows

\[ \sum F_{\text{ext}} + \sum F_{\text{int}} = \frac{d}{dt} \left( m u + \frac{dm}{dt} \right). \]  (1)

There are just two methods of motion that may apply to a body or a system, either using a net external force or through mass ejection (or accretion), thus

\[ \sum F_{\text{ext}} \neq 0 \text{ and } \sum F_{\text{int}} = 0 \text{ and } \frac{dm}{dt} = 0, \]  (2)

\[ \frac{dp}{dt} = \sum F_{\text{ext}} = m \frac{du}{dt}. \]  (3)

Furthermore, the mass ejection method yields

\[ \sum F_{\text{ext}} = 0 \text{ and } \sum F_{\text{int}} = 0, \]  (4)

\[ \frac{dm}{dt} < 0 \text{ and } u \to u_{\text{rel}} \text{ and } u_{\text{rel}} > 0, \]  (5)

\[ u \frac{dm}{dt} = -u_{\text{rel}} \frac{dm}{dt} \Rightarrow \frac{dp}{dt} = m \frac{du}{dt} = -u_{\text{rel}} \frac{dm}{dt}. \]  (6)

The rate of mass ejection corresponds to a mass transfer from the system’s interior to a point away from it (rocket) after a time interval, without affecting its center of mass. From a purely mathematical perspective, a second interpretation is also possible. An internal force causes a mass transfer between two points inside the system, resulting in a change of system’s effective inertia (reduction) after a time interval. According to Newton’s 3rd law, any action being exerted upon a part results in equal and opposite reaction force upon the rest of the system. Hence,

\[ \sum F_{\text{ext}} = 0 \text{ and } \sum F_{\text{int}} = \vec{F}_{\text{A}} + \vec{F}_{\text{R}} = 0, \]  (7)

\[ \frac{dp}{dt} = \sum F_{\text{int}} = m \frac{du}{dt} = -u_{\text{rel}} \frac{dm}{dt} = 0, \]  (8)

\[ \frac{dp}{dt} = 0 \Rightarrow u_{\text{rel}} = 0 \Rightarrow a = 0. \]  (9)

In addition to forces, the conservation of energy is given by

\[ U_s = \int_0^d F_A ds \]  (10)

\[ \int_0^d \sum F_{\text{int}} ds + \left( U_s + \int_0^d F_R ds \right) = 0, \]  (11)

where \( U_s \) is the energy stored within the system.

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FIG. 1: Proof of concept. Action–reaction forces in isolated systems. Upper: Induced internal forces (\(F_A\) and \(F_R\)). Redeployment of the center of mass (cm) and acceleration of the system. Lower: Collinear internal forces (\(F_A\) and \(F_R\)). No change in the center of mass and no acceleration of the system.

The conservation of linear momentum for the isolated system in FIG. 1, can be theoretically attributed either to a momentum exchange or a mass transfer mechanism. For collinear internal forces, implies

\[
\frac{d\vec{p}}{dt} = \sum \vec{F}_{\text{int}} = -(\vec{F}_A + \vec{F}_R) = 0 \quad \text{(collinear)},
\]

(12)

\[
m \cdot \vec{u}' - m \cdot \vec{u} + m_T \cdot \vec{u}'_{cm} - m_T \cdot \vec{u}_{cm} = 0,
\]

(13)

\[
\frac{m(u' - u)}{m} + m_T (u'_{cm} - u_{cm}) = 0,
\]

(15)

\[
\text{Momentum Exchange}
\]

\[
\frac{d\vec{p}}{dt} = m \frac{d\vec{u}}{dt} = -m_T \frac{d\vec{u}_{cm}}{dt} = 0,
\]

(16)

\[
\vec{u}' = \vec{u} \text{ and } \vec{u}'_{cm} = \vec{u}_{cm} \Rightarrow \vec{a} = 0.
\]

(17)

\[
\text{Mass Transfer}
\]

\[
\frac{dm_T}{dt} = \frac{dm}{dt},
\]

(18)

\[
\frac{d\vec{p}}{dt} = m \frac{d\vec{u}}{dt} = -\vec{u}_{rel} \frac{dm}{dt} = 0,
\]

(19)

\[
\vec{u}_{rel} = 0 \Rightarrow \vec{u}' = \vec{u} \text{ and } \vec{u}'_{cm} = \vec{u}_{cm} \Rightarrow \vec{a} = 0.
\]

(20)

In case it would be possible to curve the path of the internal reaction force, it could lead to its inversion and to a non-null relative velocity. This would have resulted in the redeployment of the center of mass and acceleration (conservation of momentum) of the system as a whole. In FIG. 1 the interacting parts are intrinsic to the system and cannot be separated from each other, therefore a potential system acceleration mechanism shouldn’t be attributed to momentum exchange. Starting from the conservation of angular momentum, the net external and internal torques in FIG. 1 - Upper, are

\[
\sum \tau_{\text{ext}} = 0 \text{ and } \vec{F}_A + \vec{F}_R = 0,
\]

(21)

\[
r_A \neq r_R \Rightarrow \vec{r}_A \neq 0 \text{ and } \vec{r}_R \neq 0,
\]

(22)

\[
\sum \tau_{\text{int}} + \vec{r}_A + \vec{r}_R = 0,
\]

(23)

\[
\sum \tau_{\text{int}} + (\vec{r}_A \times \vec{F}_A) + (\vec{r}_R \times \vec{F}_R) = 0,
\]

(24)

\[
\sum \tau_{\text{int}} + (\vec{r}_A \times \vec{F}_A) + (\vec{r}_R \times (-\vec{F}_A)) = 0,
\]

(25)

\[
\sum \tau_{\text{int}} + (\vec{r}_A - \vec{r}_R) \times \vec{F}_A = 0.
\]

(26)

An ideal translation mechanism (translation screw in FIG. 1) can maintain, amplify or reduce the magnitude of the input force by delivering the same amount of energy (no energy dissipation through friction) entering the system (energy conservation). At this point, developing a general expression for the net induced force requires the introduction of the dimensionless factor \(n_T\) (ideal mechanical advantage) along with a definition of the net induced torque. Hence, \(n_T = \frac{\omega \times (r_A - r_R)}{|u_T|} = \frac{(2\pi |r_A - r_R|)}{|r_T|}\),

(27)

\[\begin{align*}
\sum \tau_{T} & = n_T \sum \tau_{\text{int}}, \\
\sum \tau_{T} & = n_T ((r_A - r_R) \times \vec{F}_A) = 0.
\end{align*}\]

(29)

(30)

Dividing Eq.(30) by the position-vector magnitude \(|r_A - r_R|\) yields

\[
\frac{\sum \tau_{T}}{|r_A - r_R|} + \frac{n_T ((r_A - r_R) \times \vec{F}_A)}{|r_A - r_R|} = 0.
\]

(31)

Equation (33) shows when the \(\vec{F}_A\) is constant then, the angular velocity of the translation screw \(\omega\), the induced force \(\vec{F}_T\) and the translational velocity \(\vec{u}_T\) of mass \(m_T\) are also constant.

Eq.(31) can be also written as

\[
\sum \vec{F}_T = -\frac{n_T ((r_A \times \vec{F}_A) + (r_r \times \vec{F}_R))}{|r_A - r_R|},
\]

(34)

\[
\sum \vec{F}_T = -n_T (\vec{F}_{AT} + \vec{F}_{RT}) = \text{const.}
\]

(35)
Nevertheless, Eq. (31) and Eq. (33) address just the motion of mass \(m_T\). By expanding the ideal mechanical advantage to include varying angular and translational velocities, a new net force expression is derived that may apply for the motion of the system as a whole. Thus,

\[
\begin{align*}
\frac{d\omega}{dt} &= 0 : \quad \text{F}_A = \text{const.}, \quad \text{F}_R = \text{const.} \\
\neq 0 : \quad \text{F}_A \neq \text{const.}, \quad \text{F}_R \neq \text{const.} 
\end{align*}
\]

\[n_r = \frac{|d\omega \times (r_A - r_R)|}{|d\mathbf{u}_T|}, \quad (36)\]

\[
\sum \text{F}_{\text{ind}} = -n_r \left( (r_A \times \text{F}_A) + (r_R \times \text{F}_R) \right) \left( |r_A - r_R| \right) \quad (37)
\]

\[
\sum \text{F}_{\text{ind}} \propto d\omega, \quad (38)
\]

As shown in Fig. 1 - Upper, yields

\[
\begin{cases}
\omega = 0 \Rightarrow \mathbf{u}_T = \mathbf{u}_{\text{rel}} \neq 0, & \text{Eq.(39)} \\
\neq 0 \Rightarrow \mathbf{d} \mathbf{u}_T \neq 0, & \text{Eq.(40)}
\end{cases}
\]

Applying Eq.(38) to the mass transfer mechanism, yields

\[
\begin{align*}
\frac{d\omega}{dt} &\neq 0 \Rightarrow d\mathbf{u}_T = \mathbf{u}_{\text{rel}} \\
\frac{d\mathbf{p}}{dt} &= \sum \text{F}_{\text{ind}} = -n_r \left( \mathbf{F}_A + \mathbf{F}_R \right), \quad (41)
\end{align*}
\]

\[
\frac{d\mathbf{p}}{dt} = m \frac{d\mathbf{u}_T}{dt} = -n_r \mathbf{u}_{\text{rel}} \frac{dm}{dt} \neq 0 \Rightarrow a \neq 0, \quad (42)
\]

\[
\frac{d\mathbf{p}}{dt} - m \frac{d\mathbf{u}_T}{dt} = -n_r \mathbf{u}_{\text{rel}} \frac{dm}{dt} \neq 0 \Rightarrow a \neq 0, \quad (43)
\]

\[
\int_{d_1}^{d_2} \sum \text{F}_{\text{ind}} dt + U_s = 0, \quad (44)
\]

\[
U_s = I_{\text{rot}} \int_{\omega_1}^{\omega_2} \omega d\omega = \frac{I_{\text{rot}}}{2} \left( \omega_2^2 - \omega_1^2 \right) = W_{\text{rot}}, \quad (45)
\]

\[
U_s \text{ the energy stored in the system.}
\]

When the relative velocity \(\mathbf{u}_{\text{rel}}\) has constant magnitude, the effective inertia of the system is

\[
|\mathbf{u}_{\text{rel}}| = \text{const.} \Rightarrow \mathbf{u}_{\text{rel}} \neq 0 \Rightarrow \mathbf{u}_{\text{cm}} \neq \mathbf{u}_{\text{cm}}, \quad (46)
\]

\[
\frac{d\mathbf{p}}{dt} = -n_r \mathbf{u}_{\text{rel}} \frac{dm}{dt}, \quad (47)
\]

\[
\int_{m_1}^{m} dm = -\frac{1}{n_r} \mathbf{u}_{\text{rel}} \cdot \int_{0}^{p} dp, \quad (48)
\]

\[
\frac{m_1}{m} = m \left( 1 - \frac{\mathbf{p}}{n_r \cdot m \cdot \mathbf{c}} \right). \quad (49)
\]

Let us suppose that there is a theoretical quasiparticle (system) that exhibits the property whereby its effective inertia decreases as its velocity \(u\) increases. Setting \(\mathbf{u}_{\text{rel}}\) to be equal to the speed of light, Eq.(50) turns into the nonrelativistic inertia of the system:

\[
|\mathbf{u}_{\text{rel}}| = c \Rightarrow m_1 = m \left( 1 - \frac{\mathbf{p}}{n_r \cdot m \cdot c} \right). \quad (50)
\]

Alternatively, because of energy conservation, Eq.(51) becomes

\[
m_i c_2 - mc^2 = - \left( \frac{\mathbf{p}}{n_r} \right)^2 \cdot \frac{1}{2m}, \quad (52)
\]

\[
m_i c^2 - mc^2 = - \frac{1}{n_r^2} \frac{\mathbf{p}^2}{2} \Rightarrow U_s = \frac{U_k}{n_r^2}, \quad (53)
\]

\[
m_i = m \left( 1 - \frac{\mathbf{u}^2}{n_r^2 \cdot 2c^2} \right). \quad (54)
\]

The classical limit of the relativistic inertia for charged particles is obtained using the Taylor-series expansion of the Lorentz factor:

\[
\gamma = \left( 1 - \frac{u^2}{c^2} \right)^{-1/2} = \sum_{n=0}^{\infty} \left( \frac{u}{c} \right)^n \prod_{k=1}^{n} \left( \frac{2k-1}{2k} \right), \quad (55)
\]

\[
\gamma = 1 + \frac{1}{2} \left( \frac{u}{c} \right)^2 + \frac{3}{8} \left( \frac{u}{c} \right)^4 \ldots \Rightarrow u \ll c, \quad (56)
\]

\[
\gamma \approx 1 + \frac{1}{2} \left( \frac{u}{c} \right)^2, \quad (57)
\]

\[
m_i = m \gamma \approx m(1 + \frac{u^2}{2c^2}). \quad (58)
\]

Similarly, Eq.(54) is the classical limit of the relativistic inertia for the theoretical quasiparticle. The binomial series expansion of the inverse Lorentz factor yields

\[
\frac{1}{\gamma} = \left( 1 - \frac{u^2}{c^2} \right)^{1/2} = 1 - \frac{1}{2} \left( \frac{u}{c} \right)^2 - \frac{1}{8} \left( \frac{u}{c} \right)^4 \ldots, \quad (59)
\]

\[
u_n < c \Rightarrow \frac{1}{\gamma} \approx 1 - \frac{1}{2} \left( \frac{u}{c} \right)^2, \quad (60)
\]

\[
m_i = m \frac{1}{\gamma} \approx m(1 - \frac{u^2}{n_r^2 \cdot 2c^2}). \quad (61)
\]

Consequently, the relativistic inertia of the theoretical quasiparticle is

\[
m_i = \frac{m}{\gamma} \left( 1 - \frac{u^2}{n_r^2 \cdot 2c^2} \right) = \xi_c \cdot m \left( 1 - \frac{u^2}{n_r^2 \cdot 2c^2} \right)^{-1/2}, \quad (62)
\]

A general expression that incorporates Einstein’s special relativity (see Fig. 2) is derived as follows:

\[
\xi_c = \frac{u_c}{c} = \left( 1 - \frac{u_{sw}}{n_r^2 \cdot c^2} \right) \Rightarrow m_i = \xi_c \cdot m \gamma_{\text{rel}}, \quad (63)
\]

where \(u_{sw}\) is the travelling speed of the translation mechanism in the quasiparticle structure. For a particle in Einstein’s theory of special relativity, we have

\[
\sum \text{F}_{\text{ind}} = 0 \text{ and } \sum \text{F}_{\text{ext}} \geq 0, \quad (64)
\]

\[
u_{sw} = 0 \Rightarrow \xi_c = 1 \Rightarrow n_r = 1, \quad (65)
\]

\[
u_c = c \Rightarrow 0 \leq u < c, m_i = m \left( 1 - \frac{u^2}{c^2} \right)^{-1/2}, \quad (66)
\]

\[
m_i c^2 - mc^2 = U_k, \quad (67)
\]
The Lorentz transformations for particle motion in the $x$ direction are

$$x' = a_1 x + a_2 t \quad \text{where} \quad a_1 = \gamma r \quad \text{and} \quad a_2 = -\gamma r u,$$

$$y' = y \quad \text{and} \quad z' = z,$$  

$$t' = a_3 x + a_4 t \quad \text{where} \quad a_3 = -\frac{u}{c^2} \gamma r \quad \text{and} \quad a_4 = \gamma r.$$  

For the Lorentz transformations to support the existence of such a theoretical quasiparticle, the parameters $a_1$ and $a_2$ of Eq. (77) must be multiplied by $\xi_c$ and those $a_3$ and $a_4$ of Eq. (79) must be divided by $\xi_c$, leading to

$$\xi_c = \frac{u_c}{c} = \left(1 - \frac{u^2}{n^2_c \cdot c^2} \right) = 1/\gamma r,$$  

$$b_1 = \xi_c a_1 = \xi_c \gamma r \quad \text{and} \quad b_2 = \xi_c a_2 = -\xi_c \gamma r u/n_r,$$  

$$x' = b_1 x + b_2 t,$$  

$$y' = y \quad \text{and} \quad z' = z,$$  

$$t' = b_3 x + b_4 t.$$  

Consequently, the general Lorentz transformations for motion in the $x$ direction can be written in a more compact form as

$$x' = \xi_c \gamma r (x - (u/n_r)t),$$  

$$y' = y \quad \text{and} \quad z' = z,$$  

$$t' = \frac{\gamma r}{\xi_c} \left( t - \frac{u}{n_r c^2} x \right).$$  

The proper time is defined as the time measured in the rest frame of the theoretical quasiparticle, thus

$$\text{d}x' = 0 \Rightarrow \text{d}x = (u/n_r)\text{d}t,$$  

$$\text{d}t' = \text{d}t = \frac{\gamma r}{\xi_c \gamma r} \left(1 - \frac{u^2}{n^2_r c^2} \right) \text{d}t = \frac{\text{d}t}{\gamma r}. $$

Similarly, the proper length is

$$\text{d}t = 0 \Rightarrow \text{d}x' = \text{d}x' = \xi_c \gamma r \text{d}x = \frac{\text{d}x}{\gamma r}.$$
The general Lorentz transformations [Eqs. (86) and (88)] can be verified by deriving the momentum and energy using the proper time, hence

$$u_{sw} = u,$$  \hspace{1cm} (92)

$$p = \frac{dr}{dt} \cdot \frac{dt}{d\tau} = \frac{mu}{n_r} \cdot \xi_c \gamma_r,$$  \hspace{1cm} (93)

$$p = \frac{1}{\gamma_r} \frac{mu}{n_r} \sqrt{1 - \frac{u^2}{n_r^2} \cdot c^2},$$  \hspace{1cm} (94)

$$m_i c^2 = m c^2 \cdot \frac{dt}{d\tau},$$  \hspace{1cm} (95)

$$m_i c^2 = m c^2 \cdot \xi_c \gamma_r = \frac{mc^2}{\gamma_r} = mc^2 \sqrt{1 - \frac{u^2}{n_r^2} \cdot c^2}.$$.  \hspace{1cm} (96)

Note that the original expressions of Einstein and Lorentz are recovered when there are no induced internal forces (no translation mechanism) in the system:

$$\sum F_{ind} = 0 \text{ and } \sum F_{ext} \geq 0,$$  \hspace{1cm} (97)

$$u_{sw} = 0 \Rightarrow \xi_c = 1 \Rightarrow n_r = 1 \Rightarrow \gamma_r = \gamma.$$  \hspace{1cm} (98)

III. CONCLUSIONS

Newton’s laws of motion and Einstein’s theory of special relativity overlook the nature of the induced mechanical force and apparently fail to anticipate the motion of an isolated system due to induced internal forces. Besides the present results imply a paradigm shift in the way that motion is conducted, the new phenomenon whereby the effective inertia decreases while the speed of a system increases led to the discovery of a wider framework—which predicts the existence of quasiparticles with group velocities that may reach and even surpass the speed of light.

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[22] I. Newton, Roy. Soc, Philosophiae naturalis principia mathematica (1687)