Cosmological constant problem factor

ALBERTO MIRÓ MORÁN

albertomiro1991@gmail.com

A B S T R A C T

The cosmological constant problem or vacuum catastrophe is localized in the convergence between general relativity and quantum field theory, it is considered as a fundamental problem in modern physics. In this paper we find the factor which produces this difference.

Introduction

Quantum field theory (QFT) which is fundamental in modern physics show zero-point energy in space, including in areas which are in another way ‘void’ (i.e. without radiation and matter). Maybe we could think these zero-point energy give a vast vacuum energy density. On the other hand it is expect to cause an increase of cosmological constant Λ appearing in Einstein’s field equation.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (1)$$

Where $R_{\mu\nu}$ and R allude the curvature of space-time, $g_{\mu\nu}$ is the metric, G is the gravitational constant,
$T_{\mu\nu}$ is the stress-energy tensor, and c is the speed of light.
(Zinkernagel & Rugh 2000).

Equations should be agree with Newtonian theory in the limit for medium and small gravitational fields and small velocities, and this should be consonant also with the value of Λ (Zinkernagel & Rugh 2000). In fact, compare equation (1) with observations show that Λ is minuscule.

$$|\Lambda| < 10^{-56} \mathrm{cm}^2 \quad (2)$$

The Friedmann equations are a set of equations in physical cosmology that govern the expansion of space in homogeneous and isotropic models of the universe based on general relativity.(Friedman, A. 1999).There are two independent Friedmann equations, the first is:

$$H_0{}^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} + \frac{kc^2}{a^2} \qquad (3)$$

Where:

$$\rho_m + \rho_r = \rho \qquad (4)$$

$$\rho_{Vac} = \rho_\Lambda = \frac{\Lambda c^2}{8\pi G} \qquad (5)$$

H_0 is the Hubble constant.

a is the scale factor.

k is the intrinsic curvature.

Λ is the cosmological constant.

ρ_R is radiation energy density divided by the speed of light squared.

ρ_M is the matter (dark plus baryonic) energy density divided by the speed of light squared.

ρ_{Vac} is the vacuum energy density divided by the speed of light squared.

ρ_Λ is the cosmological constant energy density divided by the speed of light squared, it represent cosmological constant effect.

Vacuum energy is the background energy that exists in the universe. The effect of the vacuum energy appears in the first Friedmann equation, vacuum energy is expected to create the cosmological constant, and produce the expansion of the universe.

If $\rho = 0$ and setting the normalized spatial curvature,k, equal to zero according to the ΛCDM model when the substitutions are applied to the first of the Friedmann´s equations we find:

$$\rho_{Vac} = \rho_\Lambda = \frac{3H_0{}^2}{8\pi G} \sim 10^{-29}\frac{g}{cm^3} \mapsto \rho_{Vac}c^2 = \rho_\Lambda c^2 = \frac{3H_0{}^2c^2}{8\pi G} \sim 10^{-9}\frac{erg}{cm^3}$$

This equation usually has been understood as the vacuum energy density in gravitational theory, nevertheless, theoretical estimates the vacuum energy density in QFT exceed the observational measurement by at least 40 orders of magnitude. This contrast constitutes the cosmological constant problem (Zinkernagel & Rugh 2000).

Quantum electrodynamics is one of the principal components in the Standard Model, first, and most productive case of a working quantum field theory. We first remembrance that systems studied in non-relativistic quantum mechanics, spatial-temporal coordinates, energy and others are represented by operators in quantum electrodynamics

(Zinkernagel & Rugh 2000).

One of the most simple quantum systems is the quantum harmonic oscillator. The ground state (vacuum state) of the quantum harmonic oscillator has zero point energy:

$$E_0 = \frac{1}{2}\hbar\omega_k \qquad (6)$$

(where ω_k is the frequency of classical harmonic oscillator).

In classical electromagnetism, electromagnetic fields have values E, and B in all space-time.

The energy density in the classical electromagnetism theory is:

$$H = \frac{1}{2}(E^{-2} + B^{-2}) \qquad (7)$$

In the quantization, quantum operators take the place of classical fields, The ground state $|0>$ is the vacuum state of QED.

Whole zero-point energy of the quantum electrodynamic theory can be expressed by:

$$E = < 0|\, \hat{\mathbf{H}}_{\mathbf{i}} |0 > \qquad (8)$$

$$E = \frac{1}{2} < 0| \left(\hat{\mathbf{E}}_{\mathbf{i}}{}^{-2} + \hat{\mathbf{B}}_{\mathbf{i}}{}^{-2} \right) |0 >= \delta(0)^3 \int d^3k \frac{1}{2} \hbar\omega_k \qquad (9)$$

Where ω_k are wave-numbers and k are frequencies of a continuum of (plane-wave) modes.

The infinite delta-function $\delta^3(0)$ can be regularized introducing a cube of volume V.

This volumes (in the limit $V \rightarrow \infty$) represent the standard 'box-quantization' process for the electromagnetic field in which an artificial 'quantization volume' V is used to create an equivalence with a harmonic quantum oscillator field mode. Energy density in this approximation,

(when $\rho_\Lambda = \rho_{Vac}$) can be extract from the zero-point energy of harmonic quantum oscillator mode:

$$\rho_{Vac} c^2 = \frac{E}{v_i} = \frac{1}{V} \sum_k \frac{1}{2} \hbar\omega_k = \frac{\hbar}{2\pi^2 c^3} \int_0^{\omega_{max}} \omega^3 \, d\omega$$

$$\rho_{Vac} c^2 = \frac{\hbar}{8\pi^2 c^3} \omega_{max}{}^4 \qquad (10)$$

Where the wave vector k now refers to the so-called normal modes (of the electromagnetic field) which are compatible with the boundary conditions provided by the box volume V. The right hand side of the equation follows from the left hand side in the limit $V \rightarrow \infty$ where the energy density does not depend on the 'box quantization' volume V. If we imagine that the QFT framework is valid to the Planck energy:

$$E_{planck} = \left(\frac{\hbar c^5}{G} \right)^{\frac{1}{2}} = 10^{19} \text{ GeV} \qquad (11)$$

It is easy to see, modern physics could enter between electroweak scales and Planck scales but if the alternations are still out of the framework of quantum field theory, we could admit vacuum energy can be expressed (using Planck energy in eq.(12) with $E_{Planck} = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}} = \hbar\omega_{max}$):

$$\rho_{Vac} c^2 = 10^{114} \frac{erg}{cm^3} \qquad (12)$$

There are a difference between QED and observational result by more than ~ 120 orders of magnitude (Zinkernagel & Rugh 2000).

Proposition

On the one hand we introduce Hubble´s radius:

$$R = \frac{c}{H_0} \qquad (13)$$

Hubble´s radius (observable universe radius) is a theoretical horizon defining the limit between particles that are moving slower and faster than the speed of light. In the current expansion model FWR light emitted from the Hubble´s radius will reach us in a finite amount of time, this is observable universe limit (Earman & Moster´ın 1999; Heitler 1954).

Hubble´s radius will be our universe radius.

On the other hand we continue using the vacuum state of QED but we use zero point energy of three-dimensional harmonic oscillators in our calculus because ground state is in a three-dimensional space:

$$E_{ground} = \frac{3}{2} \hbar\omega_k \qquad (14)$$

First we calculate vacuum energy of one line, vacuum energy along R is the sum of vacuum states of three-dimensional harmonic oscillators along R.

We compare vacuum states sum along R with continuous one-dimensional wave equation. (We calculate it in two different ways and we take wave properties of the QED vacuum).

First way, standing waves in a cavity at equilibrium with its surroundings can't take just any path. They must satisfy the wave equation:

$$\frac{\partial^2 A}{\partial x^2} = \frac{\partial^2 A}{c\,\partial t^2} \tag{15}$$

The solution to the wave equation must give zero amplitude at the walls:

$$A = A_0 \sin\frac{n\pi x}{L}\sin\frac{2\pi ct}{\lambda} \tag{16}$$

Substituting this solution into the wave equation above gives:

$$\frac{n\pi}{L} = \frac{2\pi}{\lambda} \tag{17}$$

Which simplifies to:

$$n = \frac{2L}{\lambda} \tag{18}$$

We need to evaluate the number of modes n_{Mod} which can meet this condition, which amounts to counting all the possible combinations of the integer n values. An approximation can be made by treating the number of combinations as line grid of the values of n, an "n -space" with the n values specifying the coordinates along one " n " line (Department of physics and astronomy Georgia State University):

$$n_{Mod} = n = \frac{2L}{\lambda} \tag{19}$$

We calculate vacuum energy alongL:

$$\sum E_{ground} = \frac{3}{2}\hbar\overline{\omega_k}\frac{2L}{\overline{\lambda}} = \frac{3L}{2\pi c}\hbar\overline{\omega_k}^2 \tag{20}$$

If we want calculate medium value of ω_k we will take oscillators frequencies from $\omega_{min} \sim 0$ to $\omega_{max} = \omega_{Planck}$,

therefore $\overline{\omega_k} = \omega_{max}/2$:

$$\sum E_{ground} = \frac{3L\hbar\omega_{max}{}^2}{8\pi c} \tag{21}$$

Now we calculate it in the limit L → R:

$$\sum E_{ground} = \frac{3R\hbar\omega_{max}{}^2}{8\pi c} \tag{22}$$

If $\omega_{Planck} = \omega_{max}$ using (11) we find:

$$\sum E_{ground} = \frac{3Rc^4}{8\pi G} \tag{23}$$

We expose another way to obtain the same approximation.

The range of oscillators frequencies is between $\omega_{min} \sim 0$ and $\omega_{max} = \omega_{Planck}$, therefore vacuum energy along R is:

$$\sum E_{ground} \sim \delta(0)\int dk\frac{3}{2}\hbar\omega_k \tag{24}$$

Where k are frequencies of a continuum of (wave) modes.

The infinite delta-function δ(0) can be regularized in a line of length L. "Line regularization" (enclosing energy in a line of length L) implies the following substitution:

$$\delta(0) = \frac{1}{2\pi}\int e^{ikx}dx \;\rightarrow \frac{L}{2\pi} \quad \text{if} \quad k \rightarrow 0 \tag{25}$$

This L length (in the limit L →R∾∞) represent the standard 'L - quantization' process. Where the wave vector k now refers to the so-called normal modes which are compatible with the boundary conditions provided by the line L. The right hand side of the equation follows from the left hand side in the limit L → ∞ where the energy does not depend on the 'line quantization':

$$\sum E_{ground} \sim \frac{3L\hbar}{4\pi c}\int_0^{\omega_{max}}\omega\, d\omega = \frac{3L\hbar\omega_{max}{}^2}{8\pi c} \tag{26}$$

If L →R and $\omega_{Planck} = \omega_{max}$ using (11) and (26) we find:

$$\sum E_{ground} = \frac{3Rc^4}{8\pi G} \tag{27}$$

If $\rho = 0$ and setting the normalized spatial curvature,k, equal to zero according to the ΛCDM model we obtain:

$$\rho_\Lambda = \frac{3{H_0}^2}{8\pi G} \qquad (28)$$

If we divide the space in a sum of lines where the number of lines is N_{lines} , E_{Vac} could be expressed as the product of energy of one line by N_{lines} .We need calculate N_{lines} to obtain ρ_{Vac}:

$$N_{lines} = \frac{V_{universe}}{V_{Parallelepiped}} \sim \frac{\frac{4\pi R^3}{3}}{\overline{L_{oscillator}}^2 R} = \frac{4\pi}{3}\left(\frac{R}{\overline{L_{oscillator}}}\right)^2 \quad (29)$$

Where $\overline{L_{oscillator}}$ is the medium length of one vacuum state, we approach a line to a parallelepiped, parallelepiped volume is $V_{Parallelepiped}$.

If we compare vacuum state sum along R with continuous one-dimensional wave equation, the number of ground state is represented by number of modes. We calculate medium length of one vacuum state using (19) and $\overline{\omega_k} = \omega_{Planck}/2$:

$$1 = \frac{2\bar{L}}{\bar{\lambda}} = \frac{\overline{\omega_k}\,\overline{L_{oscillator}}}{\pi c} \rightarrow \overline{L_{oscillator}} = \lambda_{Planck} \qquad (30)$$

Therefore using(29) and (30) we obtain:

$$N_{lines} = \frac{4\pi}{3}\left(\frac{c}{\lambda_{Planck} H_0}\right)^2 \sim \frac{4\pi}{3}\, 10^{123} \qquad (31)$$

We calculate ρ_{Vac} using(13), (27),and (31):

$$\rho_{Vac} = \frac{N_{lines} \sum E_{ground}}{V_{universe}} = \left(\frac{3{H_0}^2}{8\pi G}\right)\left(\frac{c}{\lambda_{Planck} H_0}\right)^2$$

$$\rho_{Vac} \sim \frac{3{H_0}^2}{8\pi G} 10^{123} \qquad (32)$$

Using (28) and (36) we find:

$$\frac{\rho_{Vac}}{\rho_\Lambda} = \frac{2GN_{lines} \sum E_{ground}}{c^4 R} = \left(\frac{c}{\lambda_{Planck} H_0}\right)^2 \sim 10^{123} \qquad (33)$$

We can observe the factor 10^{123} is produce by the difference between the observable universe radius and the medium length of one vacuum state.

References

Department of physics and astronomy Georgia State University

Earman, J. and Moster´ın, J.,1999, JGPS 66, 1-49.

Friedman, A. 1999. "On the Curvature of Space". General Relativity and Gravitation.

Zinkernagel H., Rugh S. E.,2000, preprint (arXiv:hep-th/0012253).