Vibration of Yukawa Potential dependent Time and Extended Klein-Gordon Equation in Rindler Space-Time

Sangwha-Yi

Department of Math, Taejon University 300-716, South Korea

ABSTRACT
Atom’s nucleus force understand by Yukawa potential independent time. We study Yukawa potential dependent about time. We make Klein-Gordon equation is satisfied by Yukawa potential dependent about time. Yukawa potential satisfy Proca equation or Klein-Gordon equation. If we represent Yukawa potential dependent time in Rindler space-time, this Yukawa potential satisfy the extended Klein-Gordon equation in Rindler space-time. We understand Yukawa force in Rindler space-time.

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e-mail address:sangwha1@nate.com
Tel:010-2496-3953
1. Introduction

Atom’s nucleus force understand by Yukawa potential. We study Yukawa potential dependent about time. We make Klein-Gordon equation is satisfied by Yukawa potential dependent about time.

At first, Yukawa potential \( V \) describes nucleus’s combine force in semi-classical method.\[7\]

\[ V = -\frac{g}{r} \exp(-\frac{m_\pi rc}{\hbar}) \]

\( g \) is real number, \( m_\pi \) is the meson’s mass \( \text{(1)} \)

Klein-Gordon equation is satisfied by Yukawa potential \( V \).

\[-\partial \partial V + \frac{\hbar^2}{c^2} \nabla^2 V = -\hbar^2 V + \frac{m_\pi^2 c^2}{\hbar^2} V = 0\]

\[ V = -\frac{g}{r} \exp(-\frac{m_\pi rc}{\hbar}) \text{ \( \text{(2)} \)}

If we focus Klein-Gordon equation make 4-dimentional partial differential equation about Yukawa potential \( \phi \) dependent time,

\[ \frac{m_\pi^2 c^2}{\hbar^2} \phi + \partial \partial \phi = \frac{m_\pi^2 c^2}{\hbar^2} \phi + \frac{1}{c^2} \partial^2 \phi - \nabla^2 \phi = 0 \text{ \( \text{(3)} \)}

In this time, Yukawa potential \( \phi \) dependent time is.

\[ \phi = -\frac{g}{r} \exp(-\frac{m_\pi rc}{\hbar}) + A_0 \sin \omega t, \text{ \( \text{Frequency } \omega = \frac{m_\pi c^2}{\hbar} \)} \text{ \( \text{(4)} \)}

Absolutely, if we calculate, Eq(3) is satisfied by Eq(4). Yukawa potential \( \phi \) is vibrated about the amplitude \( A_0 \), but we know the nuclear strong force doesn’t vibrate about time in inertial frame..

2. Yukawa potential dependent time from Extended Klein-Gordon Equation in Rindler-Space-Time

Rindler coordinates are

\[ ct = \left( \frac{c^2}{a_0} + \xi^1 \right) \sinh \left( \frac{a_0}{c} \xi^0 \right), \quad x = \left( \frac{c^2}{a_0} + \xi^1 \right) \cosh \left( \frac{a_0}{c} \xi^0 \right) - \frac{c^2}{a_0} \]

\[ y = \xi^2, \quad z = \xi^3 \text{ \( \text{(5)} \)}

If we write Yukawa potential \( \phi \) in inertial frame,

\[ \phi = -\frac{g}{r} \exp(-\frac{m_\pi rc}{\hbar}) + A_0 \sin \omega t, \text{ \( \text{Frequency } \omega = \frac{m_\pi c^2}{\hbar} \)} \text{ \( \text{(6)} \)}

If we rewrite Yukawa potential \( \phi_\xi \) in Rindler space-time,

\[ \phi = \phi^i + \phi^\xi = \phi^i_\xi + \phi^\xi_\xi \text{ \( \text{(7)} \)}\]
\[
\phi^1 = \phi_z^1 = -\frac{g^2}{\sqrt{x^2 + y^2 + z^2}} \exp(-\frac{mc}{\hbar} \sqrt{x^2 + y^2 + z^2}) \\
= -\frac{g^2}{\sqrt{(\frac{c^2}{a_0} + \xi^2)\cosh(\frac{a_0}{c} \xi) - \frac{c^2}{a_0}}^2 + (\xi^2)^2 + (\xi^3)^2}} \exp[-\frac{mc}{\hbar} \sqrt{(\frac{c^2}{a_0} + \xi^2)\cosh(\frac{a_0}{c} \xi) - \frac{c^2}{a_0}}^2 + (\xi^2)^2 + (\xi^3)^2}] 
\]

(8)

And,

\[
\phi^2 = \phi_z^2 = A_b \sin \omega t = A_b \sin \left[ \omega \left( \frac{c}{a_0} + \frac{\xi^1}{c} \right) \sinh \left( \frac{a_0 \xi^0}{c} \right) \right] 
\]

(9)

This Yukawa potential satisfy the extended Klein-Gordon equation. At first, energy and momentum are in Rindler space-time[1],

\[
E_A = i\hbar \left( 1 + \frac{a_0 \xi^1}{c^2} \right) \frac{\partial}{\partial \xi^0}, \quad p_\xi = -i\hbar \tilde{V}_\xi 
\]

(10)

Energy-Momentum equation is in Rindler space-time[1],

\[
E_A^2 = \tilde{p}_\xi c \cdot \tilde{p}_\xi c + m^2 c^4 
\]

(11)

Hence, normal Klein-Gordon equation is in Rindler-spacetime,

\[
\frac{m^2 c^2}{\hbar^2} \phi - \frac{1}{c^2} \frac{\partial^2}{\partial (a_0 \xi^0)^2} \phi - \nabla^2 \phi = 0 
\]

(12)

In this time, we focus the gauge \( \Lambda \) equation in Rindler space-time[1],

\[
\frac{1}{c^2} \left( 1 + \frac{a_0 \xi^1}{c^2} \right)^2 \frac{\partial^2}{\partial (a_0 \xi^0)^2} \Lambda - \nabla^2 \Lambda - \frac{\partial \Lambda}{\partial a_0} \frac{1}{(1 + \frac{a_0 \xi^1}{c^2})} = 0 
\]

(13)

Hence, Eq(12) change extended Klein-Gordon equation in Rindler space-time.

Extended Klein-Gordon Equation is in Rindler space-time,

\[
\frac{m^2 c^2}{\hbar^2} \phi^1 - \frac{1}{c^2} \frac{\partial^2 \phi^1}{\partial (a_0 \xi^0)^2} - \nabla^2 \phi^1 = 0 
\]

(14)

And

\[
\frac{m^2 c^2}{\hbar^2} \phi^2 - \frac{1}{c^2} \frac{\partial^2 \phi^2}{\partial (a_0 \xi^0)^2} - \nabla^2 \phi^2 = 0 
\]

(15)
Hence,
\[
\frac{m^2 c^2}{\hbar^2} \phi_\xi + \frac{1}{C^2} \frac{\partial^2 \phi_\xi}{\partial \xi^2} - \nabla_\xi^2 \phi_\xi - \frac{\partial \phi_\xi}{\partial \xi^1} \left( \frac{a_0}{C^2} \xi^1 \right) = 0
\]  
(16)

Eq(8), Eq(9), Yukawa potentials \( \phi_\xi^1, \phi_\xi^2 \) satisfy Eq(14), Eq(15), extended Klein-Gordon equations in Rindler space-time. Therefore, Eq(7), Yukawa potential \( \phi_\xi \) satisfy Eq(16), extended Klein-Gordon equation in Rindler space-time

Yukawa force \( \vec{f} \) is

\[
\vec{f} = -\vec{\nabla} \phi = -\frac{g^2}{r^2} \left[ \exp\left( -\frac{m_x rc}{\hbar} \right) \right](1 + \frac{m_x rc}{\hbar}) \vec{r}
\]  
(17)

In this time, Yukawa force \( \vec{f}_\xi \) is Rindler space-time,

\[
\vec{f}_\xi = -\vec{\nabla}_\xi \phi_\xi = -\vec{\nabla}_\xi \phi_\xi^1 - \vec{\nabla}_\xi \phi_\xi^2 = -\frac{g^2}{r^2} \left[ \exp\left( -\frac{m_x rc}{\hbar} \right) \right](1 + \frac{m_x rc}{\hbar})(x \cosh(\frac{a_0 \xi^0}{c}), \xi^2, \xi^3)
\]

\[
-\frac{\omega}{c} A[\cos(\omega t)] \{ \sinh(\frac{a_0 \xi^0}{c}), 0, 0 \}
\]  
(18)

Hence, according to Yukawa force \( \vec{f}_\xi \) in Rindler space-time, the nuclear force strongly acts and vibrates in accelerated frame rather than inertial frame in x-axis.

3. Conclusion

We found Yukawa potential dependent time. Hence, the nuclear strong force vibrates about time in Rindler spacetime. We found Yukawa potential mechanism in Rindler Space-time. We understand nuclear force in Rindler space-time.

References
