Abstract:

A specific recursive algorithm and three formulas have been used to generate integer sequences from prime and nonprime numbers seeds. After a few generations, some growing structures have been identified in these integer sequences, whereas such structures were absent when a subset of natural numbers was used as an alternative seed. The sum of the reciprocals of primes of these integer sequences, well fitted by models of the form $a \cdot \ln(\ln(n)) + b$, were calculated. Their distances to that of the harmonic series summed only over the primes were estimated and compared to the Meissel-Mertens constant. Finally, the algorithm used with one of the three formulas led after a few iterations to the production of long primefree sequences containing large numbers and allowed to establish a so called primefree sequences conjecture.

Key Words: prime and nonprime numbers seeds, stepwise-algorithm and formulas, integer sequences structures, sum of the reciprocals of primes, $a \cdot \ln(\ln(n)) + b$ models, primefree sequences conjecture.

1- IT Tools and VBA program:
   - PC: AMD (tm) XP 2800+
     2.08 GHz. RAM: 1.00Go.
   - a VBA program has been developed for sequence calculation.
2- Recursive algorithm and formulas:

2-1 Recursive algorithm:

*Note: a complete list of abbreviations is given in annex n° 1.*

The algorithm starts with a subset of the prime numbers set containing the first 1000 primes \((2, 3, 5, 7, 11, 13, 17...7919)\) used as a seed referenced \(S_{0P}\) to produce with a formula a first sequence \(S_1\) which is then used as a new seed to produce with the same formula the next sequence \(S_2\) and so one... A larger subset of the prime numbers set containing the 20000 first primes \((2, 3, 5, 7, 11, 13, 17...224737)\) has also been used as a seed referenced \(S_{0PL}\).

A subset of the first 1000 nonprime positive integers \((4, 6, 8, 9, 10... 1197)\) and a subset of the first 1000 natural numbers \((1 \text{ to } 1000)\) have been used too as alternative seeds to the seed \(S_{0P}\). They are respectively named \(S_{0NP}\) and \(S_{0N}\). Each sequence is composed of a prime numbers subsequence containing all primes of the sequence and a nonprime numbers subsequence containing all nonprime numbers of the sequence. (e.g. the sequence \(S_1\) is composed of the prime numbers subsequence referenced \(S_{1P}\) and the nonprime numbers subsequence referenced \(S_{1NP}\).

2-2 Formulas:

**Formula n° 1**

The seed \(S_{0P}\) or \(S_{0NP}\) or \(S_{0N}\) has been filed in the first column of a 1000 rows and 21 columns matrix. Then, 20 sequences \(S_1\) to \(S_{20}\), filed in columns 2 to 21 of the matrix, were calculated with the formula:

$$t_{i+1+j, j+2} = t_{i+1+j, j+1} + t_{i+2+j, j+1} - t_{i+j, j+1} \quad (\text{formula n° 1})$$

\(i\) varying from 1 to 998-2*\(j\) and \(j\) from 0 to 19.

The term on the left side of the equal sign of the formula belongs to the sequence \(S_1\) produced from the seed \(S_{0P}\) or to one of the sequence \(S_2\) to \(S_{20}\) produced from the one which precedes it, this last being used as a seed.

The terms on the right side belong to the seed \(S_{0P}\) or to a sequence \(S_1\) to \(S_{19}\) used as a seed to produce the next sequence.
Note: as formula n° 1 leads to a loss of two terms between the seed and the first sequence and between a given sequence and the next one, the i and j ranges and the term indexes in the formula have to be set accordingly.

Formula n° 2

The same recursive algorithm was used with the seed $S_{0P}$ and the formula n° 2 below to produce a total of 13 sequences referenced $S_1$ to $S_{13}$.

$$t_{i+2+2*j, j+2} = t_{i+2+2*j, j+1} + t_{i+3+2*j, j+1} + t_{i+4+2*j, j+1} - t_{i+1+2*j, j+1} - t_{i+2*j, j+1}$$  \text{(formula n°2)}

with $i = 1$ to $996-4*j$ and $j = 0$ to $12$

Again, the first sequence $S_1$ has been produced from the seed $S_{0P}$ and the $S_2$ sequence from $S_1$ and so on…

Formula n° 3

The same recursive algorithm, again with the seed $S_{0P}$, was used with the formula n° 3 below to produce a total of 17 sequences referenced $S_1$ to $S_{17}$.

As before, $S_{0P}$ has been used to produce $S_1$ and $S_1$ to produce $S_2$ and so on…

$$t_{i+2+2*j, j+2} = t_{i+2+2*j, j+1} + t_{i+3+2*j, j+1} - t_{i+1+2*j, j+1} - t_{i+2*j, j+1}$$  \text{(formula n°:3)}

with $i = 1$ to $997-3*j$ and $j = 0$ to $16$

3- Results :

3-1 Sequences produced from the prime numbers subset $S_{0P}$ and formula n° 1:

3-1-1 Basic statistics:

The first 10 terms of the seed $S_{0P}$ and those of the sequences $S_1$ to $S_{10}$ are given in table n° 1.

- Whereas the sequences $S_1$ to $S_5$ contain only positive integers, sequences $S_6$ to $S_{20}$ contain both positive and negative integers.
So, the nonprime subsequences $S_{6np}$ to $S_{20np}$ contain both positive and negative integers.

- Figure n° 1 shows the number of terms of the seed $S_{0p}$ and those of the sequences $S_1$ to $S_{20}$, a loss of two terms between the seed and $S_1$ and between a sequence and the next one being induced by the formula.

- Figure n° 2 shows that the number of primes in the sequences decreases from $S_1$ to $S_{20}$.

- Figure n° 3 shows that the percentage of primes in the sequences decreases from $S_1$ to $S_{20}$.

- Figure n° 4 shows that the median of primes is roughly constant from prime numbers subsequences $S_{1p}$ to $S_{8p}$ but increases from $S_{9p}$ to $S_{20p}$.

- Figure n° 5 shows that the minimum prime number value is roughly constant from $S_{1p}$ to $S_{11p}$ but increases from $S_{12p}$ to $S_{20p}$, and that the maximum prime number value is roughly constant from $S_{1p}$ to $S_{8p}$ but increases from $S_{9p}$ to $S_{20p}$.

- Figure n° 6 shows that the range of prime number values is roughly constant from $S_{1p}$ to $S_{8p}$ but increases from $S_{9p}$ to $S_{20p}$.

Note: independently of this work, it has been found that the prime subsequence $S_{1p}$ of the sequence $S_1$ was filled in The On-Line Encyclopedia of Integer Sequences, published electronically at: https://oeis.org/A175873 (1).

The first 20 terms of this sequence are: 13, 17, 19, 23, 37, 47, 67, 89, 103, 107, 109, 113, 151, 173, 193, 199, 233, 239, 269...

3-1-2 Sequence structures:

Figures n° 7 to n° 27 show the seed $S_{0p}$ and the 20 prime numbers subsequences $S_{1p}$ to $S_{20p}$. Whereas figure n° 7 does not reveal any particular structure in the prime numbers seed $S_{0p}$, a more and more visible one appears in the prime numbers subsequences $S_{1p}$ to $S_{8p}$ and two distinct prime number size distributions are clearly present in $S_{7p}$ to $S_{20p}$ prime numbers subsequences.

Then, it has been checked if these identified patterns were specific or not of the generated prime subsequences. Figures n° 28 to n° 35 show
that these structures also exist in the nonprime subsequences \( S_{6np}, S_{13np} \) and \( S_{20np} \) and in the corresponding sequences \( S_6, S_{13} \) and \( S_{20} \). So, the prime numbers subset \( \{2, 3, 5, 7, 11, 13, 17, \ldots 7919\} \) generates through formula no 1 structures in integer sequences and also in both prime and nonprime subsequences of them.

3-2 Sequences produced from the nonprime numbers subset \( S_{0NP} \) and formula no 1:

3-2-1 Basic statistics:

The first 10 terms of the seed \( S_{0NP} \) and those of the sequences \( S_1 \) to \( S_{10} \) are given in table no 2.

- It can be noticed that some duplicates are produced such as 11 and 17 in sequence \( S_1 \).

- Whereas the sequences \( S_1 \) to \( S_5 \) contain only positive integers, sequences \( S_6 \) to \( S_{20} \) contain both positive and negative integers. So, the nonprime subsequences \( S_{6np} \) to \( S_{20np} \) contain both positive and negative integers.

- Figure no 36 shows that the number of primes in the sequences decreases from \( S_1 \) to \( S_{20} \).

- Figure no 37 shows that the percentage of primes in the sequences decreases from \( S_1 \) to \( S_{20} \).

- Figure no 38 shows that the median of primes is roughly constant from prime numbers subsequences \( S_{1p} \) to \( S_{9p} \) but increases from \( S_{10p} \) to \( S_{20p} \).

- Figure no 39 shows that the minimum prime number value is roughly constant from \( S_{1p} \) to \( S_{11p} \) but increases from \( S_{12p} \) to \( S_{20p} \) and that the maximum prime number value is roughly constant from \( S_{1p} \) to \( S_{8p} \) but increases from \( S_{9p} \) to \( S_{20p} \).

- Figure no 40 shows that the range of prime number values is roughly constant from \( S_{1p} \) to \( S_{8p} \) but increases from \( S_{9p} \) to \( S_{20p} \).

Overall, these observations are very similar to those seen in the case of the seed \( S_{0P} \).
3-2-2: Sequence structures:

Figures n° 41 to n° 61 show the seed $S_{0NP}$ and the 20 prime number subsequences $S_{1p}$ to $S_{20p}$. Whereas figure n°41 does not reveal any particular structure in the prime numbers seed $S_{0NP}$, a more and more visible one appears in the prime numbers subsequences $S_{1p}$ to $S_{6p}$ and two distinct prime number size distributions are clearly present in $S_{7p}$ to $S_{20p}$ subsequences. Then, it has been checked if these indentified patterns were specific or not of the generated prime subsequences. Figures n° 62 to figure n° 69 show that the structures also exist in the nonprime subsequences $S_{6np}$, $S_{13np}$ and $S_{20np}$ and in the corresponding sequences $S_{6}$, $S_{13}$ and $S_{20}$. So, the nonprime numbers subset $(4, 6, 8, 9, 10... 1197)$ similarly to the prime numbers subset $(2, 3, 5, 7, 11, 13,17...7919)$ generates through formula n° 1 structures in integer sequences and also in both prime and nonprime subsequences of them.

3-3  Sequences produced from the natural numbers set $S_{0N}$ and formula n° 1:

3-3-1 Basic statistics:

The first 10 terms of the seed $S_{0N}$ and those of the sequences $S_{1}$ to $S_{10}$ are given in table n° 3.

- To the opposite to the two previous cases for which the prime number seed $S_{0P}$ and the nonprime number seed $S_{0NP}$ were used, here none of the sequences $S_{1}$ to $S_{20}$ contain negative integers.

- Figure n° 70 shows that the number of primes in the sequences decreases from $S_{1}$ to $S_{20}$.

- Figure n° 71 shows that the percentage of primes in the sequences decreases from $S_{1}$ to $S_{20}$.

- Figure n° 72 shows that the median of primes regularly increases from prime subsequence $S_{1p}$ to $S_{20p}$ and that is a difference compared to the two previous cases where the prime number seed $S_{0p}$ and the nonprime numbers seed $S_{0NP}$ were used.

- Figure n° 73 shows that the minimum prime number value regularly increases from $S_{1p}$ to $S_{20p}$ and that the maximum prime number value is roughly constant from $S_{1p}$ to $S_{20p}$. 
- Figure n° 74 shows that the range of prime number values regularly decreases from \( S_{1p} \) to \( S_{20p} \).

Overall, these observations show that the prime numbers statistics in this case are quite different from the ones observed when the prime number seed \( S_{0p} \) and the nonprime numbers seed \( S_{0NP} \) were used.

3-3-2: Sequence structures:

Figures n° 75 to n° 95 show the seed \( S_{0N} \) and the 20 prime numbers subsequences \( S_{1p} \) to \( S_{20p} \).

Figures n° 96 to n° 103 show the nonprime numbers subsequences \( S_{6np}, S_{13np} \) and \( S_{20np} \) and the \( S_6, S_{13} \) and \( S_{20} \) sequences.

No structures have been identified in either the 20 prime numbers subsequences or in the nonprime numbers subsequences \( S_{6np}, S_{13np} \) and \( S_{20np} \) as in the corresponding sequences \( S_6, S_{13} \) and \( S_{20} \).

Therefore, this absence of pattern in the sequence and prime and nonprime numbers subsequences produced from the natural numbers seed \( S_{0N} \) and formula n° 1 is another major difference compared to the two cases for which the prime numbers seed \( S_{0p} \) and the nonprime numbers seed \( S_{0NP} \) are used together with formula n° 1.

3-4 Sequences produced from formula n° 2:

3-4-1 Basic statistics:

Note: identifying large prime numbers requires long computation times and for that reason calculations with formula n° 2 were not performed beyond sequence \( S_{13} \). However, 13 sequences was enough to make a meaningful comparison of results given by formulas n° 1 and n° 2.

The first 10 terms of the seed \( S_{0p} \) and the sequences \( S_1 \) to \( S_{10} \) are given in table n° 4.

- Whereas the sequences \( S_1 \) to \( S_3 \) contain only positive integers, sequences \( S_4 \) to \( S_{13} \) contain both positive and negative integers. So, the nonprime subsequences \( S_{4np} \) to \( S_{13np} \) contain both positive and negative integers.

- Figure n° 104 shows the number of terms of the seed \( S_{0p} \) and those of the sequences \( S_1 \) to \( S_{13} \), a loss of four terms between the seed
S₀p and S₁p and between a sequence and the next one being induced by formula n° 2.

- Figure n° 105 shows that the number of primes in the sequences decreases from S₁ to S₁₃.

- Figure n° 106 shows that the percentage of primes in the sequences decreases from S₁ to S₁₃.

- Figures n° 107 to n° 110 show the histograms of primes for the sequences S₁p, S₂p, S₃p and S₄p and a slight decrease of the density of primes along the sequences.

- Figure n° 111 shows that the median of primes is roughly constant from S₁p to S₅p and increases from S₆p to S₁₃p.

- Figure n° 112 shows that the minimum prime number value is roughly constant from S₁p to S₇p but increases from S₈p to S₁₃p and that the maximum prime number value is roughly constant from S₁p to S₂p, then increasing from S₃p to S₁₃p.

- Figure n° 113 shows that the range of prime number values increases from S₁p to S₁₃p.

3-4-2 Comparison of the basic statistics on data given by formulas n° 1 and n° 2:

- Figure n° 114 shows that formulas n° 1 and n° 2 give a similar percentage of primes for sequences S₁ to S₅ but that formula n° 1 leads to a significantly higher percentage of primes in sequences S₆ to S₁₃.

- Figure n° 115 shows that formulas n° 1 and n° 2 give a similar median of primes for subsequences S₁p to S₄p but that formula n° 2 leads to a higher median of primes in subsequences S₅p to S₁₃p.

- Figure n° 116 shows that formulas n° 1 and n° 2 give a similar range of prime number values for sequences S₁p to S₃p but that formula n° 2 leads to a higher range of primes in subsequences S₆p to S₁₃p.
3-4-3 Sequence structures:

Figures n°117 to n°130 show the seed $S_{0p}$ and the 13 prime numbers subsequences $S_{1p}$ to $S_{13p}$.
Whereas figure n°117 does not reveal any particular structure in the prime numbers seed $S_{0p}$, a more and more visible one appears in the prime numbers subsequences $S_{1p}$ to $S_{3p}$ and two distinct prime numbers size distributions are clearly present in $S_{4p}$ to $S_{13p}$ prime numbers subsequences.
Then, it has been checked if these indentified patterns were specific or not of the generated prime subsequences. Figures n°131 to figure n°136 show that the structures also exist in the nonprime numbers subsequences $S_{1np}$, $S_{6np}$, $S_{13np}$ as in the corresponding sequences $S_1$, $S_6$ and $S_{13}$.
So, the prime numbers subset (2, 3, 5, 7, 11, 13, 17…7919) generates through formula n° 2 structures in sequences and also in both prime and nonprime numbers subsequences of them.

3-5 Sequences produced from formula n° 3:

3-5-1 Basic statistics:

*Note: here and in a first step, the number of sequences has been restricted to 17 to limit the computing time for searching large prime numbers.*

The first 10 terms of the seed $S_{0p}$ and the sequences $S_1$ to $S_{17}$ are given in table n° 5.

Figures n° 137 and n° 138 show that formula n° 3 leads unlike formulas n° 1 and n° 2 to much less prime numbers, thus lower corresponding percentages.

The occurrence of primes in each sequence is given in table n° 6.

It can be noted that:

- there is 49 occurrences of the prime number 2 on a total of 53 primes.
- Sequences $S_6$ to $S_9$ and $S_{11}$ to $S_{17}$ do not contain any prime number.

On the basis of this last observation, it was worth to investigate further the absence of primes in sequences. For that purpose, a prime number seed $S_{0PL}$ containing 20000 terms from 2 to 224737 (the first 20000 primes)
was used and 28 sequences have been calculated. The $S_{0PL}$ seed was filed in the first column of a 20000 rows and 29 columns matrix and 28 sequences in the columns 2 to 29.

Figures n° 139 show the number of terms of this large seed $S_{0PL}$ and those of the 28 sequences with a loss of three terms between the seed $S_{0PL}$ and the first sequence and between a sequence and the next one; this loss being induced by formula n° 3.

Figures n° 140 and n° 141 respectively show the number of primes and the percentage of them in each sequence.

Table n° 7 gives the occurrence of primes in the 28 sequences. Compared to the set of sequences produced from formula n° 3 and a seed $S_{0P}$ containing 1000 terms (the first 1000 primes) the number of primes in the sequences is now higher due to the higher length of sequences and whereas sequences $S_6$, $S_7$ and $S_8$ were prime free in the case of the 1000 terms $S_{0P}$ prime numbers seed, they do contain 9, 2 and 1 primes in the case of the 20000 terms $S_{0PL}$ prime seed. $S_9$ contains no prime in both cases, $S_{10}$ still contains one prime (the same as in the 1000 terms $S_{0P}$ seed (1831)).

2, 7, 19 and 1831 are the four primes found. With their repeats they count for 617 primes over the 558782 terms of the 28 sequences (0.11 %) and they are concentrated in the first 10 sequences. On these 617 primes, 613 are the prime number "2" (99.35%). On 28 sequences 19 ($S_9$ and $S_{11}$ to $S_{28}$) of them containing in total 378920 terms are primefree. $S_9$ is the first and the longest primefree sequence (19973 terms). The first and smallest term of sequence n° 28 is: -429306405948700 (15 digits) and the last and largest one is: 449225647786036 (15 digits), so a range of 878532053734736.

Note: other formulas giving primefree sequences have been found and reported in the literature (see references (2) to (6)).

3-5-2 A prime free conjecture (Ref: conjecture CD-3):

Considering that:
- the number of primes decreases from $S_1$ to $S_{28}$.
- the number of prime decreases along a sequence.
- Sequences $S_9$ and $S_{11}$ to $S_{28}$ containing a total of 378920 numbers are primefree sequences.
The following conjecture (Ref: conjecture CD3 can be established:

Conjecture CD-3:
The formula below:

\[ t(i+2+2*j, j+2) = t(i+2+2*j, j+1) + t(i+3+2*j, j+1) - t(i+1+2*j, j+1) - t(i+2*j, j+1) \]

applied to the prime numbers set \( (2, 3, 5, 7, 11, 13, 17, 19\ldots ) \) used as a seed, generates a sequence which is then used as a new seed to produce the next sequence and so one. When the number of terms of the prime number set tends to \( +\infty \) and after a certain number of iterations, this recursive process leads to an infinite number of long primefree sequences containing increasingly large composite numbers.

3-6  Sum of the reciprocals of primes:

Figure n° 142 shows the continuous decrease of the reciprocals of primes.

Figure n° 143 shows the asymptotic increase of the sum of the reciprocals of primes.

Figure n° 144: shows the sum of the reciprocals of primes and the function \( \text{LN} (\text{LN}(n)) \)

The Meissel-Mertens constant often referred to as the Mertens constant is defined as the limiting difference between the sum of the reciprocals of primes \( p \leq n \) and the function \( \text{LN} (\text{LN}(n)) \) named Model-1.

Figure n° 145 shows, with the \( S_{0\text{N}} \) natural or \( S_{0\text{NP}} \) nonprime numbers seeds and formula n° 1, the sum of the reciprocal of prime numbers of the subsequence \( S_{1p} \) as a function of \( p \) and the Model-2 predicted values.

Figure n° 146 shows with the \( S_{0\text{P}} \) prime numbers seed and formula n° 1 the sum of the reciprocals of the \( S_{1p} \) subsequence prime numbers as a function of prime numbers and the Model-3 predicted values.

Figure n° 147 shows with the \( S_{0\text{P}} \) prime numbers seed and formula n° 2 the sum of the of the reciprocals of the \( S_{1p} \) subsequence prime numbers as a function of prime numbers and the Model-4 predicted values.

These last three figures show the quite good fit of Model-2, Model-3 and Model-4. The three models are of the form \( a \* \text{LN} (\text{LN}(p)) + b \), so the same form as the \( \text{LN} (\text{LN}(n)) \) with the "a" coefficient different from 1 and the "b" coefficient different from 0. The "a" and "b" coefficients of
the three models are given in table n° 8-a. The correlation and determination coefficients are also given in this table and the absolute and relative errors reported in table n° 8-b confirm the good quality of the three models.

Table n° 8-b also give the D distances (see definition in the legend of table 8-b or in annex n° 1).

Finally, figures n° 148 and n° 149 respectively give the sum \( (1/p) \) curves calculated from the \( s_{1p} \) subsequence and the sum \( (1/p) \) predicted by the models, the sum of the reciprocals of primes and the \( LN(LN(n)) \) function being shown as reference.

4- Conclusions:

- When the number of iterations of the recursive algorithm increases the number of primes in a sequence decreases and the size of its terms (number of digits) increases. 15 digits terms are found in the 28\(^{\text{th}}\) sequence produced when the \( S_{0PL} \) seed is used.

- The number of primes slightly decreases along sequences.

- When the algorithm is used with a prime or nonprime numbers seed some structures are already visible in the first sequences and their prime and nonprime numbers subsequences and their development in the next subsequences leads to two distinct number size distributions. Such patterns do not exist when the natural numbers subset is used as seed.

- When the algorithm is used with the prime numbers subset and formula n° 3, only four primes are procued in the first sequences and rapidly long primefree sequences containing large size numbers are produced. This led to establish a conjecture stating that an infinite number of primefree sequences can be produced using the standard prime numbers set and formula n° 3.

- Accurate models all of the form of sums \( (1/p) = a*LN(LN(p)) + b \) have been developed for the various type of seed/formula combinations. Finally, D distances equivalent to the Meissel-Mertens constant have been calculated for the different models.
References:


Annex n° :1 definitions and abbreviations (specific to this paper) :

- a sequence : list of positive or negative, prime or composite integers. A sequence "j" is produced from an seed or the " j-1" sequence and a formula. Sequences are noted: S_j, (j = 1, 2, 3,...).

- a prime numbers subsequence: part of a S_j sequence composed of all prime numbers of this sequence. Prime numbers subsequences are noted S_jp, (j = 1, 2, 3,...).

- a nonprime numbers subsequence: part of a S_j sequence composed of all numbers of this sequence except the primes ones. nonprimes subsequences are noted S_jnp, (j = 1, 2, 3,...).

- a term (t_index-1, index-2): an element of a sequence or a subsequence. Terms are indexed to indicate the sequence to which they belong (index-1) and their position in the sequence (index-2).

- The prime numbers seed S_0P : a subset of the standard set of prime numbers (2, 3, 5, 7, 11, 13, 17, 19, 23...7919) used as seed in the algorithm.

- The nonprime numbers seed S_0NP: a subset of the standard set of nonprime positive numbers (2, 4, 6, 8, 9,10...1197) used in the algorithm.

- The natural numbers seed S_0N : a subset of the standard set of natural numbers (1, 2, 3, 4, 5...1000) used as seed in the algorithm.
- the recursive algorithm: starts from the seed \(S_{0P} \text{ or } S_{0N} \text{ or } S_{0NP}\), to produce with a formula a first sequence \(S_1\) which is then used as a new seed to produce with the same formula the next sequence \(S_2\) and so one, each sequence \(S_{j+1}\) being produced from the previous one \(S_j\).

- a formula: it is the equation used to produce a sequence from an seed or the previous sequence.

- \(p\): prime number.

- prime \((n)\): the \(n^{th}\) prime number (e.g.: 97 is the 25\(^{th}\) prime number)

- sum \((1/p), \ p \leq n\) = sum of the reciprocals of prime numbers at \(n\).

- Meissel-Mertens constant: is defined as the difference between the sum of the reciprocals of prime numbers \(\leq n\) and the series of general term \(Ln(Ln(n))\) when \(n\) tends to \(+\infty\). Approximate value: 0,261497.

- Distance \(D\): is defined as the difference between the sum of the reciprocals of prime numbers \(\leq n\) and the sum of the reciprocal of primes numbers \(\leq n\) of a sequence generated by the algorithm and one formula as described in this paper. The "distance \(D\)" can be seen as the equivalent of the Meissel-Mertens constant.

- Absolute error: the difference, at a given \((n)\), between the sum of the reciprocals of primes \(p \leq n\) of a sequence and the corresponding model predicted value.

- Relative error: relative difference at a given \((n)\) between the sum of the reciprocals of primes \(p \leq n\) of a sequence and the corresponding model predicted value.

- \(R\): coefficient of correlation between the sum of the reciprocals of primes of a sequence and the corresponding sum calculated from a model.

- \(R^2\): coefficient of determination between the sum of the reciprocals of primes of a sequence and the corresponding sum calculated from a model.
Annex n° 2: tables

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<td>25</td>
<td>45</td>
<td>79</td>
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<td>-251</td>
<td>821</td>
<td>-941</td>
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<td>8</td>
<td>19</td>
<td>33</td>
<td>43</td>
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<td>197</td>
<td>7</td>
<td>405</td>
<td>-185</td>
<td>-579</td>
</tr>
</tbody>
</table>

Note: prime numbers are in red.

Table n° 1: first 10 terms of the prime numbers seed \( S_{0P} \) (1000 terms) and those of sequences \( S_1 \) to \( S_{10} \), formula n° 1.

<table>
<thead>
<tr>
<th>Term n°</th>
<th>( S_{0NP} )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_3 )</th>
<th>( S_4 )</th>
<th>( S_5 )</th>
<th>( S_6 )</th>
<th>( S_7 )</th>
<th>( S_8 )</th>
<th>( S_9 )</th>
<th>( S_{10} )</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>10</td>
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<td>6</td>
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<td>3</td>
<td>8</td>
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<td>18</td>
<td>20</td>
<td>22</td>
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<td>-188</td>
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</tr>
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<td>20</td>
<td>39</td>
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<td>-18</td>
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</tr>
</tbody>
</table>

Note: prime numbers are in red.

Table n° 2: first 10 terms of the nonprime numbers seed \( S_{0NP} \) (1000 terms) and those of sequences \( S_1 \) to \( S_{10} \), formula n° 1.

<table>
<thead>
<tr>
<th>Term n°</th>
<th>( S_{0N} )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_3 )</th>
<th>( S_4 )</th>
<th>( S_5 )</th>
<th>( S_6 )</th>
<th>( S_7 )</th>
<th>( S_8 )</th>
<th>( S_9 )</th>
<th>( S_{10} )</th>
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<tbody>
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<td>1</td>
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<td>4</td>
<td>7</td>
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<td>6</td>
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<td>28</td>
<td>31</td>
<td>34</td>
<td>37</td>
<td>40</td>
</tr>
</tbody>
</table>

Note: prime numbers are in red.

Table n° 3: first 10 terms of the natural numbers seed \( S_{0N} \) (1000 terms) and those of sequences \( S_1 \) to \( S_{10} \), formula n° 1.
Table n° 4: first 10 terms of the prime numbers seed $S_{OP}$ (1000 terms) and those of sequences $S_1$ to $S_{10}$, formula n° 2.

<table>
<thead>
<tr>
<th>Term n°</th>
<th>$S_{OP}$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
<th>$S_6$</th>
<th>$S_7$</th>
<th>$S_8$</th>
<th>$S_9$</th>
<th>$S_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>18</td>
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<td>652</td>
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<td>9202</td>
<td>-37558</td>
<td>161484</td>
<td>-666012</td>
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<td>6985</td>
<td>-30623</td>
<td>155727</td>
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<tr>
<td>3</td>
<td>5</td>
<td>29</td>
<td>63</td>
<td>107</td>
<td>127</td>
<td>151</td>
<td>615</td>
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<td>1259</td>
<td>33455</td>
<td>-289393</td>
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<td>31</td>
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<td>81</td>
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<td>-15911</td>
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<td>-73569</td>
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<td>99</td>
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<td>4099</td>
<td>-25137</td>
<td>109981</td>
<td>-365913</td>
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</tbody>
</table>

Note: primes numbers are in red.

Table n° 5: first 10 terms of the prime numbers seed $S_{OP}$ (1000 terms) and those of sequences $S_1$ to $S_{10}$, formula n° 3.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Prime numbers</th>
<th>Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>$S_2$</td>
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<td>32</td>
</tr>
<tr>
<td>$S_3$</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>$S_4$</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$S_5$</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$S_6$</td>
<td>19</td>
<td>1</td>
</tr>
<tr>
<td>$S_7$</td>
<td>no prime</td>
<td></td>
</tr>
<tr>
<td>$S_8$</td>
<td>no prime</td>
<td></td>
</tr>
<tr>
<td>$S_9$</td>
<td>no prime</td>
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</tr>
<tr>
<td>$S_{10}$</td>
<td>1831</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: primes are in red.

Table n° 6: occurrence of primes in sequences, prime numbers seed $S_{OP}$ (1000 terms) and formula n° 3.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Prime numbers</th>
<th>Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$ to $S_{28}$</td>
<td>no prime</td>
<td></td>
</tr>
<tr>
<td>Sequence</td>
<td>Prime numbers</td>
<td>Occurrence</td>
</tr>
<tr>
<td>----------</td>
<td>---------------</td>
<td>------------</td>
</tr>
<tr>
<td>$S_1$</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>$S_2$</td>
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<td>411</td>
</tr>
<tr>
<td>$S_2$</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>$S_3$</td>
<td>2</td>
<td>119</td>
</tr>
<tr>
<td>$S_4$</td>
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<td>2</td>
<td>23</td>
</tr>
<tr>
<td>$S_5$</td>
<td>19</td>
<td>1</td>
</tr>
<tr>
<td>$S_6$</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>$S_7$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$S_8$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$S_9$</td>
<td>----</td>
<td>no prime</td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>1831</td>
<td>1</td>
</tr>
<tr>
<td>$S_{11}$ to $S_{28}$</td>
<td>----</td>
<td>no prime</td>
</tr>
</tbody>
</table>

Table n° 7: occurrence of primes in the sequences, *prime numbers seed $S_{0P}$ (2x10⁴ terms) and formula n° 3.*

<table>
<thead>
<tr>
<th>Model inputs</th>
<th>Model</th>
<th>Correlation coefficient (R)</th>
<th>Determination coefficient ($R^2$)</th>
</tr>
</thead>
</table>
| Prime numbers | Model-1 :  
Sum (1/p)= LN(LN(n)) | 0.999 | 0.997 |
| Natural or nonprime numbers seed $S_{0N}$ or $S_{0NP}$| Model-2 :  
Prime numbers subsequence $S_{1p}$  
Formula n° 1 | 0.999 | 0.998 |
| Prime numbers seed $S_{0P}$  
Prime numbers subsequence $S_{1p}$  
Formula n° 1 | Model-3 :  
Prime numbers seed $S_{0P}$  
Prime numbers subsequence $S_{1p}$  
Formula n° 2 | 0.978 | 0.957 |
| Prime numbers seed $S_{0P}$  
Prime numbers subsequence $S_{1p}$  
Formula n° 2 | Model-4 :  
Prime numbers seed $S_{0P}$  
Prime numbers subsequence $S_{1p}$  
Formula n° 2 | 0.997 | 0.994 |

Table n° 8-a: models of sums of the reciprocals of prime numbers.
<table>
<thead>
<tr>
<th>Model</th>
<th>Model inputs</th>
<th>Absolute error</th>
<th>Relative error</th>
<th>Distance D *</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model-1</td>
<td>Prime numbers</td>
<td>not appropriate</td>
<td>not appropriate</td>
<td>0,26</td>
</tr>
<tr>
<td>Model-2</td>
<td>Natural or nonprime numbers seed $S_{ON}$ or $S_{ONP}$ prime subsequence $S_{1p}$ formula n° 1</td>
<td>0,00115</td>
<td>0,06</td>
<td>0,83</td>
</tr>
<tr>
<td>Model-3</td>
<td>Prime numbers $S_{0p}$ subsequence $S_{1p}$, formula n° 1</td>
<td>-0,0053</td>
<td>0,86</td>
<td>2,16</td>
</tr>
<tr>
<td>Model-4</td>
<td>Prime numbers seed $S_{0p}$ Prime numbers subsequence $S_{1p}$ formula n° 2</td>
<td>-0,0037</td>
<td>0,80</td>
<td>2,32</td>
</tr>
</tbody>
</table>

Table n° 8-b: absolute and relative errors of the models and distance $D$, (at the prime number 224729 of the subsequence $S_{1p}$ for models n° 1, n° 2, n° 3 and at the prime number 224611 of the subsequence $S_{1p}$ for model n° 4; the closest prime number from the prime number 224729.)

*Distance D : difference between the sum of the reciprocals of prime numbers and the sum of the reciprocals of prime numbers of the $S_{1p}$ subsequence.
Annex n° 3 : figures

Figures from the prime numbers seed $S_{0P}$ and formula n° 1

Figure n° 1: numbers of terms in the seed $S_{0P}$ and the sequences prime numbers seeds and formula n° 1

Figure n° 2: number of primes in each sequence prime numbers seed $S_{0P}$ and formuma n° 1

Figure n° 3: percentage of prime numbers in each sequence prime numbers seed $S_{0P}$ and formula n° 1
Figure n° 4: median of primes in $S_{op}$ and each prime numbers subsequence.

Figure n° 5: prime number minimum and maximum values in $S_{op}$ and each prime numbers subsequence.

Figure n° 6: prime number range values for $S_{op}$ and each prime numbers subsequence.
Figure n° 7: prime numbers seed $S_{op}$
formula n° 1

Figure n° 8: prime numbers subsequence $S_{1p}$
prime numbers seed $S_{op}$ and formula n° 1

Figure n° 9: prime numbers subsequence $S_{2p}$
prime numbers seed $S_{op}$ and formula n° 1
Figure n° 10: prime numbers subsequence $S_{3p}$
prime numbers seed $S_{3p}$ and formula n° 1

Figure n° 11: prime numbers subsequence $S_{4p}$
prime numbers seed $S_{4p}$ and formula n° 1

Figure n° 12: prime numbers subsequence $S_{5p}$
prime numbers seed $S_{5p}$ and formula n° 1
Figure n° 13: prime numbers subsequence $S_{0p}$
prime numbers seed $S_{0p}$ and formula n° 1

Figure n° 14: prime numbers subsequence $S_{7p}$
prime numbers seed $S_{0p}$ and formula n° 1

Figure n° 15: prime numbers subsequence $S_{8p}$
prime numbers seed $S_{0p}$ and formula n° 1
Figure n° 16: prime numbers subsequence $S_{9p}$
prime numbers seed $S_{9p}$ and formula n° 1

Figure n° 17: prime numbers subsequence $S_{10p}$
prime numbers seed $S_{10p}$ and formula n° 1

Figure n° 18: prime numbers subsequence $S_{11p}$
prime numbers seed $S_{11p}$ and formula n° 1
Figure n° 19: prime numbers subsequence $S_{12p}$
prime numbers seed $S_{0p}$ and formula n° 1

Figure n° 20: prime numbers subsequence $S_{13p}$
prime numbers seed $S_{0p}$ and formula n° 1

Figure n° 21: prime numbers subsequence $S_{14p}$
prime numbers seed $S_{0p}$ and formula n° 1
Figure n° 22: prime numbers subsequence \( S_{15p} \)
prime numbers seed \( S_{0p} \) and formula n° 1

Figure n° 23: prime numbers subsequence \( S_{16p} \)
prime numbers seed \( S_{0p} \) and formula n° 1

Figure n° 24: prime numbers subsequence \( S_{17p} \)
prime numbers seed \( S_{0p} \) and formula n° 1
Figure n° 25: prime numbers subsequence $S_{18p}$
prime numbers seed $S_{0p}$ and formula n° 1

Figure n° 26: prime numbers subsequence $S_{19p}$
prime numbers seed $S_{0p}$ and formula n° 1

Figure n° 27: prime numbers subsequence $S_{20p}$
prime numbers seed $S_{0p}$ and formula n° 1
Figure n° 28: nonprime numbers subsequence $S_{1np}$ prime numbers seed $S_{0p}$ and formula n° 1

Figure n° 29: sequence $S_t$ prime numbers seed $S_{0p}$ and formula n°:1

Figure n° 30: nonprime numbers subsequence $S_{0np}$ prime numbers seed $S_{0p}$ and formula n° 1

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Prime (n) and nonrime (n)

Figure n° 31: sequence $S_n$
prime numbers seed $S_{0P}$ and formula n° 1

Nonprime (n)

Figure n° 32: nonprime numbers subsequence $S_{13np}$
prime numbers seed $S_{0P}$ and formula n° 1

Prime and nonprimes (n)

Figure n° 33: sequence $S_{13}$
prime numbers seed $S_{0P}$ and formula n° 1
Figure n° 34: nonprime numbers subsequence $S_{20np}$
prime numbers seed $S_{0p}$ and formula n° 1

Figure n° 35: sequence $S_{20}$
prime numbers seed $S_{0p}$ and formula n° 1
Figures from the nonprime numbers seed $S_{0NP}$ and formula $n^o:1$

Figure n° 36: number of prime numbers in each sequence, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 37: percentage of prime numbers in each sequence, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 38: median of prime numbers in each prime numbers subsequence, nonprime numbers seed $S_{0NP}$ and formula n° 1
Figure n° 39: prime number minimum and maximum values in each prime numbers subsequence, nonprime numbers seed $S_{OP}$ and formula n° 1

![Prime number minimum and maximum values](image)

Figure n° 40: prime number range values for each prime numbers subsequence nonprime numbers seed $S_{OP}$ and formula n° 1

![Prime number range values](image)

Figure n° 41: nonprime numbers seed $S_{OP}$

![Nonprime numbers seed](image)
Figure n° 42: prime numbers subsequence $S_{1p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 43: prime numbers subsequence $S_{2p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 44: prime numbers subsequence $S_{3p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1
Figure n° 45: prime numbers subsequence $S_{np}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 46: prime numbers subsequence $S_{5p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 47: prime numbers subsequence $S_{6p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1
Figure n° 51: prime numbers subsequence $S_{10p}$, nonprime numbers seed $S_{0np}$ and formula n° 1

Figure n° 52: prime numbers sequence $S_{11p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 53: prime numbers subsequence $S_{12p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1
Figure n° 54: prime numbers subsequence $S_{13p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 55: prime numbers subsequence $S_{14p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 56: prime numbers subsequence $S_{15p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1
Figure n° 57: prime numbers subsequence $S_{16p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1.

Figure n° 58: prime numbers subsequence $S_{17p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1.

Figure n° 59: prime numbers subsequence $S_{18p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1.
Figure n° 60: prime numbers subsequence $S_{1np}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 61: prime numbers subsequence $S_{2np}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 62: nonprime numbers subsequence $S_{1np}$, nonprime numbers seed $S_{0NP}$ and formula n° 1
Figure n° 63: Sequence $S_1$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 64: nonprime numbers subsequence $S_{6np}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 65: Sequence $S_{6p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1
Figure n° 66: nonprime numbers subsequence $S_{13np}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 67: Sequence $S_{13p}$, nonprime numbers seed $S_{0NP}$ and formula n° 1

Figure n° 68: nonprime numbers subsequence $S_{20np}$, nonprime numbers seed $S_{0NP}$ and formula n° 1
Figure n° 69: Sequence $S_{20}$, nonprime numbers seed $S_{0NP}$ and formula n° 1.
Figures from the natural numbers seed and formula n° 1

Figure n° 70: number of primes in the seed $S_{O_N}$ and in each sequence, natural numbers seed $S_{O_N}$ and formula n° 1

Natural numbers seed $S_{O_N}$ and sequences

Figure n° 71: percentage of primes in each sequence, natural numbers seed $S_{O_N}$ and formula n° 1

Natural numbers seed $S_{O_N}$ and sequences

Figure n° 72: median of primes in the seed $S_{O_N}$ and in each prime numbers subsequence, natural numbers seed $S_{O_N}$ and formula n° 1

Natural numbers seed $S_{O_N}$ and subsequences
Prime minimum and maximum values in each prime numbers subsequence:

Natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 73

Prime range values for the natural numbers seed $S_{0N}$ and each prime numbers subsequence:

Natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 74

Natural number seed $S_{0N}$

Figure n° 75
Figure n° 76: prime numbers subsequence $S_{1p}$, natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 77: prime numbers subsequence $S_{2p}$, natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 78: prime numbers subsequence $S_{3p}$, natural numbers seed $S_{0N}$ and formula n° 1
Figure n° 79: prime numbers subsequence $S_{4p}$, natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 80: prime numbers subsequence $S_{5p}$, natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 81: prime numbers subsequence $S_{6p}$, natural numbers seed $S_{0N}$ and formula n° 1
Figure n° 82: prime numbers subsequence $S_{7p}$, natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 83: prime numbers subsequence $S_{8p}$, natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 84: prime numbers subsequence $S_{9p}$, natural numbers seed $S_{0N}$ and formula n° 1
Figure n° 85: prime numbers subsequence $S_{10p}$
natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 86: prime numbers subsequence $S_{13p}$
natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 87: prime numbers sequence $S_{12p}$
natural numbers seed $S_{0N}$ and formula n° 1
Figure n° 88: prime numbers subsequence $S_{13p}$
- natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 89: prime numbers subsequence $S_{14p}$
- natural numbers seed $S_{0N}$ and formula n° 1

Figure n° 90: prime numbers subsequence $S_{15p}$
- natural numbers seed $S_{0N}$ and formula n° 1
Figure n° 91: prime numbers subsequence $S_{16p}$, natural number seed $S_{0N}$ and formula n° 1

Figure n° 92: prime numbers subsequence $S_{17p}$, natural number seed $S_{0N}$ and formula n° 1

Figure n° 93: prime numbers subsequence $S_{18p}$, and natural number seed and formula n° 1
Figure n° 94: prime numbers subsequence $S_{10p}$, natural number seed $S_{0N}$ and formula n° 1

Figure n° 95: prime numbers subsequence $S_{20p}$, natural number seed $S_{0N}$ and formula n° 1

Figure n° 96: nonprime numbers subsequence $S_{1}$, natural number seed $S_{0N}$ and formula n° 1
Figure n° 97: Sequence $S_1$, natural number seed $S_{0N}$ and formula $n^0:1$

Figure n° 98: nonprime numbers subsequence $S_6$, natural number seed $S_{0N}$ and formula $n^0:1$

Figure n° 99: Sequence $S_{65}$, natural number seed $S_{0N}$ and formula $n^0:1$
Figure n° 100: nonprime numbers subsequence $S_{13}$, natural number seed $S_{0N}$ and formula n° 1

Figure n° 101: Sequence $S_{13}$, natural number seed $S_{0N}$ and formula n° 1

Figure n° 102: nonprime numbers subsequence $S_{29}$, natural number seed $S_{0N}$ and formula n° 1
Figure n° 103: Sequence $S_{20^p}$, natural number seed $S_{0N}$ and formula n° 1
Figures from the prime numbers seed $S_{0P}$ and formula n° 2

Figure n° 104: number of terms in the seed $S_{0P}$ and sequences

Prime numbers seed $S_{0P}$ and sequences

Figure n° 105: number of prime numbers in $S_{0P}$ and each sequence,

Prime numbers seed $S_{0P}$ and sequences

Figure n° 106: percentage of prime numbers in $S_{0P}$ and each sequence,

Prime numbers seed $S_{0P}$ and sequences
Figure n° 107: histogram of prime numbers in the $S_{1p}$ subsequence, prime numbers seed $S_{0p}$ and formula n° 2.

Figure n° 108: histogram of prime numbers in the $S_{2p}$ subsequence, prime numbers seed $S_{0p}$ and formula n° 2.

Figure n° 109: histogram of prime numbers in the $S_{3p}$ subsequence, prime numbers seed $S_{0p}$ and formula n° 2.
Figure n° 110: histogram of prime numbers in the $S_{4p}$ subsequence, prime numbers seed $S_{0p}$ and formula n° 2

![Histogram of prime numbers](image)

Class of term values

Number of primes per class

Figure n° 111: median of prime numbers in $S_{0p}$ and each prime numbers subsequence, prime numbers seed $S_{0p}$ and formula n° 2

![Median of prime numbers](image)

Prime numbers seed $S_{0p}$ and prime numbers subsequences

Figure n° 112: prime number minimum and maximum values in $S_{0p}$ and each prime numbers subsequence, prime numbers seed $S_{0p}$ and formula n° 2

![Minimum and maximum values](image)

Prime numbers seed $S_{0p}$ and prime numbers subsequences
Prime numbers seed $S_{0p}$ and prime numbers subsequences

Figure n°113: prime number range values for $S_{0p}$ and each prime numbers subsequence, prime numbers seed $S_{0p}$ and formula n° 2

Figure n°114: percentage of primes in each sequence, prime numbers seed $S_{0p}$, comparison formulas n° 1 and n° 2

Figure n°115: median of primes for each subsequence, prime numbers seed $S_{0p}$, comparison formulas n° 1 and n° 2
Figure n° 116: prime number range values for each subsequence, prime numbers seed $S_{0p}$, comparison formulas n° 1 and n° 2

- Formula n° 1
- Formula n° 2

Figure n° 117: prime numbers seed $S_{0p}$ and formula n° 2

Figure n° 118: prime numbers subsequence $S_{1P}$, prime numbers seed $S_{0p}$ and formula n° 2
Figure n° 119: prime numbers subsequence $S_{2p}$, prime numbers seed $S_{0p}$ and formula n° 2

Figure n° 120: prime numbers subsequence $S_{3p}$ (prime numbers seed $S_{0p}$ and formula n° 2)

Figure n° 121: prime numbers subsequence $S_{4p}$, prime numbers seed $S_{0p}$ and formula n° 2
Figure n° 122: prime numbers subsequence $S_{5p}$, prime numbers seed $S_{0p}$ and formula n° 2.

Figure n° 123: prime numbers subsequence $S_{6p}$, prime numbers seed $S_{0p}$ and formula n° 2.

Figure n° 124: prime numbers subsequence $S_{7p}$, prime numbers seed $S_{0p}$ and formula n° 2.
Figure n° 125: prime numbers subsequence $S_{8p}$, prime numbers seed $S_{0p}$ and formula n° 2

Figure n° 126: prime numbers subsequence $S_{9p}$, prime numbers seed $S_{0p}$ and formula n° 2

Figure n° 127: prime numbers subsequence $S_{10p}$, prime numbers seed $S_{0p}$ and formula n° 2
Figure n°128: prime numbers subsequence $S_{11p}$, prime numbers seed $S_{0p}$ and formula n° 2

Figure n°129: prime numbers subsequence $S_{12p}$, prime numbers seed $S_{0p}$ and formula n° 2

Figure n°130: prime numbers subsequence $S_{13p}$, prime numbers seed $S_{0p}$ and formula n° 2
Figure n°131: nonprime number subsequence $S_{n_{op}}$
prime numbers seed $S_{op}$ and formulas n°2

Figure n° 132: sequence $S_{r}$
prime numbers seed $S_{op}$ and formula n° 2

Figure n° 133: nonprime numbers subsequence $S_{6op}$
prime numbers seed $S_{op}$ and formula n° 2
Figure n° 134: sequence $S_{0p}$
prime numbers seed $S_{0p}$ and formula n° 2

Figure n° 135: nonprime numbers subsequence $S_{13pp}$
prime numbers seed $S_{0p}$ and formula n° 2

Figure n° 136: sequence $S_{13}$
prime numbers seed $S_{0p}$ and formula n° 2
Figures from the prime numbers seeds $S_{0P}$ and $S_{0PL}$ and formula n° 3

Figure n° 137: number of prime numbers in each sequence, prime numbers seed $S_{0P}$ (1000 terms) and formula n° 3

Figure n° 138: percentage of prime numbers in each sequence prime numbers seed $S_{0P}$ (1000 terms) and formula n° 3

Figure n° 139: number of terms of each sequence prime numbers seed $S_{0PL}$ ($2\times10^4$ terms), formula n° 3
Figure n° 140: number of primes in each sequence, Prime number seed $S_{opt}$ ($2 \times 10^4$ terms), formula n° 3

Figure n° 141: percentage of primes in each sequence prime number seed $S_{opt}$ ($2 \times 10^4$ terms), formula n° 3
Figures of the sums of the reciprocals of prime numbers:

Figure n° 142: subsequence $S_{1pl}$, reciprocals of primes $(1/p) = f(p)$, prime numbers seed $S_{0pl}$, formula n° 1
First 50 terms.

Figure n° 143: subsequence $S_{1pl}$, sum of the reciprocals of primes; sum of $(1/p) = f(p)$, prime numbers seed $S_{0pl}$, formula n° 1
First 50 terms.

Figure n° 144: sum of the reciprocals of prime numbers and Model-1
Figure no. 145: sum of the reciprocals of prime numbers of the subsequence $S_{1,p}$ and corresponding Model-2 predicted values.
Natural or nonprime seeds $S_{0NL}$ or $S_{0NPL}$ and formula no. 1

Natural or nonprime seeds $S_{0NL}$ or $S_{0NPL}$ and formula no. 1

Figure no. 146: sum of the reciprocals of prime numbers of the subsequence $S_{1,p}$ and corresponding Model-3 predicted values.
Prime numbers seed $S_{0PL}$ and formula no. 1

Prime numbers seed $S_{0PL}$ and formula no. 1

Figure no. 147: sum of the reciprocals of prime numbers of the subsequence $S_{1,p}$ and corresponding Model-4 predicted values.
Prime numbers seed $S_{0PL}$ and formula no. 2

Prime numbers seed $S_{0PL}$ and formula no. 2
Figure no 148: sum of the reciprocals of prime numbers of the subsequence $S_{1p}$.

- Prime numbers
- Natural or nonprime seed S0NL or S0NPL, formula n° 1
- Prime numbers seed S0PL, formula n° 1
- Prime numbers seed S0PL, formula n° 2
Figure n° 149: models of sum of the reciprocals of prime numbers \((1/p) = f(p)\),

- **Model-1**: \(\text{LN} \left( \text{LN} (p) \right)\), prime numbers
- **Model-2**: natural or nonprime numbers seed 
  \(\text{S0NL}\) or \(\text{S0NPL}\), formula n° 1
- **Model-3**: prime numbers seed \(\text{S0PL}\), formula n° 1
- **Model-4**: prime numbers seed \(\text{S0PL}\), formula n° 2

Data points show the behavior of the sum of the reciprocals of prime numbers on a logarithmic scale.