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Goldbach Conjecture Proved

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Abstract

The Goldbach Conjecture states that every even integer greater than 4 can be expressed as the sum of two odd primes. In this paper, the proof of Goldbach conjecture is guided by the approach for finding Goldbach partitions. This approach leads directly to evidence that every even integer greater than 4 is the sum of two odd primes. The main principle for finding Goldbach partitions from a known partition is the application of the addition axiom to a Goldbach partition equation. It is shown that given an equation for a Goldbach partition, one can produce a Goldbach partition for any even integer greater than 4. Beginning with the partition equation, 6 = 3 + 3, and applying the addition of a 2 to both sides of this equation, and subsequent equations, one obtained Goldbach partitions for over 180 consecutive even integers. A consequent generalized procedure also produced Goldbach partitions for the non-consecutive even integers, 100; 1000; 372,131,740; and 400,000,001,1000. An equation derived for the Goldbach partition shows that every even integer greater than 4 can be written as the sum of two odd prime integers.

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Preliminaries

Introduction

The following is from the first page of the author's high school practical physics note book: Science is the systematic observation of what happens in nature and the building up of body of laws and theories to describe the natural world. Scientific knowledge is being extended and applied to everyday life. The basis of this growing knowledge is experimental work. To prove Goldbach conjecture, one would be guided by the approach for finding Goldbach partitions.

Finding Goldbach Partitions

For communication purposes, let E_n be a positive even integer, and let P_r and P_s be two odd prime integers, where the subscript n is an even integer and the subscripts r and s are odd prime integers.; and $r + s = n$. Also ,$E_n = P_r + P_s$.
The main principle for finding a Goldbach partition for a consecutive even integer from a known partition is the application of the addition axiom which states that if equal quantities are added to equal quantities, the sums are equal. One will begin with the Goldbach partition, $6 = 3 + 3$, and apply the addition axiom to this equation to obtain partitions for larger even integers. In obtaining partitions for consecutive even integers, one will add 2 to both sides of the equation. For the next consecutive even integer, one will add a 2 to the present even integer, and one will also add a 2 to the right side of the equation. For the addition of a 2 to the right side of the equation, there are a number of possibilities.

Possibility 1 (desirable)**:** The addition of a 2 to any of the terms results in two prime integers.

Possibility 2: One will inspect the two terms, P_r and P_s to determine which term if not prime would become prime when 2 is added to it, and add to it accordingly.

Possibility 3: If on adding a 2, to say, P_r one obtains a composite integer, one may add another 2 or more 2's until one obtains a prime integer; but for compensation, one would have to subtract the extra 2 or the extra 2's (added) from the other term P_s, noting that one is adding a single 2 to the left side and a single 2 to the right side of the equation. For examples, if one repeats the addition of 2 once, one will subtract a single 2 from the other term, If one repeats the addition twice, one will subtract 4 from the other term, If one repeats the 2-addition a third time, one will subtract a 6 from the other term. With experience, one would be able to determine which term to add a 2 or 2;s to in order to avoid the extra 2-addition. For the difference in the subtraction, the resulting difference cannot be less than 3. The difference must be a prime number.
An important skill for finding Goldbach partitions is changing composite integer sums to prime integer sums. With respect to the addends in the following equations, change any composite integer to a prime integer without changing the sum of the addends. **1.** $70 = 31 + 39$;
2, $68 = 5 + 63$; **3.** $100 = 31 + 69$
Solution: For #1: 31 is prime but 39 is composite, To make 39 prime add 2 to 39 to obtain 41 which is prime, and perform the opposite operation on 31, that is, subtract 2 from 31 to obtain 29 which is also prime.
Solutions

Given: $70 = 31 + 39$	**Given**: $68 = 5 + 63$	**Given:** $100 = 31 + 69$
$70 = (31 - 2) + (39 + 2)$	$68 = (5 + 2) + (63 - 2))$	$100 = (31 - 2) + (69 + 2)$
$\boxed{70 = 29 + 41}$	$\boxed{68 = 7 + 61}$	$\boxed{100 = 29 + 71}$
(**Note**: both 29 and 41 are prime	**Note**: both 7 and 61 are prime	**Note**: both 29 and 71 are prime

Examples on Finding Goldbach Partitions

Example 1a.
Given: 3, 5, 7
Required: Show that each of the even integers, 6,8,10 is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution: $E_n = P_r + P_s,\ r + s = n$

1. $E_6 = P_3 + P_3;\ \ 6 = 3 + 3$
2. $E_8 = P_5 + P_3;\ \ 8 = 5 + 3$
3. $E_{10} = P_7 + P_3;\ 10 = 7 + 3$

Details

From 1 to 2:
On both sides: add 2 to 6 to obtain 8; add 2 to the first 3 to obtain 5 which is a prime; and keep the other 3 unchanged

From 2 to 3:
On both sides, add 2 to 8 to obtain 10; and add 2 to 5 to obtain 7 which is a prime number and keep the 3 unchanged

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Example 1b.
Given: $3,\ 5,\ 7,\ \underbrace{11,\ 13,\ 17,\ 19}_{\text{Added}}$.
Required: Show that each of the even integers, 12,14,16.18,20 is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution: ,

1. $E_{12} = P_7 + P_5;\ \ 12 = 7 + 5$
2. $E_{14} = P_{11} + P_3;\ \ 14 = 11 + 3$
3. $E_{16} = P_{13} + P_3;\ \ 16 = 13 + 3$
4. $E_{18} = P_{13} + P_5;\ \ 18 = 13 + 5$
5. $E_{20} = P_{17} + P_3;\ \ 20 = 17 + 3 = 13 + 7$

Details for 1b

From Ia to Ib:
Add 2 to the 3 and keep the 7.

From 1 to 2:
On the left side , add 2 to 12 to obtain 14, On the right side if one adds 2 to 7, one obtains 9 which is not prime and one adds another 2 to obtain 11, which is prime; and to compensate for the extra 2, one subtracts 2 from the 5 on the right side to obtain 3. Note above that one could also add the 2 to the 5 to obtain 7 to result in 14 = 7 +7

Details

From 2 to 3:
On the left side , add 2 to 14 to obtain 16, On the right side, add 2 to 11 to obtain 13, which is prime and desirable; one keeps the 3 unchanged.

From 3 to 4:
On the left side , add 2 to 16 to obtain 18, On the right side, keep the 13 (which is prime) and add 2 to the 3 to obtain 5 which is prime. **Note** if one added 2 to the 13, one would obtain 15, which is not prime; one would have to add another 2 to obtain 17 which is prime; and one would have to subtract a 2 from the 3 to obtain 1 which is not prime.

From 4 to 5
Two options:
Option 1: : On the left side , add 2 to 18 to obtain 20, On the right side if one adds 2 to 13, one obtains 15 which is not prime and one adds another 2 to obtain 17, which is prime; and to compensate for the extra 2, one subtracts 2 from the 5 on the right side to obtain 3.

Option 2: On the right side, keep the 13 and add 2 to the 5 to obtain 7, which is prime and desirable.. Option 2 is faster.

Example 2	Example 3
Given: $3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ \underbrace{23,\ 29}_{\text{added}}$	**Given**: $3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ \underbrace{31,\ 37}_{\text{added}}$
Required: Show that each of the even integers, $22,\ 24,\ 26,\ 28,\ 30.$ is either the sum of two of the above prime numbers or is twice any of the above prime numbers	**Required:** Show that each of the even integers, $32,\ 34,\ 36,\ 38,\ 40.$ is either the sum of two of the above prime numbers or is twice any of the above prime numbers
Solution: 1. $E_{22} = P_{19} + P_3;\quad 22 = 19 + 3$ 2. $E_{24} = P_{19} + P_5;\quad 24 = 19 + 5$ 3. $E_{26} = P_{19} + P_7;\quad 26 = 19 + 7$ 4. $E_{28} = P_{23} + P_5;\quad 28 = 23 + 5$ 5. $E_{30} = P_{23} + P_7;\quad 30 = 23 + 7$	**Solution:** 1. $E_{32} = P_{29} + P_3;\ 32 = 29 + 3$ 2. $E_{34} = P_{31} + P_3;\quad 34 = 31 + 3$ 3. $E_{36} = P_{31} + P_5;\quad 36 = 31 + 5$ 4. $E_{38} = P_{31} + P_7;\quad 38 = 31 + 7$ 5. $E_{40} = P_{37} + P_3;\quad 40 = 37 + 3$
From Ib to 2: Add 2 to 20; add 2 the 17 to obtain 19 and keep the 3. **From 1 to 2** Add 2 to 22,, keep the 19, and add 2 to 3 obtain 5. **From 2 to 3:** Add 2 to 24 to obtain 26;,, keep the 19, and add 2 to 5 obtain 7. **From 3 to 4:** On the left side , add 2 to 26 to obtain 28, On the right side, if one adds 2 to 19, one obtains 21 which is not prime; add another 2 to obtain 23, which is prime; to compensate for the extra 2, one subtracts 2 from the 7 on the right side to obtain 5 **From 4 to 5:** Add 2 to 28 to obtain 30;,, keep the 23, and add 2 to 5 obtain 7.	**From Ex 2 to Ex 3:** Add 2 to the 30 to obtain 32 ; add 6 (why?) to 23 to obtain 29; subtract the extra 4 added from 7 to obtain 3. **From 1 to 2** Add 2 to 32; add 2 to 29, keep the 3 **From 2 to 3:** Add 2 to 34; keep the 31, and add 2 to 3 obtain 5. **From 3 to 4:** Add 2 to 36; keep 31 & add 2 to 5. **From 4 to 5:** Add 2 to 38 to obtain 40. On the right side, add 6 to 31 to obtain 37; to compensate for the extra 4 added, subtract 4 from the 7 on the right side to obtain 3., **Note**: One skipped the composites 33 and 35.

Example 4

Given: $3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ \underbrace{41,\ 43,\ 47}_{\text{added}}$

Required: Show that each of the even integers $42,\ 44,\ 46,\ 48,\ 50.$ is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution:

1. $E_{42} = P_{37} + P_5;\quad 42 = 37 + 5$
2. $E_{44} = P_{41} + P_3;\quad 44 = 41 + 3$
3. $E_{46} = P_{43} + P_3;\quad 46 = 43 + 3$
4. $E_{48} = P_{43} + P_5;\quad 48 = 43 + 5$
5. $E_{50} = P_{47} + P_3;\quad 50 = 47 + 3$

Example 5

Given: $3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,\ 43,\ 47,\ \underbrace{53,\ 59}_{\text{added}}$

Required: Show that each of the even integers $52,\ 54,\ 56,\ 58,\ 60.$is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution:

1. $E_{52} = P_{47} + P_5;\quad 52 = 47 + 5$
2. $E_{54} = P_{47} + P_7;\quad 54 = 47 + 7$
3. $E_{56} = P_{53} + P_3;\quad 56 = 53 + 3$
4. $E_{58} = P_{53} + P_5;\quad 58 = 53 + 5$
5. $E_{60} = P_{53} + P_7;\quad 60 = 53 + 7$

Example 6

Given: $3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,\ 43,\ 47,\ 53,\ 59,\ \underbrace{61,\ 67}_{\text{added}}$

Required: Show that each of the even integers $62,\ 64,\ 66,\ 68,\ 70.$is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution:

1. $E_{62} = P_{59} + P_3;\quad 62 = 59 + 3$
2. $E_{64} = P_{61} + P_3;\quad 64 = 61 + 3$
3. $E_{66} = P_{61} + P_5;\quad 66 = 61 + 5$
4. $E_{68} = P_{61} + P_7;\quad 68 = 61 + 7$
5. $E_{70} = P_{67} + P_3;\quad 70 = 67 + 3$

Example 7

Given: $3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,\ 43,\ 47,\ 53,\ 59,\ 61,\ 67,\ \underbrace{71,\ 73,\ 79}_{\text{added}}$

Required: Show that each of the even integers $72,\ 74,\ 76,\ 78,\ 80.$is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution:

1. $E_{72} = P_{67} + P_5;\quad 72 = 67 + 5$
2. $E_{74} = P_{71} + P_3;\quad 74 = 71 + 3$
3. $E_{76} = P_{73} + P_3;\quad 76 = 73 + 3$
4. $E_{78} = P_{73} + P_5;\quad 78 = 73 + 5$
5. $E_{80} = P_{73} + P_7;\quad 80 = 73 + 7$

Example 8 **Given**: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71,73,79 $\underbrace{83, 89}_{\text{added}}$ **Required:** Show that each of the even integers 82, 84, 86, 88, 90. is either the sum of two of the above prime numbers or is twice any of the above prime numbers **Solution:** 1. $E_{82} = P_{79} + P_3$; $82 = 79 + 3$ 2. $E_{84} = P_{79} + P_5$; $84 = 79 + 5$ 3. $E_{86} = P_{83} + P_3$; $86 = 83 + 3$ 4. $E_{88} = P_{83} + P_5$; $88 = 83 + 5$ 5. $E_{90} = P_{83} + P_7$; $90 = 83 + 7$.	**Example 9** **Given**: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59,61,67,71,73,79, 83, 89, $\underbrace{97}_{\text{added}}$ **Required:** Show that each of the even integers 92, 94, 96, 98, 100.is either the sum of two of the above prime numbers or is twice any of the above prime numbers **Solution:** **1.** $E_{92} = P_{89} + P_3$; $92 = 89 + 3$ 2. $E_{94} = P_{89} + P_5$; $94 = 89 + 5$ 3. $E_{96} = P_{89} + P_7$; $96 = 89 + 7$ 4. $E_{98} = P_{79} + P_{19}$; $98 = 79 + 19$ 5. $E_{100} = P_{97} + P_3$; $100 = 97 + 3$
Example 10 **Given**: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59,61,67,71,73,79, 83, 89, 97, $\underbrace{101, 103, 107, 109}_{\text{added}}$ **Required:** Show that each of the even integers 102,104,106,108,110.is either the sum of two of the above prime numbers or is twice any of the above prime numbers **Solution:** **1.** $E_{102} = P_{97} + P_5$; $102 = 97 + 5$ 2. $E_{104} = P_{101} + P_3$; $104 = 101 + 3$ 3. $E_{106} = P_{103} + P_3$; $106 = 103 + 3$ 4. $E_{108} = P_{103} + P_5$; $108 = 103 + 5$ 5. $E_{110} = P_{103} + P_7$; $110 = 103 + 7$	**Example 11** **Given**: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59 ,61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, $\underbrace{113}_{\text{added}}$ **Required:** Show that each of the even integers112,114,116,118,120.is either the sum of two of the above prime numbers or is twice any of the above prime numbers **Solution:** **1.** $E_{112} = P_{109} + P_3$; $112 = 109 + 3$ 2. $E_{114} = P_{109} + P_5$; $114 = 109 + 5$ 3. $E_{116} = P_{113} + P_3$; $116 = 113 + 3$ 4. $E_{118} = P_{113} + P_5$; $118 = 113 + 5$ 5. $E_{120} = P_{113} + P_7$; $120 = 113 + 7$

Example 12

Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, $\underbrace{127}_{\text{added}}$

Required: Show that each of the even integers 122,124,126,128,130.is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution:

1. $E_{122} = P_{109} + P_{13}$; $122 = 109 + 13$

2. $E_{124} = P_{113} + P_{11}$; $124 = 113 + 11$

3. $E_{126} = P_{113} + P_{13}$; $126 = 113 + 13$

4. $E_{128} = P_{109} + P_{19}$; $128 = 109 + 19$

5. $E_{130} = P_{127} + P_{3}$; $130 = 127 + 3$

Example 13

Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, 127, $\underbrace{131,\ 137,\ 139}_{\text{added}}$

Required: Show that each of the even integers132,134,136,138,140. is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution:

1. $E_{132} = P_{127} + P_{5}$; $132 = 127 + 5$

2. $E_{134} = P_{131} + P_{3}$; $134 = 131 + 3$

3. $E_{136} = P_{131} + P_{5}$; $136 = 131 + 5$

4. $E_{138} = P_{131} + P_{7}$; $138 = 131 + 7$

5. $E_{140} = P_{137} + P_{3}$; $140 = 137 + 3$

Example 14

Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, $\underbrace{149}_{\text{added}}$

Required: Show that each of the even integers 142, 144, 146, 148, 150. is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution:

1. $E_{142} = P_{139} + P_{3}$; $142 = 139 + 3$

2. $E_{144} = P_{139} + P_{5}$; $144 = 139 + 5$

3. $E_{146} = P_{139} + P_{7}$; $146 = 139 + 7$

4. $E_{148} = P_{137} + P_{11}$; $148 = 137 + 11$

5. $E_{150} = P_{139} + P_{11}$; $150 = 139 + 11$

Example 15

Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, $\underbrace{151,\ 157}_{\text{added}}$

Required: Show that each of the even integers152, 154, 156, 158, 160.is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution:

1. $E_{152} = P_{149} + P_{3}$; $152 = 149 + 3$

2. $E_{154} = P_{151} + P_{3}$; $154 = 151 + 3$

3. $E_{156} = P_{151} + P_{5}$; $156 = 151 + 5$

4. $E_{158} = P_{151} + P_{7}$; $158 = 151 + 7$

5. $E_{160} = P_{157} + P_{3}$; $160 = 157 + 3$

Example 16 **Given**: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157 $\underbrace{163, 167}_{\text{added}}$ **Required:** Show that each of the even integers 162, 164, 166, 168, 170. is either the sum of two of the above prime numbers or is twice any of the above prime numbers **Solution:** **1.** $E_{162} = P_{157} + P_5$; $162 = 157 + 5$ 2. $E_{164} = P_{157} + P_7$; $164 = 157 + 7$ 3. $E_{166} = P_{163} + P_3$; $166 = 163 + 3$ 4. $E_{168} = P_{163} + P_5$; $168 = 163 + 5$ 5. $E_{170} = P_{167} + P_3$; $170 = 167 + 3$	**Example 17** **Given**: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, $\underbrace{173, 179}_{\text{added}}$ **Required:** Show that each of the even integers 172, 174, 176, 178, 180. is either the sum of two of the above prime numbers or is twice any of the above prime numbers **Solution:** **1.** $E_{172} = P_{167} + P_5$; $172 = 167 + 5$ 2. $E_{174} = P_{167} + P_7$; $174 = 167 + 7$ 3. $E_{176} = P_{173} + P_3$; $176 = 173 + 3$ 4. $E_{178} = P_{173} + P_5$; $178 = 173 + 5$ 5. $E_{180} = P_{173} + P_7$; $180 = 173 + 7$

Example 18	**Example Extra 1:T 1,000**
Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, $\underbrace{181}_{\text{added}}$ **Required:** Show that each of the even integers 182, 184, 186, 188, 190. is either the sum of two of the above prime numbers or is twice any of the above prime numbers **Solution:** **1.** $E_{182} = P_{179} + P_3;\ 182 = 179 + 3$ 2. $E_{184} = P_{181} + P_3;\ 184 = 181 + 3$ 3. $E_{186} = P_{181} + P_5;\ 186 = 181 + 5$ 4. $E_{188} = P_{181} + P_7;\ 188 = 181 + 7$ 5. $E_{190} = P_{179} + P_{11};\ 190 = 179 + 11$	Here, one will skip the even integers 192-940, and jump to the even integers 992, 994, 996, 998, 1000. **Given**: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179,181......... $\underbrace{977, 983, 991, 997}_{\text{added}}$ **Required:** Show that each of the even integers 992, 994, 996, 998, 1000. is either the sum of two of the above prime numbers or is twice any of the above prime numbers **Solution:** **1.** $E_{992} = P_{919} + P_{73};\ 992 = 919 + 73$ **2.** $E_{994} = P_{991} + P_3;\ 994 = 991 + 3$ **3.** $E_{996} = P_{991} + P_5;\ 996 = 991 + 5$ **4.** $E_{998} = P_{991} + P_7;\ 998 = 991 + 7$ **5.** $E_{1000} = P_{997} + P_3;\ 1000 = 997 + 3$ Observe also that we used the first odd prime number to change 997 to 1000,

Example Extra 2: To 1,000,000
Here, one will skip the even integers 1,002-999990, and jump to the even integers 999992, 999994, 999996, 999998,1,000,000.

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59, 61, 67,71,73,79, 83,
89, 97, 101, 103, 107, 109,
113, 127, 131, 137, 139, 149, 151,
157, 163, 167, 173, 179,181...
977, 983, 991,997...

$$\underbrace{999931\ 999953,\ 999959,\ 999961,999983}_{\text{added}}$$

Required: Show that each of the even integers 999992, 999994, 999996, 999998,1,000,000.,is either the sum of two of the above prime numbers or is twice any of the above prime numbers

Solution:.

1. $E_{999,992} = P_{999,979} + P_{13};\ \ 999{,}992 = 999{,}979 + 13$
2. $E_{999,994} = P_{999,983} + P_{11};\ \ 999{,}994 = 999{,}983 + 11$
3. $E_{999,996} = P_{999,983} + P_{13};\ \ 999{,}996 = 999{,}983 + 13$
4. $E_{999,998} = P_{999,979} + P_{19};\ \ 999{,}998 = 999{,}979 + 19$
5. $E_{1,000,000} = P_{999,983} + P_{17};\ \ 1{,}000{,}000 = 999{,}983 + 17$

Observe that we used the sixth odd prime number, 17, to change 999,983 to 1000,000.

Example Extra 3a :To 399,999,842
Here, one will skip the even integers..........and
jump to the even integers 399,999,834; 399,999,834; 399,999,838;
399,999,840; 399,999,842

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59, 61, 67,71,73,79, 83,
89, 97, 101, 103, 107, 109,
113, 127, 131, 137, 139, 149, 151,
157, 163, 167, 173, 179,181...
977, 983, 991,997...999931 999953,
999959, 999961,999983,

$$\underbrace{399{,}999{,}781;\ 399{,}999{,}823;\ 399{,}999{,}827;\ 399{,}999{,}829, 399{,}999{,}839}_{\text{added}}$$

Required: Show that each of the
even integers 399,999,834; 399,999,836; 399,999,838; 399,999,840; 399,999,842
is either the sum of two of the above
prime numbers or is
twice any of the above prime numbers
Solution:

1. $E_{399,999,834} = P_{399,999,829} + P_5$; 399,999,834 = 399,999,829 + 5
2. $E_{399,999,836} = P_{399,999,829} + P_7$; 399,999,836 = 399,999,829 + 7
3. $E_{399,999,838} = P_{399,999,827} + P_{11}$; 399,999,838 = 399,999,827 + 11
4. $E_{399,999,840} = P_{399,999,829} + P_{11}$; 399,999,840 = 399,999,829 + 11
5. $E_{399,999,842} = P_{399,999,839} + P_3$; 399,999,842 = 399,999,839 + 3

Observe that we used the first odd prime number, 3,
to change 399,999,839 to 399,999,842

Extra 3b

1. $E_{3,399,999,834} = P_{3,399,999,797} + P_{37}$; 3,399,999,834 = 399,999,797 + 37
2. $E_{3,399,999,836} = P_{3,399,999,763} + P_{73}$; 3,399,999,836 = 3,399,999,763 + 73
3. $E_{3,399,999,838} = P_{3,399,999,797} + P_{41}$; 3,399,999,838 = 3,399,999,797 + 41
4. $E_{3,399,999,840} = P_{3,399,999,797} + P_{43}$; 3,399,999,840 = 3,399,999,797 + 43
5. $E_{3,399,999,842} = P_{399,999,763} + P_{79}$; 3,399,999,842 = 3,399,999,763 + 79

5. $E_{3,399,999,842} = P_{399,999,763} + P_{79}$; 3,399,999,842 = 3,399,999,763 + 79

6. $E_{500,000,000,246} = P_{500,000,000,243} + P_3$; 500,000,000,246 = 500,000,000,243 + 3
7. $E_{400,000,001,000} = P_{400,000,000,997} + P_3$; 400,000,001,000 = 400,000,000,997 + 3
8. $E_{372,131,740} = P_{372,131,737} + P_3$; 372,131,740 = 372,131,737 + 3

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Condensed Goldbach Partition Production
Consecutive Descendants

The following table condenses the processes involved in examples on page 4-12, and will show that every even integer greater than 4 can be expressed as the sum of two odd primes, because the process will not terminate and can continue indefinitely. Review the instructions on page 3, and review the examples on page 4-12. One will begin with the partition equation $6 = 3 + 3$, and apply the addition of 2 to both sides of the equation to produce the partition for the next even number, 8. From the partition equation, $8 = 5 + 3$, one will repeat the 2-addition process to obtain the partition for the next even integer, 10. From the partition for 10, the process will continue indefinitely and every even integer would be partitioned as the sum of two odd primes.

Basis: $,n = 2k = 6,\ k = 3$ From $6 = 3 + 3$

1. $6 = 3 + 3$	**26.** $56 = 53 + 3 = 37 + 19$
2. $8 = 5 + 3$	**27.** $58 = 53 + 5 = 41 + 17$
3. $10 = 7 + 3$	**28.** $60 = 53 + 7 = 43 + 17 = 41 + 19$
4. $12 = 7 + 5$	**29.** $62 = 59 + 3 = 43 + 19$
5. $14 = 7 + 7 = 11 + 3$	**30.** $64 = 59 + 5 = 47 + 17$
6. $16 = 11 + 5 = 13 + 3$	**31.** $66 = 59 + 7 = 47 + 19$
7. $18 = 13 + 5$	**32.** $68 = 61 + 7$
8. $20 = 13 + 7$	**33.** $70 = 67 + 3 = 59 + 11$
9. $22 = 17 + 5 = 11 + 11$	**34.** $72 = 67 + 5 = 59 + 13$
10. $24 = 19 + 5 = 13 + 11$	**35.** $74 = 71 + 3 = 61 + 13 = 67 + 7$
11. $26 = 19 + 7 = 23 + 3 = 13 + 13$	**36.** $76 = 71 + 5 = 59 + 17$
12. $28 = 23 + 5 = 17 + 11$	**37.** $78 = 71 + 7 = 61 + 17$
13. $30 = 23 + 7 = 17 + 13$	**38.** $80 = 73 + 7 = 61 + 19 =$
14. $32 = 29 + 3 = 21 + 11$	**39.** $82 = 79 + 3 = 59 + 23$
15. $34 = 31 + 3 = 29 + 5$	**40.** $84 = 79 + 5 = 61 + 23$
16. $36 = 31 + 5 = 29 + 7$	**41.** $86 = 79 + 7$
17. $38 = 31 + 7$	**42.** $88 = 83 + 5$
18. $40 = 37 + 3 = 29 + 11$	**43.** $90 = 83 + 7$
19. $42 = 37 + 5 = 29 + 13$	**44.** $92 = 89 + 3$
20. $44 = 37 + 7 = 31 + 13$	**45.** $94 = 89 + 5$
21. $46 = 41 + 5 = 29 + 17$	**46.** $96 = 89 + 7$
22. $48 = 43 + 5 = 29 + 19$	**47.** $98 = 79 + 19$
23. $50 = 43 + 7 = 31 + 19$	**48.** $100 = 83 + 17$
24. $52 = 47 + 5 = 29 + 23$	**49.** $102 = 83 + 19$
25. $54 = 47 + 7 = 31 + 23$	**50.** $104 = 97 + 7$

Above, beginning with the partition equation , $6 = 3 + 3$, the partition equations #2 to #50 were produced. Also, knowing partition #5, above, one can also obtain partition #39 and vice versa. In the table, there are fifty Goldbach partitions. As shown below, by skipping partitions for $8–82$, above, one can also obtain partitions for 84**, 1**00 as well as partitions for 1000, $372{,}131{,}740$, and $400{,}000{,}001{,}000$,

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Finding partitions for 84, 100, 1000, 372,131,740, and 400,000,001,000 and others

1. From 6 to 84

$6 = 3 + 3$

$6 + (84 - 6) = 3 + (3 + 84 - 6)$

$6 + (78) = 3 + (3 + 78)$

$84 = 3 + 81$ (Note : 81 is not prime)

$84 = (3 + 2) + (81 - 2)$

$\mathbf{84 = 5 + 79}$ or $\mathbf{79 + 5}$. Note: 79 is prime

(---Note above that if one adds 2 to 81, one obtains 83 which is prime, but on subtracting 2 from 3, one would get 1, which is not prime)

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2. From 6 to 100

$6 = 3 + 3$

$6 + (100 - 6) = 3 + (3 + 100 - 6)$

$6 + (94) = 3 + (3 + 94)$

$\mathbf{100 = 3 + 97}$ **Note** : 97 is prime.

3a. From 6 to 1000

$6 = 3 + 3$

$6 + (1000 - 6) = 3 + (3 + (1000 - 6)$

$6 + 994 = 3 + (3 + 994)$

$1000 = 3 + (3 + 994)$

$\mathbf{1000 = 3 + 997}$ Note : 997 is prime

3b From 84 to 6

$84 = 5 + 79$

$84 + (6 - 84) = 5 + (79 + (6 - 84))$

$84 - 78 = 5 + 1$ Note : 1 is not prime

$\mathbf{6 = 3 + 3}$ (Add 2 to 1 and subtract 2 from 5)

3c From 92 to 6

$92 = 3 + 89$

$92 + (6 - 92) = 3 + (89 + 6 - 92)$

$92 - 86 = 3 + 89 - 86$

$\mathbf{6 = 3 + 3}$

4. From 6 to 372,131,740

4.

Three hundred seventy-two million,one hundred thirty-one thousand, seven hundred,forty

$6 + (372{,}131{,}740 - 6) = 3 + (3 + (372{,}131{,}740 - 6)$

$6 + (372{,}131{,}734) = 3 + (3 + (372{,}131{,}734)$

$\mathbf{372{,}131{,}740 = 3 + 372{,}131{,}737}$

Note : 372,131,737 is prime

5. From 6 to 400,000,001,000

5. Four hundred billion, one thousand

$6 = 3 + 3$

$6 + (400{,}000{,}001{,}000 - 6) = 3 + (3 + 400{,}000{,}001{,}000 - 6)$

$6 + (400{,}000{,}000{,}994) = 3 + (3 + (400{,}000{,}000{,}994)$

$\mathbf{400{,}000{,}001{,}000 = 3 + 400{,}000{,}000{,}997}$

Note : 400,000,000,997 is prime

Goldbach Conjecture Proved

Given: **1**. A known Goldbach partition of the even integer $E_1 = P_1 + P_2$, where P_1 and P_2 are prime odd integers. **2**. The even integer, E_2, whose partition is to be determined..

Required: To show that the even integer E_2 has a Goldbach partition ,

Plan: Let $E_2 = P_r + P_s$, where P_r and P_s are prime odd integers. The proof would be complete after finding a formula for E_2 which can be used to find a Goldbach partition for every even integer.

Proof

Statements	Reasons
1. E_1 is an even integer	**1** ..Given
2. P_1 and P_2 are prime odd integers	**2.** Given
3. $E_1 = P_1 + P_2$	**3.** .Given
4. $E_1 + (E_2 - E_1) = P_1 + (P_2 + (E_2 - E_1))$	**4.** Addition axiom (Equals added to equals)
5. $E_2 = P_1 + (P_2 + E_2 - E_1)$	**5,** Simplifying the left-side
6. In statement 5, above, P_1 is an odd prime integer	**6**. Given
7. Case A: $(P_2 + E_2 - E_1)$ is prime (desirable), $P_r = P_1$ and $P_s = (P_2 + E_2 - E_1)$. and the proof is complete;	
Case B: $(P_2 + E_2 - E_1)$ is **not** prime. The addition or subtraction of a 2 or 2 's would make $(P_2 + E_2 - E_1)$ become prime, However, the 2 or 2's added or subtracted must be subtracted from or added to P_1.	**7.** There are infinitely many primes and the addition or subtraction would ultimately make $(P_2 + E_2 - E_1)$ become prime.
Case B1: $P_1 = 3$; $p_s = (P_2 + E_2 - E_1) - 2t$; $p_r = 3 + 2t$, where t is the number of times 2 is subtracted before primality occurs.	
.Case B2: $P_1 > 3$; $P_s = (P_2 + E_2 - E_1) \pm 2t$ and $P_r = P_1 \pm 2t$	

After the inevitable successful changes in P_1 and $(P_2 + E_2 - E_1)$, the even number E_2 would have been expressed as the sum of two primes, and the proof would be complete.

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Discussion

For examples on Case A of the proof, see Example 2, 3a, 4 and 5 on page 14. For Case B1, see Example 1, p.14. For Case B2, see Example 3, page 5 ("From 4 to 5"). For more examples on the above cases, see pages 3-12

Interestingly, one can also obtain the partition equation $6 = 3 + 3$ from any other partition equation, as in examples 3b and 3c, p.14 . Such a result is very convincing that every even integer can be written as the sum of two odd primes. Thus, given a Goldbach partition of an even integer, by hand, one can find, without exception, a Goldbach partition for any other even integer, within minutes.

Conclusion

It was shown in this paper that every even integer greater than 4 can be expressed as the sum of two odd primes. The proof was guided by the approach the author used in finding Goldbach partitions. The approach for finding the partitions led directly to evidence that every even integer greater than 4 is the sum of two odd primes. The main principle for finding Goldbach partitions from a known partition is the application of the addition axiom to a Goldbach partition equation. It was shown that given an equation for a Goldbach partition, one can produce a Goldbach partition for any even integer. Beginning with the partition equation, $6 = 3 + 3$, and applying the addition of a 2 to both sides of this equation sequentially and repeatedly, one obtained Goldbach partitions for over 180 consecutive even integers. Also, a derived formula was used successfully to find partitions for the non-consecutive even integers, 100; 1000; 372,131,740; and 400,000,001,1000.

It is concluded that, knowing a single Goldbach partition equation for an even number, one can by hand, find a Goldbach partition quickly for any other even integer.

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References

1. The approach used in covering Goldbach conjecture in this paper is similar to the approach the author used in proving Beal Conjecture (vixra: 2001.0694).

2. In covering the Goldbach conjecture, one must have quick access to the list of the prime numbers, Some places on the web for the lists of prime numbers are at: 1.www.mathsfun.com; 2. CalculatorSoup.com.

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