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In Search of the fourth dimension of space

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ABSTRACT

This is a scientific speculation that presents an unusual, but at the same time intuitive, idea of a cosmological model.

The mathematical construction foresees a universe of finite dimensions, in expansion, homogeneous and isotropic. Here all the points are equivalent and from each one there are no preferential directions. Gravity is needed to clarify some aspects of the model but is treated only at a qualitative level.

An explanation is given to Galactic recession and the Lorentz transformations but math used is fairly simple. Definitions, data and formulas used can be easily found on the internet. To complete the whole, there is also a simple verification based on astronomical observations.

In this geometry, expansion mechanism infers Special relativity. The next step would be to add gravity, building a physical model with matter and energy.

WHY THIS SEARCH

From the theory of Big Bang we consider universe what is occupied by matter and not an empty container of infinite dimensions in which matter expands. There are various cosmological models that respect both the laws of physics and the astronomical observations. Starting from a new point of view, this speculation concerns the analysis of a new different one.

Simply to accept a scientific evidence or to investigate it, depends on what it is about and on the way of thinking of each of us. What made me think is superluminal motion, possible in galactic recession. With the latter and relative motion in mind, I looked for a geometry from which both the principle of relativity and the recession mechanism arise together.

This was the target but, about real motivations, I am convinced that with another dimension it is explained isotropy and homogeneity. With this idea as a new starting point, I looked then for a model in which the metric of the universe is not what appears to us but it is only the result of our perception of a four dimensional space.

These are the reasons that led me to this search.

By accepting the idea of a fourth dimension, we would consider what it is entails, for a three-dimensional observer, studying a four-dimensional universe. Eventually we should change, in agreement, the laws of our physics, applying the older, as we usually do, only as approximations.

The way chosen to set the problem is very rudimentary.

For simplicity, we can think to an observer who can move in one dimension, only along the circumference of a circle: that is the universe he perceives. Now imagine that the circle gets larger over time: for that universe, the present is on the circumference, the future outside it and the past inside. An arc belonging to the past is longer when measured in the present.

Let's apply this idea to our universe, so that it lies on the surface of a hypersphere whose radius continues to stretch. We cannot observe recent galaxies if these are far away, as their rays of light haven't reached us yet. We can instead observe images of the older ones that, born closer to the center, now lay on the surface. Speeds higher than light are possible but here nothing is moving: is the hypersphere inflating.

By analogy with the surface of a sphere, all points in this hypersphere's surface are equivalent and from each one there are no preferential directions. This geometry gives a space homogeneous and isotropic.

In this hypothesis no changes are needed, all our physics can be applied locally in the whole universe even if the whole universe moves, expanding over time.

We cannot directly observe the fourth dimension of space, e.g. the radius in this geometry, simply because it does not belong to the universe.

VELOCITY–DISTANCE RELATION AMONG GALAXIES

To introduce the discussion of the hypersphere is useful to consider an intuitive solution in 2d or 3d spaces. Here is immediate the linear relation between recessional velocity and distance, as from of the Hubble's law:

$s = r\Theta \quad v_r = \Theta dr/dt$ where s is the distance of the galaxy and Θ is constant over time

For a 4-sphere in polar coordinates it holds: [1]

$x_1 = r \cos(\Theta_1)$

$x_2 = r \sin(\Theta_1) \cos(\Theta_2)$

$x_3 = r \sin(\Theta_1) \sin(\Theta_2) \cos(\Theta_3)$

$x_4 = r \sin(\Theta_1) \sin(\Theta_2) \sin(\Theta_3) \sin(\Theta_4)$

We have $x_{Oi}(\Theta_i)$ for the galaxy of the observer and $x_{Fi}(\varphi_i)$ for the faraway galaxy. If we make a couple of axis rotations to set $\Theta_1 = \varphi_2 = 0$ what remains ($with\ \phi_1 = \varphi_1 - \Theta_1$) is:

x_{O1} = r

x_{O2} = 0

x_{F1}= r cos (ϕ_1)

x_{F2} = r sin (ϕ_1)

which brings us back, as we might expected (even if it was not obvious), to the case of the arc in a 2d circle.

To travel the arc to us, from a faraway galaxy, the ray of light started from a distant past. As we will see, the redshift refers to that remote instant but the recession velocity was the same than now.

GALACTIC RECESSION

Distances increase with the passage of time but, apart from the galactic recession, we do not measure other appreciable differences in lengths. Through the Hubble constant we can measure a stretch of $7.35 * 10^{-8}\ m\ year^{-1}$ on $10^6\ km$: the effect is not negligible. It is reasonable then to assume that gravity, within its action range, effectively counteract the expansion to the point of canceling its effect. In a binary system in equilibrium, the two stars, while moving away from us, should maintain the same distance between them. Recession due to expansion is in no way counteracted in the large zones of intergalactic vacuum.

Now we consider the radius and we put $r = v_r\, t = ct$, where we assume $v_r\,(t) = const$ and name the constant c. Next step is trying to assign a value to this constant:

velocity c = speed of light in vacuum.

The 4-sphere's geometry, then, suggest a linear relation between the arc angle Θ and the galactic recession, in this way constant over time ($ds/dt = c\Theta$). Otherwise, in Hubble's recession $v(z) = Hl$, the redshift z is proportional with distance l (the arc length not the angle)

$$z \propto l$$

However, for the Hubble constant *H,* measurement sampling, obtained with the Hubble Space Telescope HST, is based on stars (Cepheids) within 20 Megaparsec from us. For those relatively small distances we can use the Doppler redshift to obtain the present proper distance.

If now we consider the relation (we assumed $v_r\,(t) = const,\quad \partial z/\partial t = 0$ during period concerned)

$$z \propto \Theta$$

the Hubble's law is preserved unchanged and gives a recession velocity constant over time. The calculated recession velocity from Doppler redshift (its radial component from special relativity formula) at the time the ray of light started is the same as now and not needs any correction due to expansion.

The fact that a galaxy moves away at superluminal speed should not suggest that we can observe it: its rays of light will never reach us. That galaxy is an object in the elsewhere zone, as it always has been from the distant past: in this geometry nothing crosses the relativistic light cone.

THE LORENTZ TRANSFORMATIONS

In this context, the space we know is a frame of reference, consisting of three Cartesian axes, always tangent to the expanding 4-sphere. An exact solution seems to be impractical due to its extreme complexity. However, if we neglect the effects of curvature but have the foresight to consider the effect due to expansion, the error is negligible at least for regions of space close to us.

Now we look at the geometry: everything is bound to a 3d-surface in which geodesics are 4-sphere's arcs.

A ray of light always travels the shortest path along a circumference arc $s = r\Theta$ at the maximum possible speed $ds/dt = c$. Tangential speeds greater than the radial velocity c are not possible because in this geometry this would entail abandoning the surface, falling inside the 4-sphere and entering the past universe.

To apply Special relativity we must verify the Lorenz transformations. In our case the simplest and most straightforward method is to remember that the latter were obtained to satisfy

$$\alpha(v + c) = c$$

<u>But, in the 4-sphere, this is the condition on the tangential velocity not to reverse the arrow of time!</u>

What we have achieved with this geometry is a universe where the laws of Special relativity are deduced and never violated. This also applies when we expect the presence of superluminal motion for some farthest object. We can foresee but not observe it.

EVIDENCES FROM OBSERVATIONS

The time has come to do a simple check (*Mpc* stays for Megaparsec, *ly* for light years).

The assumption is that $r = ct$ where c is light speed in vacuum:

1 Mpc = $3.09 * 10^{19}\ Km$

Time elapsed from Big Bang = $1.38 * 10^{10}\ years = 4.35 * 10^{17} s$ [2]

Light velocity c = $3 * 10^{8}\ m\ s^{-1} = 3.17 * 10^{-8}\ ly\ s^{-1}$

we have:

$r = 4.23 * 10^{3}\ Mpc$ (c * Time elapsed from Big Bang)

$\Theta_{1\,Mpc} = 1\ Mpc\ /\ r = 2.36 * 10^{-4}\ rad$

4-sphere's recessional velocity $H_{sphere} = c\Theta_{1\,Mpc} = 70{,}9\ Km\ s^{-1}\ (per\ \Theta_{1\,Mpc})$

Hubble's recessional velocity $H = 72\ Km\ s^{-1}\ Mpc^{-1}$

Even if rough, 4-sphere recessional velocity H_{sphere} seems a quite good result.

APPLYING 4-SPHERE'S FORMULAS TO GALACTIC RECESSION

Calculating 4-sphere recession velocity from the radial relativistic Doppler's redshift (for a Minkowski space) we have: [3]

$1 + z = (1 + \beta)^{1/2}(1 - \beta)^{-1/2}\ where\ \beta = v/c$ and $\beta = ((1 + z)^2 - 1)/((1 + z)^2 + 1)$.

Applying 4-sphere's formulas to the farthest known galaxy GN-z11: [4]

Spectroscopic redshift $z = 11.09$

Calculated $\beta = 0.986$

Calculated $\Theta = v/c = \beta = 0.986\ rad$

Distance $r\Theta = 4.17 * 10^3\ Mpc$

The present proper distance of $4.17 * 10^3\ Mpc$ against a distance of our antipodal point ($\Theta = \pi$) of $1.33 * 10^4\ Mpc$ seems good. A recessional velocity < c and an arc $\Theta < 1\ rad$ are proper of an object in the observable zone. This passes the test too.

To roughly test the age of a galaxy (getting a time between its birth and dead) we can use the time spent by the light ray to travel the arc Θ

$\Delta r = \Delta s = c\Theta t_0$ — t_0 is the time the ray started

$\Delta t = \Delta r/c = t_1 - t_0 = \Theta t_0$ — Δt is the travel time and t_1 is now

We have

$t_0 = t_1/(1 + \Theta)$

For GN-z11 $t_0 = 7 * 10^9\ years$

The last result is too rough, maybe useless but not wrong. All the above results seem consistent. A birth around 200 to 900 million years after the Big Bang and a lifespan not less than 6-7 billion years is acceptable for a very old galaxy. [5]

Note that, also if we were able to perform astronomical observations at even greater distances, finding galaxies even further away, we should be not able to find GN-z11 (with a different recessional velocity) by looking in the exactly opposite direction.

Assuming as valid the Hubble's law even for GN-z11 in the opposite direction:

$\Theta = 2\pi - 0.97 = 5.31\, rad$

Distance = $2.25 * 10^4\ Mpc$

Hypothesized recessional speed = $1.59 * 10^6\ Km\, s^{-1}$

The resulting $\Theta > 1$ with its corresponding speed > c puts the galaxy in the relativistic elsewhere zone, out of our possible observations. No galaxy can be observed in either direction.

References from Wikipedia:

[1] - https://en.wikipedia.org/wiki/N-sphere

[2] - https://en.wikipedia.org/wiki/Big_Bang

[3] - https://en.wikipedia.org/wiki/Redshift

[4] - https://en.wikipedia.org/wiki/GN-z11

[5] - https://en.wikipedia.org/wiki/Chronology_of_the_universe