Comment on "Perturbative operator approach to high-precision light-pulse atom interferometry"

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An anomaly of the Earth gravitational field could increase the second-order gravity-gradient tensor more than an order of magnitude and third-order gravity-gradient tensor more than 4 orders of magnitudes. As a result estimates of the systematic errors in the atomic gravimetry considered in the articles [1, 2] should be proportionally enlarged.

The article [1] is devoted to a new method to obtain the phase of an atom interferometer (AI), which is proposed to be used in a precision atomic interferometry. As an application of this method, the AI phase in the Earth's gravitational field was considered. The contributions due to gravity-gradient tensors of the first and second order, $\Gamma^{(1)}$ and $\Gamma^{(2)}$, are included in this consideration.. To obtain these tensors, it is assumed that the earth is a ball with a radius $R \simeq 6\times10^6$m. The same assumption regarding the Earth gravitational field was made in the papers [2–4]. The article [2] even took into account the contribution from the gravity-gradient tensor of the third order S.

A more realistic field model was used in the patent [5], in which the ball was replaced with Geoid and an anomaly part of the gravitational potential $\Phi_a(\vec{x})$ was also simulated. We will show here that the presence of a gravitational anomaly could increase by one or more orders of magnitude the second- and third-order gravitational tensors, $\Gamma^{(2)}$ and S.

Indeed, the earth's gravitational field anomaly itself, $\vec{g}_a = -\partial_{\vec{x}}\Phi_a(\vec{x})$ is small, the root-mean-square (rms) of this anomaly is $|g_a| \sim 30$mGal [6]. But since the anomaly is caused by the deviation of the terrain from the surface of the Geoid and changes at distance L_a much smaller than the Earth's radius R, the anomaly's contribution to gravity-gradient tensors of higher orders may prove to be dominant.

Let's evaluate this contribution. We failed to find data on the distance L_a. But, however, we found data for the first-order gravitational gradient tensor $\Gamma^{(1)}_{azz}$ [7]. From the Plate 5 in the article [7], one can conclude that $\Gamma^{(1)}_{azz}$ rms is of the order of

$$\left|\Gamma^{(1)}_{azz}\right| \sim 100E, \tag{1}$$

and, consequently, the characteristic size of the gravitational field anomaly change is of the order of

$$L_a \sim |g_a| / \left|\Gamma^{(1)}_{azz}\right| \sim 3\text{km}. \tag{2}$$

Then for the gradient tensors of higher orders one gets estimates

$$\left|\Gamma^{(2)}_{azzz}\right| \sim |g_a|/L_a^2 \simeq 3\times10^{-11}\text{m}^{-1}\text{s}^{-2}, \tag{3a}$$

$$|S_{azzzz}| \sim |g_a|/L_a^3 \simeq 10^{-14}\text{m}^{-2}\text{s}^{-2}. \tag{3b}$$

One sees that, due to a gravitational anomaly, the Earth's field could have the second-order gradient (3a), which is 25 times larger than the gradient used in [1] (compare Eqs. (3a) and (6) in [1]), and a third-order gradient (3b), which is 1.4×10^4 times greater than the gradient, used in the last three lines in table 1 in the article [2].

This comment is particularly important in the context of the relative standard deviation of the measurement of the atomic accelerations difference 7×10^{-12} achieved in [8],, since the systematic error in measuring the gravitational field due to the tensor $\Gamma^{(2)}$ will be 25 times more than the error shown in figure 1 in the article [1] and would reach the level 3×10^{-10}. At the same time, the error associated with the gradient tensor S would increase from the value 2.64×10^{-16} in [2] to the level 4×10^{-12}.

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[1] C. Ufrecht and E. Giese, Perturbative operator approach to high-precision light-pulse atom interferometry, Phys. Rev. A **101**, 053615 (2020).

[2] J. M. Hogan, D. M. S. Johnson, M. A. Kasevich, Light-pulse atom interferometry, arXiv:0806.3261 [physics.atom-ph], appear in the Proceedings of the International Summer School of Physics "Enrico Fermi" on Atom Optics and Space Physics (Varenna, July 2007).

[3] B. Dubetsky and M. A. Kasevich, Atom interferometer as a selective sensor of rotation or gravity, Phys. Rev. A **74**, 023615 (2006).

[4] K. Bongs, R. Launay, and M. A. Kasevich, High-order inertial phase shifts for time-domain atom interferometers, Appl. Phys. B **84**, 599 (2006).

[5] M. A. Kasevich and B. Dubetsky, Kinematic Sensors Employing Atom Interferometer Phases, US Patent 7,317,184 (2005).

[6] Committee on Earth Gravity from Space, J. O. Dickey, Chair, Satellite Gravity and the Geosphere: Contributions to the Study of the Solid Earth and Its Fluid En-

velopes. Washington, DC: The National Academies Press. https://doi.org/10.17226/5767, National Research Council 1997, p. 13.

[7] Y. M. Wang, GSFCO0 mean sea surface, gravity anomaly, and vertical gravity gradient from satellite altimeter data, J. Geophys. Res. **106**, 31167 (2001).

[8] P. Asenbaum, C. Overstreet, M. Kim, J. Curti, M. A. Kasevich, Atom-interferometric test of the equivalence principle at the 10^{-12} level, arXiv:2005.11624v1 [physics.atom-ph].