Action–reaction symmetry breaking by induced internal forces

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Abstract. Newton’s third law of motion would be the cornerstone of physics were it not for certain experimental results that demonstrate a conflict with conservation of momentum under certain special circumstances. Such conceptual conflicts appear in systems in which the interacting parts of the system are mediated by a nonequilibrium environment that gives rise to nonreciprocal forces. The present theoretical study of a new mechanism of motion that utilizes induced internal forces in an isolated system (internally powered), addresses the probable cause behind the breaking of action–reaction symmetry and explores the potential implications of these findings for Einstein’s special relativity and Lorentz transformations.

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Introduction. In classical mechanics, any realization of the action–reaction principle presupposes two bodies exerting equal and opposite forces on each other. The notion of an internally powered isolated system moving in the absence of external forces, touches upon the extraordinary idea that object A is attracted to object B while object B is simultaneously repelled by object A. Nonreciprocal forces associated with the breaking of action–reaction symmetry is a subject being addressed by various disciplines in physics as in statistical mechanics [1–3], condensed matter [4–9], high energy physics [13], relativistic mechanics [14–16] and optics [17–21]. The present paper proposes and reveals the potentially missing part of Newton’s [22] third law of motion by simultaneously establishing the breaking of New-ton’s action–reaction symmetry as a fundamental law of nature. The acceleration of an isolated system that utilizes induced internal forces leads inevitably to a new phenomenon, namely the reduction of the effective inertia, with profound implications for Einstein’s [23] special relativity.

Methods. As shown in FIG. 1, an applied internal force \( \overrightarrow{F_A} \) exerted on part \( m_A \) of an ideal isolated system causes a reaction internal force on part \( m_R \) along the line connecting them (the mutual actions are linearly entangled). Thus,

\[
\sum F_{\text{ext}} = 0 \quad \text{and} \quad \overrightarrow{F_A} + \overrightarrow{F_R} = 0, \quad (1)
\]

\[
U_s = \int_0^d F_A \, ds \quad \text{and} \quad \sum F_{\text{int}} + (\overrightarrow{F_A} + \overrightarrow{F_R}) = 0, \quad (2)
\]

\[
\int_0^d \sum F_{\text{int}} \, ds + \left( U_s + \int_0^d F_R \, ds \right) = 0, \quad (3)
\]

where \( U_s \) is the energy stored within the system.

When the mutual actions are not linearly but helically entangled, the net internal force (induced) is expected to be not null. Starting from the conservation of angular momentum, the net external and internal torques are

\[
\sum \tau_{\text{ext}} = 0 \quad \text{and} \quad \overrightarrow{\tau_A} + \overrightarrow{\tau_R} = 0, \quad (4)
\]

\[
r_A \neq r_R \Rightarrow \tau_A \neq 0 \quad \text{and} \quad \tau_R \neq 0, \quad (5)
\]

\[
\sum \tau_{\text{int}} + \tau_A + \tau_R = 0, \quad (6)
\]

\[
\sum \tau_{\text{int}} + (r_A \times \overrightarrow{F_A}) + (r_R \times \overrightarrow{F_R}) = 0, \quad (7)
\]

\[
\sum \tau_{\text{int}} + (r_A \times \overrightarrow{F_A}) + (r_R \times (-\overrightarrow{F_A})) = 0, \quad (8)
\]

\[
\sum \tau_{\text{int}} + ((r_A - r_R) \times \overrightarrow{F_A}) = 0. \quad (9)
\]

A translation mechanism (translation screw in FIG. 2) can maintain, amplify or reduce the magnitude of the input force by delivering the same amount of energy (ideally) entering the system (energy conservation).
At this point, developing a general expression for the net induced force requires the introduction of the dimensionless factor \( n_r \) (mechanical advantage) along with a definition of the net induced torque. Hence,

\[
n_r = \frac{\omega \times (r_A - r_R)}{|\Delta u|} = \frac{2\pi|r_A - r_R|}{|\Delta d|},
\]

\[
\sum \tau_{\text{ind}} + n_r (r_A - r_R) \times \vec{F}_A = 0,
\]

\[
\sum \tau_{\text{int}} = n_r \sum \tau_{\text{int}},
\]

\[
\sum \tau_{\text{ind}} + n_r (r_A - r_R) \times \vec{F}_A = 0. \tag{13}
\]

Dividing Eq. (13) by the position-vector magnitude \(|r_A - r_R|\) yields

\[
\frac{\sum \tau_{\text{ind}}}{|r_A - r_R|} + n_r \frac{((r_A - r_R) \times \vec{F}_A)}{|\vec{F}_A - r_R|} = 0, \tag{14}
\]

\[
U_s = \int_0^{2\pi} ((r_A - r_R) \times \vec{F}_A) \, d\phi, \tag{15}
\]

\( U_s \) is the energy stored (1 cyc) within the system.

The net induced force (internal) is not null, which implies that the isolated system starts to accelerate from rest, something that Newton’s laws of motion do not anticipate. Thus,

\[
\sum \vec{F}_{\text{ind}} = -n_r (r_A - r_R) \times \vec{F}_A \neq 0, \tag{16}
\]

\[
\sum \vec{F}_{\text{ind}} = \frac{d\vec{p}}{dt} = -\left(\vec{F}_A + \vec{F}_R\right) \neq 0, \tag{17}
\]

\[
\sum \vec{F}_{\text{ind}} = \frac{d\vec{p}}{dt} = -(\vec{F}_A + \vec{F}_R) \neq 0, \tag{18}
\]

\[
\sum \vec{F}_{\text{ind}} = \frac{d\vec{p}}{dt} = -(\vec{F}_A + \vec{F}_R) \neq 0, \tag{19}
\]

\[
\vec{F}_A : \frac{d\vec{p}}{dt} = -(\vec{F}_A + \vec{F}_R) \neq 0. \tag{20}
\]

An ideal isolated system (see FIG. 1 and FIG. 2) could acquire non-zero momentum if and only if, any change in momentum of mass \( m_A \) results in the non-zero change in momentum of system as a whole. In the case of collinear internal action–reaction forces, the conservation of momentum is

\[
\frac{d\vec{p}}{dt} = -(\vec{F}_A + \vec{F}_R) = 0 \quad \text{(collinear)}, \tag{21}
\]

\[
\vec{p} = (m_A + m_R + m_R') \vec{u}_\text{cm}, \tag{22}
\]

\[
\vec{p} = - (m_A \vec{u}_\text{cm}' - m_A \vec{u}_\text{cm}), \tag{23}
\]

\[
\vec{p} = m_A (\vec{u}_\text{cm}' - \vec{u}_\text{cm}), \tag{24}
\]

\[
\frac{d\vec{p}}{dt} = -n_r \Delta \vec{u} \frac{dm_A}{dt} \neq 0 \Rightarrow \vec{u}_\text{cm}' = \vec{u}_\text{cm}. \tag{25}
\]

Then, for the induced internal action–reaction forces, the conservation of momentum becomes

\[
\frac{d\vec{p}}{dt} = -(\vec{F}_A + \vec{F}_R) \neq 0 \quad \text{(induced)}, \tag{26}
\]

\[
\Delta \vec{u} = \vec{u}_\text{cm}' - \vec{u}_\text{cm}, \tag{27}
\]

\[
\vec{p} = -n_t m_A (\vec{u}_\text{cm}' - \vec{u}_\text{cm}) = -n_t m_A \Delta \vec{u}, \tag{28}
\]

\[
\frac{d\vec{p}}{dt} = -n_t \Delta \vec{u} \frac{dm_A}{dt} \neq 0 \Rightarrow \vec{u}_\text{cm}' = \vec{u}_\text{cm}. \tag{29}
\]

When the change in velocity \( \Delta \vec{u} \) has constant magnitude, the effective inertia of the system becomes

\[
|\Delta \vec{u}| = \text{const.} \Rightarrow \vec{u}_\text{cm}' = \vec{u}_\text{cm} \neq 0. \tag{30}
\]

\[
m = m_A + m_R + m_R' \tag{31}
\]

\[
dm/dt = dm_A/dt + dm_R/dt + dm_R'/dt \tag{32}
\]

but \( dm_R/dt = dm_R'/dt = 0 \Rightarrow dm = dm_A, \tag{33}
\]

\[
dm = dm_A \Rightarrow \frac{d\vec{p}}{dt} = -n_t (\vec{u}_\text{cm}' - \vec{u}_\text{cm}) \frac{dm}{dt}, \tag{34}
\]

\[
\frac{d\vec{p}}{dt} = -n_t (\vec{u}_\text{cm}' - \vec{u}_\text{cm}) \frac{dm}{dt}, \tag{35}
\]

\[
\int_{m}^{m_1} dm = - \frac{1}{n_t (\vec{u}_\text{cm}' - \vec{u}_\text{cm})} \int_0^p dp, \tag{36}
\]

\[
m_1 = m \left(1 - \frac{p}{n_t \cdot m \cdot \Delta \vec{u}}\right), \tag{37}
\]

\[
m_1 = m \left(1 - \frac{p}{n_t \cdot m \cdot \Delta \vec{u}}\right), \tag{38}
\]

\[
m_1 = m \left(1 - \frac{p}{n_t \cdot m \cdot \Delta \vec{u}}\right). \tag{39}
\]
Let us suppose that there is a theoretical quasiparticle (system) that exhibits the property whereby its effective inertia decreases as its velocity $u_{sw}$ or momentum $p$ increases. Setting $\Delta u$ to be equal to the speed of light, Eq. (39) becomes the nonrelativistic inertia of the system. Thus,

$$\Delta u = c \Rightarrow m_i = m \left(1 - \frac{p}{n_c \cdot m \cdot c}\right). \quad (40)$$

Alternatively, because of energy conservation, Eq. (40) becomes

$$u_{sw} = u \Rightarrow m_i c^2 - mc^2 = -\left(\frac{p}{n_c}\right)^2 \cdot \frac{1}{2m}, \quad (41)$$

$$m_i c^2 - mc^2 = \frac{1}{n_c^2} m u^2 = \frac{U_k}{n_c} \Rightarrow U_k = \frac{U_k}{n_c^2}, \quad (42)$$

$$m_i = m \left(1 - \frac{u^2}{n_c^2 \cdot 2c^2}\right). \quad (43)$$

The classical limit of the relativistic inertia for charged particles is obtained using the Taylor-series expansion of the Lorentz factor:

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2} = \sum_{n=0}^\infty \left(\frac{u}{c}\right)^n \prod_{k=1}^{2n} \frac{2k-1}{2k}, \quad (44)$$

$$\gamma = 1 + \frac{1}{2} \left(\frac{u}{c}\right)^2 + \frac{1}{8} \left(\frac{u}{c}\right)^4 \cdots \Rightarrow u \ll c, \quad (45)$$

$$\gamma \approx 1 + \frac{1}{2} \left(\frac{u}{c}\right)^2, \quad (46)$$

$$m_i = m \gamma \approx m \left(1 + \frac{u^2}{2c^2}\right). \quad (47)$$

Similarly, Eq. (43) is the classical limit of the relativistic inertia for the theoretical quasiparticle.

The binomial series expansion of the inverse Lorentz factor yields

$$u_n = \frac{u}{n_c}, \quad (48)$$

$$\frac{1}{\gamma r} = \left(1 - \frac{u_n^2}{c^2}\right)^{1/2} = 1 - \frac{1}{2} \left(\frac{u_n}{c}\right)^2 - \frac{1}{8} \left(\frac{u_n}{c}\right)^4 \cdots \quad (49)$$

$$\frac{u_n}{c} << c \Rightarrow \frac{1}{\gamma r} \approx 1 - \frac{1}{2} \left(\frac{u_n}{c}\right)^2, \quad (50)$$

$$m_i = \frac{m}{\gamma r} \approx m \left(1 - \frac{u^2}{n_c^2 \cdot 2c^2}\right). \quad (51)$$

Consequently, the relativistic inertia of the theoretical quasiparticle is

$$m_i = \frac{m}{\gamma r} = m \left(1 - \frac{u^2}{n_c^2 \cdot c^2}\right)^{1/2} = \xi_c \cdot m \left(1 - \frac{u^2}{n_c^2 \cdot c^2}\right)^{-1/2}. \quad (52)$$

A general expression that incorporates Einstein’s special relativity (see FIG. 3) is derived as follows:

$$\xi_c = \frac{u c}{\gamma r} = \left(1 - \frac{u_{sw}^2}{n_c^2 \cdot c^2}\right) \Rightarrow m_i = \xi_c \cdot m \gamma r, \quad (53)$$

where $u_{sw}$ is the travelling speed of the translation mechanism in the quasiparticle structure.

For a particle in Einstein’s theory of special relativity (see FIG. 3 and FIG. 4), we have

$$\sum F_{\text{ind}} = 0 \quad \text{and} \quad \sum F_{\text{ext}} \geq 0, \quad (53)$$

$$u_{sw} = 0 \Rightarrow \xi_c = 1 \Rightarrow n_t = 1, \quad (54)$$

$$u_c = c \Rightarrow 0 \leq u < c, m_i = m \left(1 - \frac{u^2}{c^2}\right)^{-1/2}, \quad (55)$$

$$m_i c^2 - mc^2 = U_k, \quad (56)$$
\[ mc^2 \left( 1 - \frac{u^2}{c^2} \right)^{-1/2} - mc^2 = U_k, \quad (57) \]
\[ u = c \sqrt{1 - \frac{1}{\left( 1 + \frac{mc^2}{E} \right)^2}}. \quad (58) \]

For the theoretical quasiparticle (see FIG. 3 and FIG. 4), we have
\[ \sum F_{\text{ind}} \geq 0 \quad \text{and} \quad \sum F_{\text{ext}} = 0, \quad (59) \]
\[ u_{\text{sw}} = u \Rightarrow \xi \leq 1 \Rightarrow n_t \geq 1, \quad (60) \]
\[ 0 \leq u \leq n_t \cdot c \quad \text{and} \quad 0 \leq u_{\text{sw}} \leq c, \quad (61) \]
\[ m_i = m \left( 1 - \frac{u^2}{n_t^2 \cdot c^2} \right)^{1/2}, \quad (62) \]
\[ m_i c^2 - mc^2 = -U_k / n_t^2, \quad (63) \]
\[ mc^2 \left( 1 - \frac{u^2}{n_t^2 \cdot c^2} \right)^{1/2} = mc^2 - U_k / n_t^2, \quad (64) \]
\[ u = n_t \cdot c \sqrt{1 - \left( \frac{U_k}{n_t^2 \cdot mc^2} \right)^2}. \quad (65) \]

The Lorentz transformations for particle motion in the \( x \) direction are
\[ x' = a_1 x + a_2 t \quad \text{where} \quad a_1 = \gamma_r \quad \text{and} \quad a_2 = -\gamma u, \quad (66) \]
\[ y' = y \quad \text{and} \quad z' = z, \quad (67) \]
\[ t' = a_3 x + a_4 t \quad \text{where} \quad a_3 = -\frac{u}{c^2} \gamma_r \quad \text{and} \quad a_4 = \gamma_r. \quad (68) \]

For the Lorentz transformations to support the existence of such a theoretical quasiparticle, the parameters \( a_1 \) and \( a_2 \) of Eq.(66) must be multiplied by \( \xi \) and those \( (a_3 \) and \( a_4 \) of Eq.(68) must be divided by \( \xi \), leading to
\[ \xi = \frac{u_c}{c} = \left( 1 - \frac{u^2}{n_t^2 \cdot c^2} \right) = 1 / \gamma^2, \quad (69) \]
\[ b_1 = \xi c a_1 = \xi c \gamma_r \quad \text{and} \quad b_2 = \xi c a_2 = -\xi c \gamma u / n_t, \quad (70) \]
\[ x' = b_1 x + b_2 t, \quad (71) \]
\[ y' = y \quad \text{and} \quad z' = z, \quad (72) \]
\[ t' = b_3 x + b_4 t, \quad (73) \]
\[ b_3 = \frac{a_3}{\xi c} = -\frac{u}{n_t c^2} \gamma \gamma_r \quad \text{and} \quad b_4 = \frac{a_4}{\xi c} = \frac{\gamma}{\xi c}. \quad (74) \]

Consequently, the general Lorentz transformations for motion in the \( x \) direction can be written in a more compact form as
\[ x' = \xi c \gamma_r (x - (u/n_t) t), \quad (75) \]
\[ y' = y \quad \text{and} \quad z' = z, \quad (76) \]
\[ t' = \frac{\gamma}{\xi c} \left( t - \frac{u}{n_t c^2} x \right). \quad (77) \]

The proper time is defined as the time measured in the rest frame of the theoretical quasiparticle, thus
\[ dt' = 0 \Rightarrow dt = (u/n_t) dt, \quad (78) \]
\[ dt = dt' = \frac{\gamma}{\xi c} \left( 1 - \frac{u^2}{n_t^2 c^2} \right) dt = \frac{dt}{\xi c \gamma_r} = \gamma_r dt. \quad (79) \]

Similarly, the proper length is
\[ dt = 0 \Rightarrow dx_r = dx' = \xi c \gamma_r dx = \frac{dx}{\gamma_r}. \quad (80) \]

The general Lorentz transformations [Eqs. (75) and (77)] can be verified by deriving the momentum and energy using the proper time, hence
\[ p = \frac{dx_r}{dt} = \frac{\mu u}{n_t} \xi c \gamma_r, \quad (81) \]
\[ p = \frac{1}{n_t} \xi c \gamma_r \left( 1 - \frac{u^2}{n_t^2 c^2} \right), \quad (82) \]
\[ p = \frac{1}{n_t} \xi c \gamma_r \left( 1 - \frac{u^2}{n_t^2 c^2} \right), \quad (83) \]
\[ m_i c^2 = mc^2 \frac{dt}{dt_r}, \quad (84) \]
\[ m_i c^2 = mc^2 \frac{\gamma_r}{\xi c} \left( 1 - \frac{u^2}{n_t^2 c^2} \right). \quad (85) \]

Note that the original expressions of Einstein and Lorentz are recovered when there are no induced internal forces (no translation mechanism) in the system, thus
\[ \sum F_{\text{ind}} = 0 \quad \text{and} \quad \sum F_{\text{ext}} \geq 0, \quad (86) \]
\[ u_{\text{sw}} = 0 \Rightarrow \xi \geq 1 \Rightarrow n_t = 1 \Rightarrow \gamma_r = \gamma. \quad (87) \]

**Conclusions.** Newton’s laws of motion and Einstein’s theory of special relativity overlook the nature of the induced mechanical force and apparently fail to anticipate the motion of an isolated system due to induced internal forces. Besides the present results imply a paradigm shift in the way that motion is conducted, the new phenomenon whereby the effective inertia decreases while the speed of a system increases led to the discovery of a wider framework—which predicts the existence of quasiparticles with group velocities that may reach and even surpass the speed of light.

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[22] I. Newton, Roy. Soc, Philosophiae naturalis principia mathematica (1687)