# The Riemann Hypothesis Proof

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## Abstract

Many people are using the term “Assigned Value” or “Analytic Continuation” for divergent series. But this explanation is so lacking and can be replaced with a much easier and simpler term of explanation. For me (as I see it) when I am looking at the zeta function I dont see (or use) the term “Assigned Value” or “Analytic Continuation”. Instead I see “spirals” all around the grid.

# The Riemann Hypothesis Proof

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But this explanation is so lacking and can be replaced with a much easier and simpler term of explanation

For me (as I see it) when I am looking at the zeta function I dont see (or use) the term “Assigned Value” or “Analytic Continuation”
Instead I see “spirals” all around the grid

The simplest way is to first look at the Complex plane $\zeta(s)=\zeta(x+iy)=a+ib$ where $s>1$ and the behavior of convergent points
The spiral swirls around inwards to an unique point which the series Converges - Same goes for the other way around!

When I look at the Complex plane $\zeta(s)=\zeta(x+iy)=a+ib$ where $s<1$ and the behavior of divergent points
The spiral swirls around outwards but if you look closely you will notice that the spiral has a “center point” or an “origin”
and that “origin” is the “Assigned Value” everyone is talking about

when I first started to read about the zeta function I didn’t know what are those “Assigned Values” or “Analytic Continuation”
and how and why people are trying to give a value for divergent series And why that specific value and not something else?
I wanted an explanation other then “because the formula says so” and without going deeper into all the “Analytic Continuation stuff".

Those “origin points” did the trick!

If you are assigning a value for a series that decreases to a specific value (case #1)
Then you can assigning a value for a series that increases from a specific value (case #2)

Other then those two cases there is one more
This is when the spiral at some point start to spin around a specific value with a “fixed radius”
those cases appears at the zeta function $\zeta(s)=\zeta(x+iy)=a+ib$ when $x=1$ and the radius will be $1/y$
meaning that this is a divergent series with a “fixed radius”

This was a small intro for the eta function spirals

Its true that the zeta function spirals have 3 cases but they are all spirals with **one arm**
Now at the eta function the spirals have **two arms** (that is because of the +/- swapping) with the same 3 cases

By the way the “fixed radius” appears at the eta function $\eta(s)=\eta(x+iy)=a+ib$ when $x=0$

If you like to know more I am providing further details at http://myzeta.125mb.com

# Removing the Riemann hypothesis from the Complex plane

I am going to show that $\boxed{\frac{1}{k^{(a+ib)}} = \frac{\cos(b\cdot\ln k)}{k^a} - i\cdot\frac{\sin(b\cdot\ln k)}{k^a}}$ by using binomial theorem and exponential function (you can skip those 3 pages if you like)

$\boxed{e^{\theta} = \lim_{n\to\infty}\left(1+\frac{\theta}{n}\right)^n}$ (exponential function)

$$\theta = -ib\cdot\ln k$$

$$e^{\theta} = e^{-ib\cdot\ln k} = k^{-ib}$$

$$\frac{1}{k^{(a+ib)}} = \frac{1}{k^a}\cdot k^{-ib} = \frac{1}{k^a}\cdot e^{\theta} = \frac{1}{k^a}\cdot\lim_{n\to\infty}\left(1+\frac{\theta}{n}\right)^n = \frac{1}{k^a}\cdot\lim_{n\to\infty}\left(1-i\cdot\frac{b\cdot\ln k}{n}\right)^n$$

$\boxed{(x+y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \ldots + \binom{n}{n-1}x^1 y^{n-1} + \binom{n}{n}x^0 y^n}$ (binomial theorem)

$$(x-iy)^n = x^n - i\binom{n}{1}x^{n-1}y^1 - \binom{n}{2}x^{n-2}y^2 + i\binom{n}{3}x^{n-3}y^3 + \binom{n}{4}x^{n-4}y^4 - i\binom{n}{5}x^{n-5}y^5 - \binom{n}{6}x^{n-6}y^6 + i\binom{n}{7}x^{n-7}y^7 + \binom{n}{8}x^{n-8}y^8 - \ldots \pm \binom{n}{n}x^0(-iy)^n$$

$$\lim_{n\to\infty}(x-iy)^n = \lim_{n\to\infty}\left[x^n - i\binom{n}{1}x^{n-1}y^1 - \binom{n}{2}x^{n-2}y^2 + i\binom{n}{3}x^{n-3}y^3 + \binom{n}{4}x^{n-4}y^4 - i\binom{n}{5}x^{n-5}y^5 - \binom{n}{6}x^{n-6}y^6 + i\binom{n}{7}x^{n-7}y^7 + \binom{n}{8}x^{n-8}y^8 - \ldots\right]$$

$$\lim_{n\to\infty}(x-iy)^n = \lim_{n\to\infty}\left[x^n - \binom{n}{2}x^{n-2}y^2 + \binom{n}{4}x^{n-4}y^4 - \binom{n}{6}x^{n-6}y^6 + \binom{n}{8}x^{n-8}y^8 + \ldots\right] + i\lim_{n\to\infty}\left[-\binom{n}{1}x^{n-1}y^1 + \binom{n}{3}x^{n-3}y^3 - \binom{n}{5}x^{n-5}y^5 + \binom{n}{7}x^{n-7}y^7 + \ldots\right]$$

$\boxed{\binom{n}{k} = \frac{n!}{(n-k)!k!}}$ (binomial coefficient formula)

$$\lim_{n\to\infty}(x-iy)^n = \lim_{n\to\infty}\left[x^n - \frac{n!x^{n-2}y^2}{(n-2)!2!} + \frac{n!x^{n-4}y^4}{(n-4)!4!} - \frac{n!x^{n-6}y^6}{(n-6)!6!} + \frac{n!x^{n-8}y^8}{(n-8)!8!} + \ldots\right] + i\lim_{n\to\infty}\left[-\frac{n!x^{n-1}y^1}{(n-1)!1!} + \frac{n!x^{n-3}y^3}{(n-3)!3!} - \frac{n!x^{n-5}y^5}{(n-5)!5!} + \frac{n!x^{n-7}y^7}{(n-7)!7!} - \ldots\right]$$

lets replace $x=1, y=\frac{b\cdot\ln k}{n}$

$$\lim_{n\to\infty}\left(1-i\cdot\frac{b\cdot\ln k}{n}\right)^n=\lim_{n\to\infty}\left[1-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^2}{(n-2)!2!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^4}{(n-4)!4!}-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^6}{(n-6)!6!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^8}{(n-8)!8!}+\ldots\right]+i\lim_{n\to\infty}\left[-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^1}{(n-1)!1!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^3}{(n-3)!3!}-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^5}{(n-5)!5!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^7}{(n-7)!7!}-\ldots\right]$$

now lets multiply by $k^{-a}$

$$\frac{1}{k^{(a+ib)}}=\frac{1}{k^a}\cdot\lim_{n\to\infty}\left(1-i\cdot\frac{b\cdot\ln k}{n}\right)^n=$$

$$\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[1-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^2}{(n-2)!2!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^4}{(n-4)!4!}-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^6}{(n-6)!6!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^8}{(n-8)!8!}+\ldots\right]+i\cdot\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^1}{(n-1)!1!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^3}{(n-3)!3!}-\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^5}{(n-5)!5!}+\frac{n!\left(\frac{b\cdot\ln k}{n}\right)^7}{(n-7)!7!}-\ldots\right]$$

$$\frac{1}{k^{(a+ib)}}=\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[1-\frac{n!(b\cdot\ln k)^2}{n^2(n-2)!2!}+\frac{n!(b\cdot\ln k)^4}{n^4(n-4)!4!}-\frac{n!(b\cdot\ln k)^6}{n^6(n-6)!6!}+\frac{n!(b\cdot\ln k)^8}{n^8(n-8)!8!}+\ldots\right]+i\cdot\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[-\frac{n!(b\cdot\ln k)^1}{n^1(n-1)!1!}+\frac{n!(b\cdot\ln k)^3}{n^3(n-3)!3!}-\frac{n!(b\cdot\ln k)^5}{n^5(n-5)!5!}+\frac{n!(b\cdot\ln k)^7}{n^7(n-7)!7!}-\ldots\right]$$

if $1\le m$ then

$$\lim_{n\to\infty}\frac{n!(b\cdot\ln k)^m}{n^m(n-m)!m!}=\frac{(b\cdot\ln k)^m}{m!}\cdot\lim_{n\to\infty}\frac{n!}{n^m(n-m)!}=$$

$$\frac{(b\cdot\ln k)^m}{m!}\cdot\lim_{n\to\infty}\frac{1}{n^m}\cdot\frac{1\cdot2\cdot3\cdot\ldots\cdot(n-m)\cdot\ldots\cdot(n-2)(n-1)n}{1\cdot2\cdot3\cdot\ldots\cdot(n-m)}=$$

$$\frac{(b\cdot\ln k)^m}{m!}\cdot\lim_{n\to\infty}\frac{(n-(m-1))\cdot\ldots\cdot(n-2)(n-1)(n-0)}{n^m}=$$

$$\frac{(b\cdot\ln k)^m}{m!}\cdot\lim_{n\to\infty}\frac{(n-(m-1))}{n}\cdot\ldots\cdot\frac{(n-2)}{n}\cdot\frac{(n-1)}{n}\cdot\frac{(n)}{n}=\frac{(b\cdot\ln k)^m}{m!}$$

$$\boxed{\lim_{n\to\infty}\frac{n!(b\cdot\ln k)^m}{n^m(n-m)!m!}=\frac{(b\cdot\ln k)^m}{m!}}$$

$$\frac{1}{k^{(a+ib)}}=\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[1-\frac{n!(b\cdot\ln k)^2}{n^2(n-2)!2!}+\frac{n!(b\cdot\ln k)^4}{n^4(n-4)!4!}-\frac{n!(b\cdot\ln k)^6}{n^6(n-6)!6!}+\frac{n!(b\cdot\ln k)^8}{n^8(n-8)!8!}+\ldots\right]+i\cdot\frac{1}{k^a}\cdot\lim_{n\to\infty}\left[-\frac{n!(b\cdot\ln k)^1}{n^1(n-1)!1!}+\frac{n!(b\cdot\ln k)^3}{n^3(n-3)!3!}-\frac{n!(b\cdot\ln k)^5}{n^5(n-5)!5!}+\frac{n!(b\cdot\ln k)^7}{n^7(n-7)!7!}-\ldots\right]$$

$$\frac{1}{k^{(a+ib)}}=\frac{1}{k^a}\cdot\left[1-\frac{(b\cdot\ln k)^2}{2!}+\frac{(b\cdot\ln k)^4}{4!}-\frac{(b\cdot\ln k)^6}{6!}+\frac{(b\cdot\ln k)^8}{8!}+\ldots\right]+i\cdot\frac{1}{k^a}\cdot\left[-\frac{(b\cdot\ln k)^1}{1!}+\frac{(b\cdot\ln k)^3}{3!}-\frac{(b\cdot\ln k)^5}{5!}+\frac{(b\cdot\ln k)^7}{7!}-\ldots\right]$$

$$\boxed{\cos(x)=\frac{1}{0!}-\frac{(x)^2}{2!}+\frac{(x)^4}{4!}-\frac{(x)^6}{6!}+\frac{(x)^8}{8!}-\ldots}\qquad\boxed{\sin(x)=\frac{(x)^1}{1!}-\frac{(x)^3}{3!}+\frac{(x)^5}{5!}-\frac{(x)^7}{7!}-\ldots}$$

$$\cos(b\cdot\ln k)=\left[1-\frac{(b\cdot\ln k)^2}{2!}+\frac{(b\cdot\ln k)^4}{4!}-\frac{(b\cdot\ln k)^6}{6!}+\frac{(b\cdot\ln k)^8}{8!}-\ldots\right]\qquad\sin(b\cdot\ln k)=\left[\frac{(b\cdot\ln k)^1}{1!}-\frac{(b\cdot\ln k)^3}{3!}+\frac{(b\cdot\ln k)^5}{5!}-\frac{(b\cdot\ln k)^7}{7!}-\ldots\right]$$

$$\boxed{\frac{1}{k^{(a+ib)}}=\frac{\cos(b\cdot\ln k)}{k^a}-i\cdot\frac{\sin(b\cdot\ln k)}{k^a}}$$

$$\boxed{\eta(a+ib)=\frac{1}{1^{(a+ib)}}-\frac{1}{2^{(a+ib)}}+\frac{1}{3^{(a+ib)}}-\frac{1}{4^{(a+ib)}}+\ldots}$$

$$+\frac{1}{1^{(a+ib)}}=\left[+\frac{\cos(b\cdot\ln 1)}{1^a}\right]+i\cdot\left[-\frac{\sin(b\cdot\ln 1)}{1^a}\right]$$

$$-\frac{1}{2^{(a+ib)}}=\left[-\frac{\cos(b\cdot\ln 2)}{2^a}\right]+i\cdot\left[+\frac{\sin(b\cdot\ln 2)}{2^a}\right]$$

$$+\frac{1}{3^{(a+ib)}}=\left[+\frac{\cos(b\cdot\ln 3)}{3^a}\right]+i\cdot\left[-\frac{\sin(b\cdot\ln 3)}{3^a}\right]$$

$$-\frac{1}{4^{(a+ib)}}=\left[-\frac{\cos(b\cdot\ln 4)}{4^a}\right]+i\cdot\left[+\frac{\sin(b\cdot\ln 4)}{4^a}\right]$$

$$\eta(a+ib)=\frac{1}{1^{(a+ib)}}-\frac{1}{2^{(a+ib)}}+\frac{1}{3^{(a+ib)}}-\frac{1}{4^{(a+ib)}}+\ldots=\left[\frac{\cos(b\ln 1)}{1^a}-\frac{\cos(b\ln 2)}{2^a}+\frac{\cos(b\ln 3)}{3^a}-\frac{\cos(b\ln 4)}{4^a}+\ldots\right]+i\cdot\left[-\frac{\sin(b\ln 1)}{1^a}+\frac{\sin(b\ln 2)}{2^a}-\frac{\sin(b\ln 3)}{3^a}+\frac{\sin(b\ln 4)}{4^a}+\ldots\right]$$

another way (and much more easier way) to look at this is:

$$\eta(a+ib)=\left[\frac{1}{1^a}\cdot\cos(-b\ln 1)-\frac{1}{2^a}\cdot\cos(-b\ln 2)+\frac{1}{3^a}\cdot\cos(-b\ln 3)-\frac{1}{4^a}\cdot\cos(-b\ln 4)+....\right]+\left[\frac{1}{1^a}\cdot\sin(-b\ln 1)-\frac{1}{2^a}\cdot\sin(-b\ln 2)+\frac{1}{3^a}\cdot\sin(-b\ln 3)-\frac{1}{4^a}\cdot\sin(-b\ln 4)+....\right]\cdot i$$

$$\vec{V}_k=\frac{1}{1^k} \qquad \theta_k=-b\ln k$$

$$\eta(a+ib)=\left[\vec{V}_1\cdot\cos(\theta_1)-\vec{V}_2\cdot\cos(\theta_2)+\vec{V}_3\cdot\cos(\theta_3)-\vec{V}_4\cdot\cos(\theta_4)+....\right]+\left[\vec{V}_1\cdot\sin(\theta_1)-\vec{V}_2\cdot\sin(\theta_2)+\vec{V}_3\cdot\sin(\theta_3)-\vec{V}_4\cdot\sin(\theta_4)+....\right]\cdot i$$

moving on the xAxis $\vec{V}_1\cdot\cos(\theta_1)-\vec{V}_2\cdot\cos(\theta_2)+\vec{V}_3\cdot\cos(\theta_3)-\vec{V}_4\cdot\cos(\theta_4)+....$

moving on the yAxis $\vec{V}_1\cdot\sin(\theta_1)-\vec{V}_2\cdot\sin(\theta_2)+\vec{V}_3\cdot\sin(\theta_3)-\vec{V}_4\cdot\sin(\theta_4)+....$

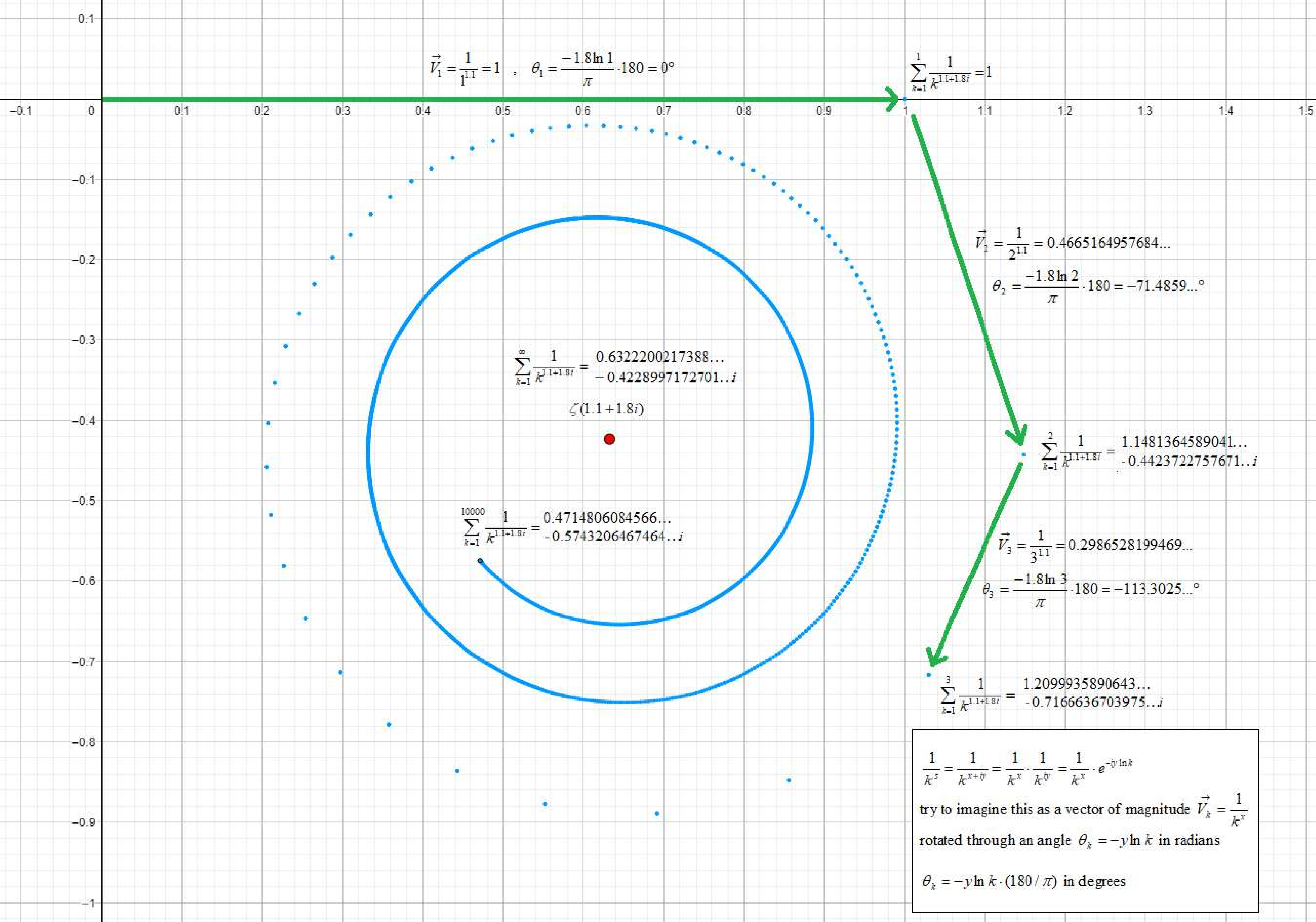
when xAxis=0 and yAxis=0 then $\eta(s)=0$ meaning that also $\xi(s)=\dfrac{\eta(s)}{(1-2^{1-s})}=0$

this helps extend the zeta function from Re(s) > 1 to the larger domain

The Riemann hypothesis equivalent to:

$$\boxed{0=\frac{\cos(b\ln 1)}{1^a}-\frac{\cos(b\ln 2)}{2^a}+\frac{\cos(b\ln 3)}{3^a}-\frac{\cos(b\ln 4)}{4^a}+....} \text{ and } \boxed{0=\frac{\sin(b\ln 1)}{1^a}-\frac{\sin(b\ln 2)}{2^a}+\frac{\sin(b\ln 3)}{3^a}-\frac{\sin(b\ln 4)}{4^a}+....}$$

where a and b are real numbers and the only solution is when $a=\dfrac{1}{2}$



$$\vec{V}_1 = \frac{1}{1^{1.1}} = 1 \quad , \quad \theta_1 = \frac{-1.8\ln 1}{\pi}\cdot 180 = 0°$$

$$\sum_{k=1}^{1}\frac{1}{k^{1.1+1.8i}} = 1$$

$$\vec{V}_2 = \frac{1}{2^{1.1}} = 0.4665164957684\ldots$$

$$\theta_2 = \frac{-1.8\ln 2}{\pi}\cdot 180 = -71.4859\ldots°$$

$$\sum_{k=1}^{2}\frac{1}{k^{1.1+1.8i}} = \begin{array}{l}1.1481364589041\ldots\\ -0.4423722757671\ldots i\end{array}$$

$$\vec{V}_3 = \frac{1}{3^{1.1}} = 0.2986528199469\ldots$$

$$\theta_3 = \frac{-1.8\ln 3}{\pi}\cdot 180 = -113.3025\ldots°$$

$$\sum_{k=1}^{3}\frac{1}{k^{1.1+1.8i}} = \begin{array}{l}1.2099935890643\ldots\\ -0.7166636703975\ldots i\end{array}$$

$$\sum_{k=1}^{\infty}\frac{1}{k^{1.1+1.8i}} = \begin{array}{l}0.6322200217388\ldots\\ -0.4228997172701\ldots i\end{array}$$

$$\zeta(1.1+1.8i)$$

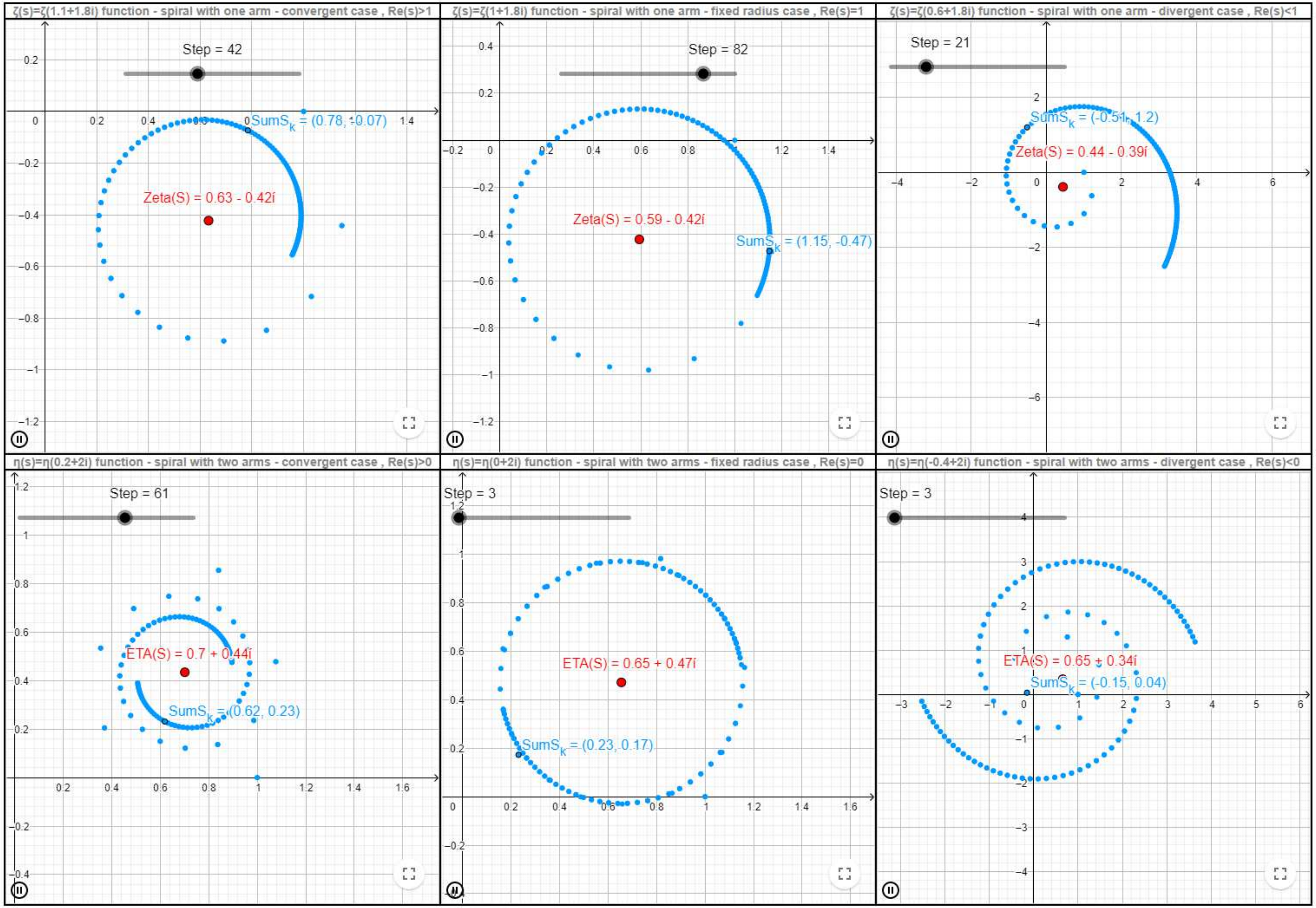
$$\sum_{k=1}^{10000}\frac{1}{k^{1.1+1.8i}} = \begin{array}{l}0.4714806084566\ldots\\ -0.5743206467464\ldots i\end{array}$$

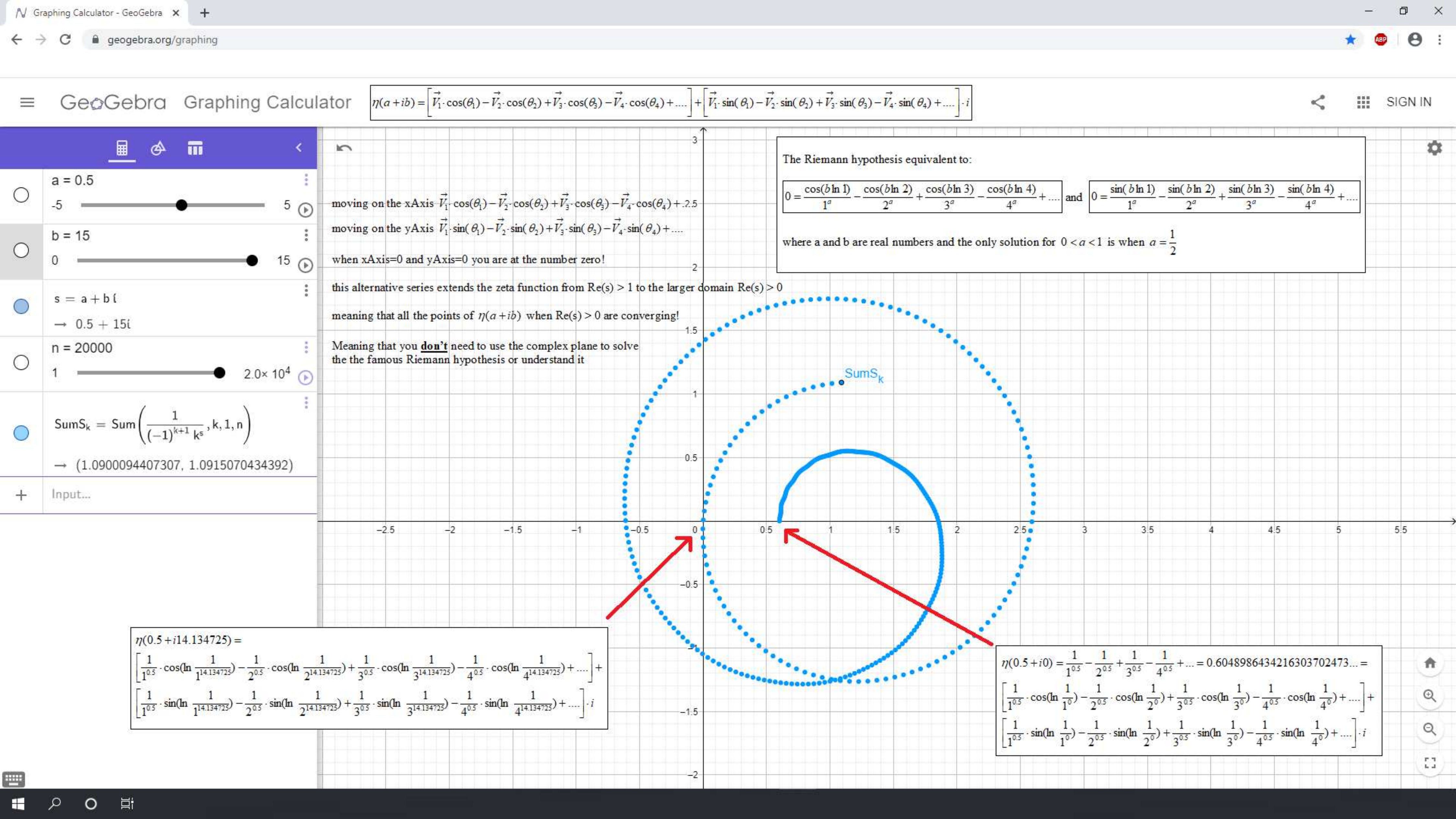
$$\frac{1}{k^s} = \frac{1}{k^{x+iy}} = \frac{1}{k^x}\cdot\frac{1}{k^{iy}} = \frac{1}{k^x}\cdot e^{-iy\ln k}$$

try to imagine this as a vector of magnitude $\vec{V}_k = \frac{1}{k^x}$

rotated through an angle $\theta_k = -y\ln k$ in radians

$\theta_k = -y\ln k\cdot(180/\pi)$ in degrees


ζ(s)=ζ(1.1+1.8i) function - spiral with one arm - convergent case , Re(s)>1
Step = 42
SumS$_k$ = (0.78, -0.07)
Zeta(S) = 0.63 - 0.42i
ζ(s)=ζ(1+1.8i) function - spiral with one arm - fixed radius case , Re(s)=1
Step = 82
Zeta(S) = 0.59 - 0.42i
SumS$_k$ = (1.15, -0.47)
ζ(s)=ζ(0.6+1.8i) function - spiral with one arm - divergent case , Re(s)<1
Step = 21
SumS$_k$ = (-0.51, 1.2)
Zeta(S) = 0.44 - 0.39i
η(s)=η(0.2+2i) function - spiral with two arms - convergent case , Re(s)>0
Step = 61
ETA(S) = 0.7 + 0.44i
SumS$_k$ = (0.62, 0.23)
η(s)=η(0+2i) function - spiral with two arms - fixed radius case , Re(s)=0
Step = 3
ETA(S) = 0.65 + 0.47i
SumS$_k$ = (0.23, 0.17)
η(s)=η(-0.4+2i) function - spiral with two arms - divergent case , Re(s)<0
Step = 3
ETA(S) = 0.65 + 0.34i
SumS$_k$ = (-0.15, 0.04)



$$\eta(a+ib)=\left[\vec{V_1}\cdot\cos(\theta_1)-\vec{V_2}\cdot\cos(\theta_2)+\vec{V_3}\cdot\cos(\theta_3)-\vec{V_4}\cdot\cos(\theta_4)+....\right]+\left[\vec{V_1}\cdot\sin(\theta_1)-\vec{V_2}\cdot\sin(\theta_2)+\vec{V_3}\cdot\sin(\theta_3)-\vec{V_4}\cdot\sin(\theta_4)+....\right]\cdot i$$

a = 0.5

-5 5

b = 15

0 15

s = a + b ί

→ 0.5 + 15ί

n = 20000

1 $2.0\times10^4$

$\text{SumS}_k = \text{Sum}\left(\frac{1}{(-1)^{k+1}\,k^s}, k, 1, n\right)$

→ (1.0900094407307, 1.0915070434392)

moving on the xAxis $\vec{V_1}\cdot\cos(\theta_1)-\vec{V_2}\cdot\cos(\theta_2)+\vec{V_3}\cdot\cos(\theta_3)-\vec{V_4}\cdot\cos(\theta_4)+...$

moving on the yAxis $\vec{V_1}\cdot\sin(\theta_1)-\vec{V_2}\cdot\sin(\theta_2)+\vec{V_3}\cdot\sin(\theta_3)-\vec{V_4}\cdot\sin(\theta_4)+....$

when xAxis=0 and yAxis=0 you are at the number zero!

this alternative series extends the zeta function from Re(s) > 1 to the larger domain Re(s) > 0

meaning that all the points of $\eta(a+ib)$ when Re(s) > 0 are converging!

Meaning that you **don't** need to use the complex plane to solve
the the famous Riemann hypothesis or understand it

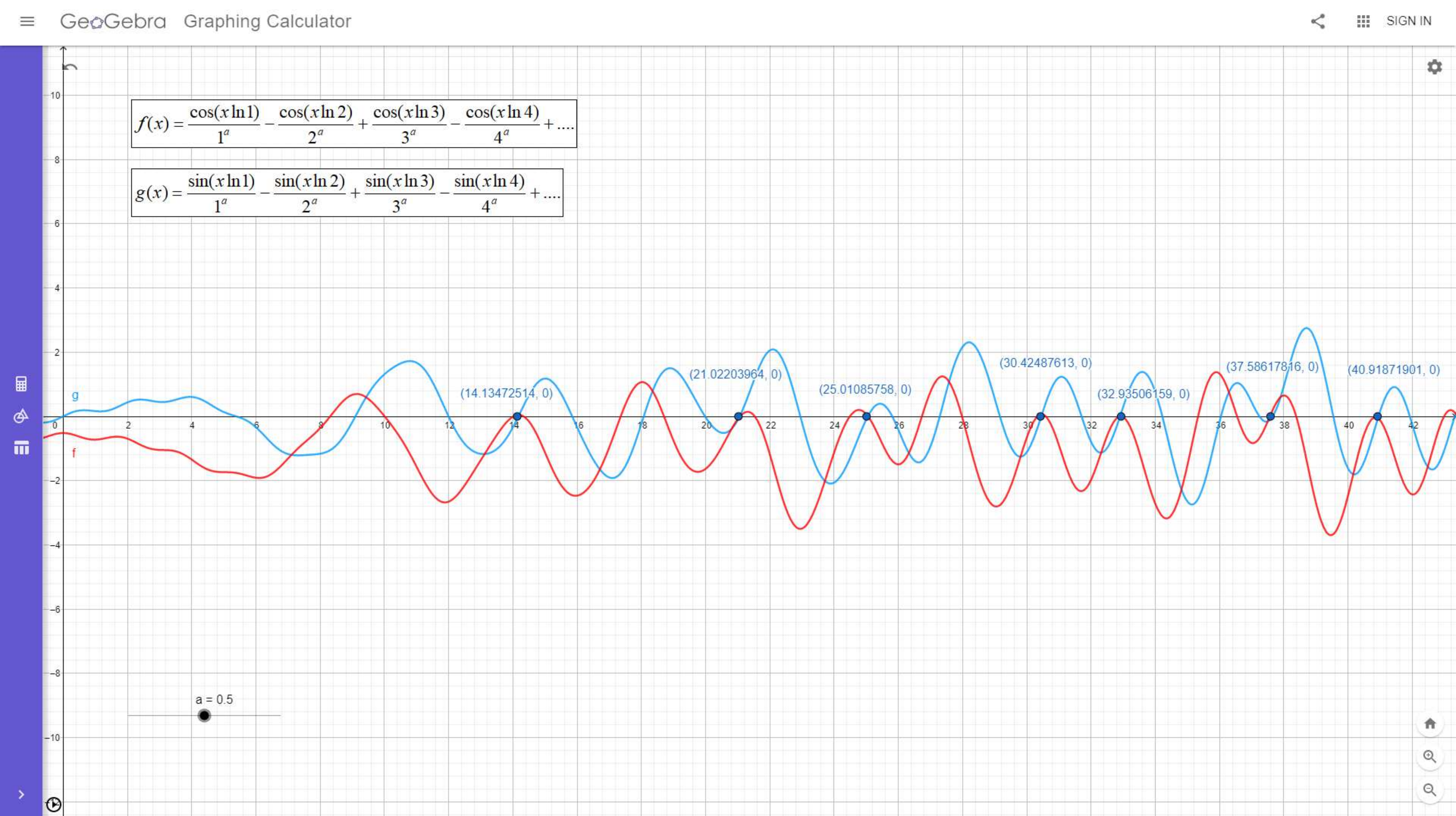
The Riemann hypothesis equivalent to:

$$\boxed{0=\frac{\cos(b\ln 1)}{1^a}-\frac{\cos(b\ln 2)}{2^a}+\frac{\cos(b\ln 3)}{3^a}-\frac{\cos(b\ln 4)}{4^a}+....}\text{ and }\boxed{0=\frac{\sin(b\ln 1)}{1^a}-\frac{\sin(b\ln 2)}{2^a}+\frac{\sin(b\ln 3)}{3^a}-\frac{\sin(b\ln 4)}{4^a}+....}$$

where a and b are real numbers and the only solution for $0<a<1$ is when $a=\frac{1}{2}$

$$\eta(0.5+i14.134725)=$$
$$\left[\frac{1}{1^{0.5}}\cdot\cos(\ln\frac{1}{1^{14.134725}})-\frac{1}{2^{0.5}}\cdot\cos(\ln\frac{1}{2^{14.134725}})+\frac{1}{3^{0.5}}\cdot\cos(\ln\frac{1}{3^{14.134725}})-\frac{1}{4^{0.5}}\cdot\cos(\ln\frac{1}{4^{14.134725}})+....\right]+$$
$$\left[\frac{1}{1^{0.5}}\cdot\sin(\ln\frac{1}{1^{14.134725}})-\frac{1}{2^{0.5}}\cdot\sin(\ln\frac{1}{2^{14.134725}})+\frac{1}{3^{0.5}}\cdot\sin(\ln\frac{1}{3^{14.134725}})-\frac{1}{4^{0.5}}\cdot\sin(\ln\frac{1}{4^{14.134725}})+....\right]\cdot i$$

$$\eta(0.5+i0)=\frac{1}{1^{0.5}}-\frac{1}{2^{0.5}}+\frac{1}{3^{0.5}}-\frac{1}{4^{0.5}}+...=0.6048986434216303702473...=$$
$$\left[\frac{1}{1^{0.5}}\cdot\cos(\ln\frac{1}{1^{0}})-\frac{1}{2^{0.5}}\cdot\cos(\ln\frac{1}{2^{0}})+\frac{1}{3^{0.5}}\cdot\cos(\ln\frac{1}{3^{0}})-\frac{1}{4^{0.5}}\cdot\cos(\ln\frac{1}{4^{0}})+....\right]+$$
$$\left[\frac{1}{1^{0.5}}\cdot\sin(\ln\frac{1}{1^{0}})-\frac{1}{2^{0.5}}\cdot\sin(\ln\frac{1}{2^{0}})+\frac{1}{3^{0.5}}\cdot\sin(\ln\frac{1}{3^{0}})-\frac{1}{4^{0.5}}\cdot\sin(\ln\frac{1}{4^{0}})+....\right]\cdot i$$



$$f(x) = \frac{\cos(x\ln 1)}{1^a} - \frac{\cos(x\ln 2)}{2^a} + \frac{\cos(x\ln 3)}{3^a} - \frac{\cos(x\ln 4)}{4^a} + \ldots.$$

$$g(x) = \frac{\sin(x\ln 1)}{1^a} - \frac{\sin(x\ln 2)}{2^a} + \frac{\sin(x\ln 3)}{3^a} - \frac{\sin(x\ln 4)}{4^a} + \ldots.$$

a = 0.5

$$\boxed{f(x)=\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}-\frac{\cos(x\ln 4)}{4^a}+....}$$

$$\boxed{g(x)=\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+....}$$

I am going to make a new function $h(x)$ that will include both cases and will be 0 only when both functions $f(x)$ and $g(x)$ are 0 as well

The simplest way is to have $h(x)=f(x)f(x)+g(x)g(x)$ where $h(x)\geq 0$ and $h(x)=0$ only when you have those non-trival zeros

$$\boxed{f(x)=\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}-\frac{\cos(x\ln 4)}{4^a}+....}$$

$$f(x)f(x)=?$$

$$\boxed{\begin{aligned}
&+\frac{\cos(x\ln 1)}{1^a}\cdot\left(\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}-\frac{\cos(x\ln 4)}{4^a}+...\right)\\
&-\frac{\cos(x\ln 2)}{2^a}\cdot\left(\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}-\frac{\cos(x\ln 4)}{4^a}+...\right)\\
&+\frac{\cos(x\ln 3)}{3^a}\cdot\left(\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}-\frac{\cos(x\ln 4)}{4^a}+...\right)\\
&-\frac{\cos(x\ln 4)}{4^a}\cdot\left(\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}-\frac{\cos(x\ln 4)}{4^a}+...\right)
\end{aligned}}$$

$$+\frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 3)}{3^a}-\frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 4)}{4^a}+...$$

$$-\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 1)}{1^a}+\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 2)}{2^a}-\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 3)}{3^a}+\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 4)}{4^a}-...$$

$$+\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 3)}{3^a}-\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 4)}{4^a}+...$$

$$-\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 1)}{1^a}+\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 2)}{2^a}-\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 3)}{3^a}+\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 4)}{4^a}-...$$

$$+\frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 1)}{1^a}$$

$$-2\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 1)}{1^a}+\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 2)}{2^a}$$

$$+2\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 1)}{1^a}-2\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 3)}{3^a}$$

$$-2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 1)}{1^a}+2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 2)}{2^a}-2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 3)}{3^a}+\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 4)}{4^a}-...$$

$$\boxed{g(x)=\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+....}$$

$$g(x)g(x)=?$$

$$\boxed{\begin{aligned}
&+\frac{\sin(x\ln 1)}{1^a}\cdot\left(\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+...\right)\\
&-\frac{\sin(x\ln 2)}{2^a}\cdot\left(\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+...\right)\\
&+\frac{\sin(x\ln 3)}{3^a}\cdot\left(\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+...\right)\\
&-\frac{\sin(x\ln 4)}{4^a}\cdot\left(\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+...\right)
\end{aligned}}$$

$$\begin{aligned}
&+\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 4)}{4^a}+...\\
&-\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 1)}{1^a}+\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 2)}{2^a}-\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 3)}{3^a}+\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 4)}{4^a}-...\\
&+\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 4)}{4^a}+...\\
&-\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 1)}{1^a}+\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 2)}{2^a}-\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 3)}{3^a}+\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 4)}{4^a}-...
\end{aligned}$$

$$\begin{aligned}
&+\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 1)}{1^a}\\
&-2\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 1)}{1^a}+\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 2)}{2^a}\\
&+2\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 1)}{1^a}-2\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 3)}{3^a}\\
&-2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 1)}{1^a}+2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 2)}{2^a}-2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 3)}{3^a}+2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 4)}{4^a}-...
\end{aligned}$$

$$h(x) = f(x)f(x) + g(x)g(x) = ?$$

now lets combine the two functions

$$
\begin{aligned}
&+\frac{\cos(x\ln 1)}{1^a}\cdot\frac{\cos(x\ln 1)}{1^a}\\
&-2\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 1)}{1^a}+\frac{\cos(x\ln 2)}{2^a}\cdot\frac{\cos(x\ln 2)}{2^a}\\
&+2\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 1)}{1^a}-2\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}\cdot\frac{\cos(x\ln 3)}{3^a}\\
&-2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 1)}{1^a}+2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 2)}{2^a}-2\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 3)}{3^a}+\frac{\cos(x\ln 4)}{4^a}\cdot\frac{\cos(x\ln 4)}{4^a}-\ldots
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\sin(x\ln 1)}{1^a}\cdot\frac{\sin(x\ln 1)}{1^a}\\
&-2\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 1)}{1^a}+\frac{\sin(x\ln 2)}{2^a}\cdot\frac{\sin(x\ln 2)}{2^a}\\
&+2\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 1)}{1^a}-2\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}\cdot\frac{\sin(x\ln 3)}{3^a}\\
&-2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 1)}{1^a}+2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 2)}{2^a}-2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 3)}{3^a}+2\frac{\sin(x\ln 4)}{4^a}\cdot\frac{\sin(x\ln 4)}{4^a}-\ldots
\end{aligned}
$$

$$\boxed{\cos(a-b)=\cos(a)\cos(b)+\sin(a)\sin(b)}$$

now lets merge the two functions

$$
\begin{aligned}
&+\frac{\cos(x\ln 1-x\ln 1)}{1^a\cdot 1^a}\\
&-2\frac{\cos(x\ln 2-x\ln 1)}{2^a\cdot 1^a}+\frac{\cos(x\ln 2-x\ln 2)}{2^a\cdot 2^a}\\
&+2\frac{\cos(x\ln 3-x\ln 1)}{3^a\cdot 1^a}-2\frac{\cos(x\ln 3-x\ln 2)}{3^a\cdot 2^a}+\frac{\cos(x\ln 3-x\ln 3)}{3^a\cdot 3^a}\\
&-2\frac{\cos(x\ln 4-x\ln 1)}{4^a\cdot 1^a}+2\frac{\cos(x\ln 4-x\ln 2)}{4^a\cdot 2^a}-2\frac{\cos(x\ln 4-x\ln 3)}{4^a\cdot 3^a}+\frac{\cos(x\ln 4-x\ln 4)}{4^a\cdot 4^a}
\end{aligned}
$$

$$
\begin{array}{llll}
+\frac{\cos(x\ln 1/1)}{1^a\cdot 1^a} & & & \\
-2\frac{\cos(x\ln 2/1)}{2^a\cdot 1^a} & +\frac{\cos(x\ln 2/2)}{2^a\cdot 2^a} & & \\
+2\frac{\cos(x\ln 3/1)}{3^a\cdot 1^a} & -2\frac{\cos(x\ln 3/2)}{3^a\cdot 2^a} & +\frac{\cos(x\ln 3/3)}{3^a\cdot 3^a} & \\
-2\frac{\cos(x\ln 4/1)}{4^a\cdot 1^a} & +2\frac{\cos(x\ln 4/2)}{4^a\cdot 2^a} & -2\frac{\cos(x\ln 4/3)}{4^a\cdot 3^a} & +\frac{\cos(x\ln 4/4)}{4^a\cdot 4^a}
\end{array}
$$

$$
\begin{array}{llll}
+\frac{1}{1^a\cdot 1^a} & & & \\
-2\frac{\cos(x\ln 2/1)}{2^a\cdot 1^a} & +\frac{1}{2^a\cdot 2^a} & & \\
+2\frac{\cos(x\ln 3/1)}{3^a\cdot 1^a} & -2\frac{\cos(x\ln 3/2)}{3^a\cdot 2^a} & +\frac{1}{3^a\cdot 3^a} & \\
-2\frac{\cos(x\ln 4/1)}{4^a\cdot 1^a} & +2\frac{\cos(x\ln 4/2)}{4^a\cdot 2^a} & -2\frac{\cos(x\ln 4/3)}{4^a\cdot 3^a} & +\frac{1}{4^a\cdot 4^a}
\end{array}
$$

$$
\begin{array}{llll}
+\frac{1}{1^a\cdot 1^a} & & & \\
+\frac{1}{2^a\cdot 2^a} & -2\frac{\cos(x\ln 2/1)}{2^a\cdot 1^a} & & \\
+\frac{1}{3^a\cdot 3^a} & +2\frac{\cos(x\ln 3/1)}{3^a\cdot 1^a} & -2\frac{\cos(x\ln 3/2)}{3^a\cdot 2^a} & \\
+\frac{1}{4^a\cdot 4^a} & -2\frac{\cos(x\ln 4/1)}{4^a\cdot 1^a} & +2\frac{\cos(x\ln 4/2)}{4^a\cdot 2^a} & -2\frac{\cos(x\ln 4/3)}{4^a\cdot 3^a}
\end{array}
$$

$$
\zeta(2a)+\boxed{\begin{array}{l}
-2\frac{\cos(x\ln 2/1)}{2^a\cdot 1^a} \\
+2\frac{\cos(x\ln 3/1)}{3^a\cdot 1^a}-2\frac{\cos(x\ln 3/2)}{3^a\cdot 2^a} \\
-2\frac{\cos(x\ln 4/1)}{4^a\cdot 1^a}+2\frac{\cos(x\ln 4/2)}{4^a\cdot 2^a}-2\frac{\cos(x\ln 4/3)}{4^a\cdot 3^a} \\
+2\frac{\cos(x\ln 5/1)}{5^a\cdot 1^a}-2\frac{\cos(x\ln 5/2)}{5^a\cdot 2^a}+2\frac{\cos(x\ln 5/3)}{5^a\cdot 3^a}-2\frac{\cos(x\ln 5/4)}{5^a\cdot 4^a}
\end{array}}
$$

$$\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n}\frac{2\cos(x\ln(n/k))}{(nk)^a}=\boxed{\begin{array}{l}
-2\dfrac{\cos(x\ln 2/1)}{2^a\cdot 1^a}\\
+2\dfrac{\cos(x\ln 3/1)}{3^a\cdot 1^a}-2\dfrac{\cos(x\ln 3/2)}{3^a\cdot 2^a}\\
-2\dfrac{\cos(x\ln 4/1)}{4^a\cdot 1^a}+2\dfrac{\cos(x\ln 4/2)}{4^a\cdot 2^a}-2\dfrac{\cos(x\ln 4/3)}{4^a\cdot 3^a}\\
+2\dfrac{\cos(x\ln 5/1)}{5^a\cdot 1^a}-2\dfrac{\cos(x\ln 5/2)}{5^a\cdot 2^a}+2\dfrac{\cos(x\ln 5/3)}{5^a\cdot 3^a}-2\dfrac{\cos(x\ln 5/4)}{5^a\cdot 4^a}\\
-\ldots\\
+\ldots\\
-\ldots
\end{array}}$$

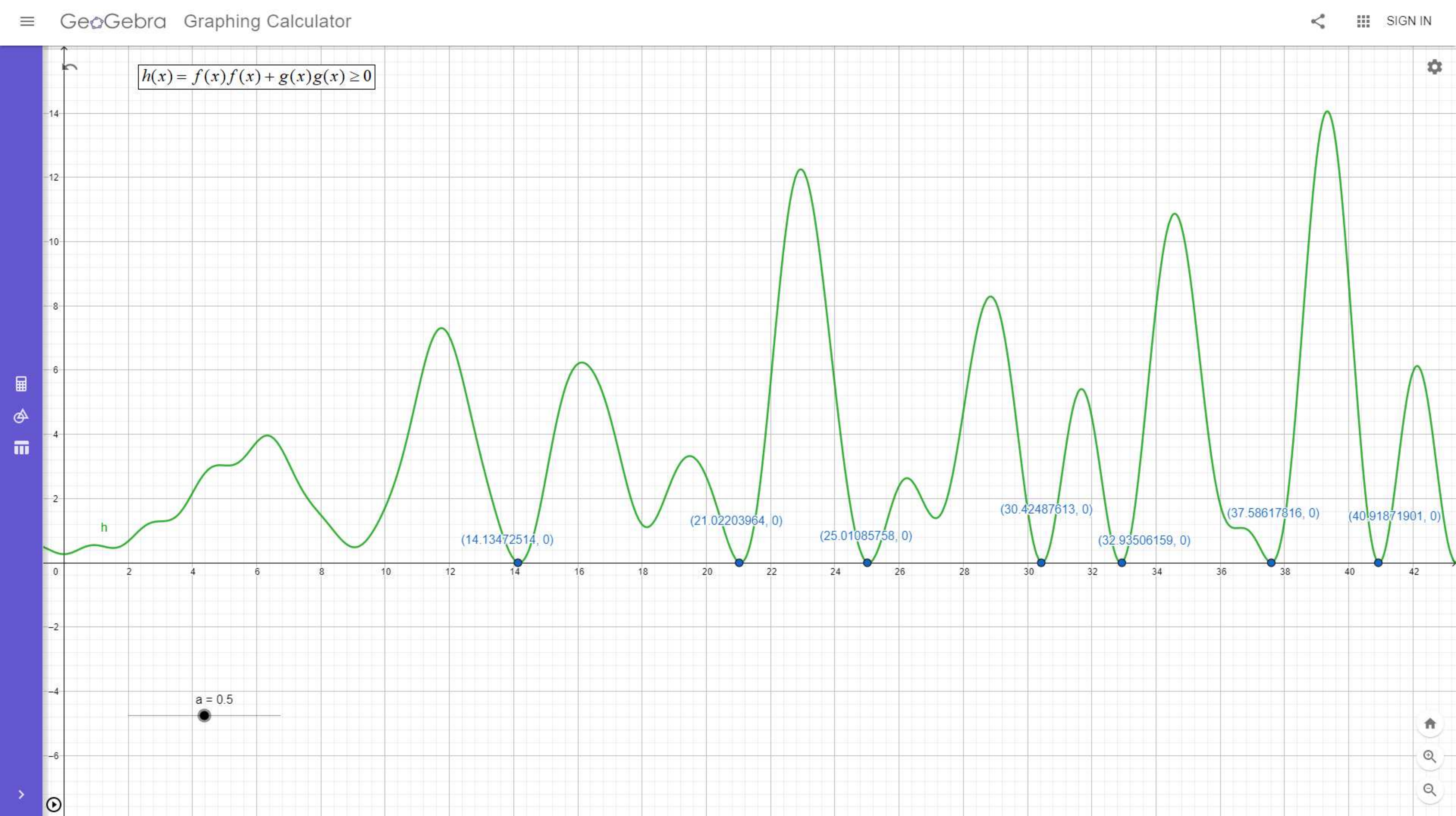
$$h(x)=f(x)f(x)+g(x)g(x)=\zeta(2a)+\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n}\frac{2\cos(x\ln(n/k))}{(nk)^a}$$

$$\boxed{0\le h(x)=\zeta(2a)+\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n}\frac{2\cos(x\ln(n/k))}{(nk)^a}}$$

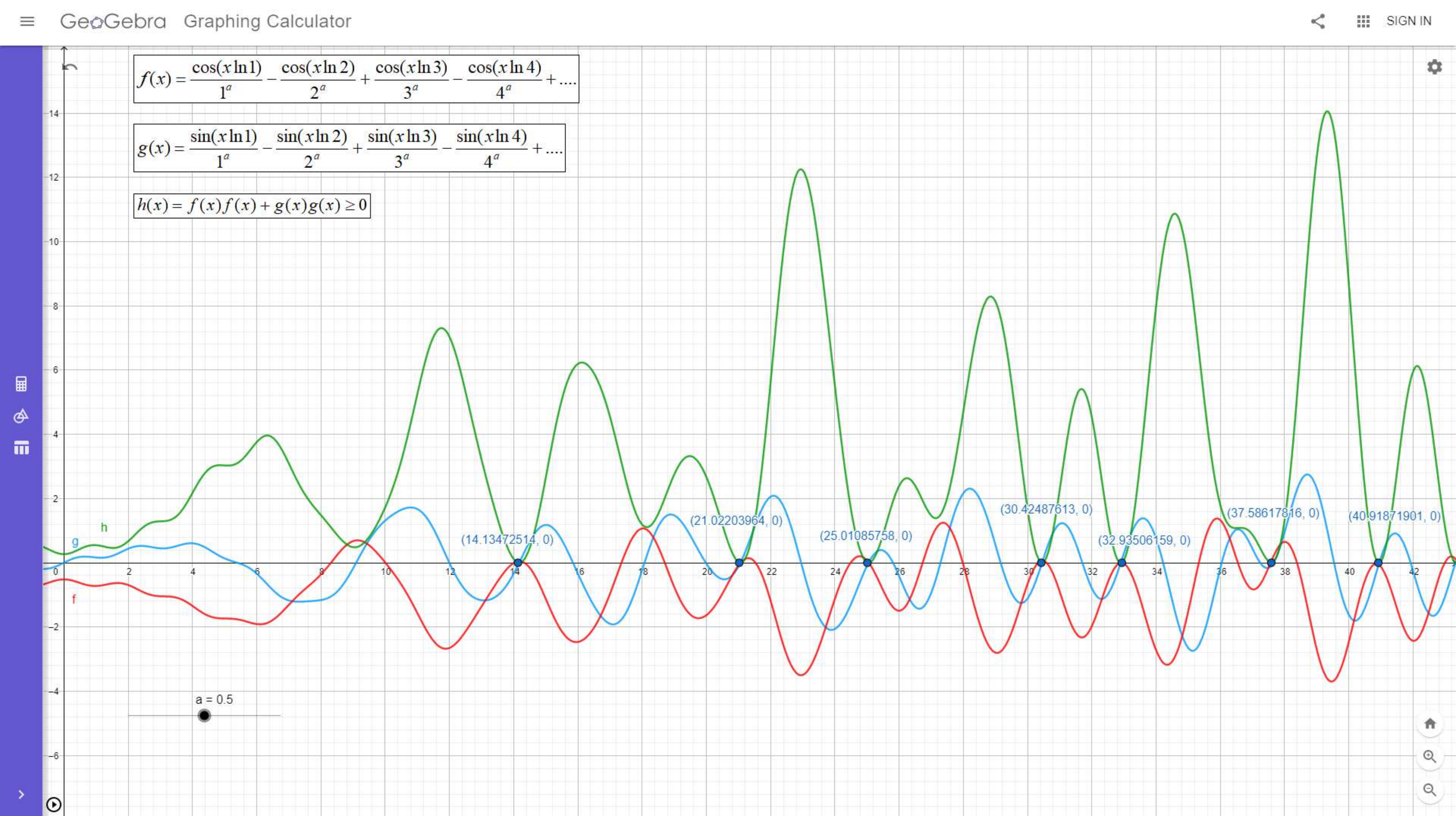
$$-\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n}\frac{2\cos(x\ln(n/k))}{(nk)^a}\le\zeta(2a)$$

$$\boxed{q(x)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(x\ln(n/k))}{(nk)^a}\le\zeta(2a)}$$

When $q(b)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\dfrac{2\cos(b\ln(n/k))}{(nk)^a}=\zeta(2a)$ then $\zeta(a+ib)$ is a non trivial zero(because $h(b)=0$)



$h(x)=f(x)f(x)+g(x)g(x)\geq 0$

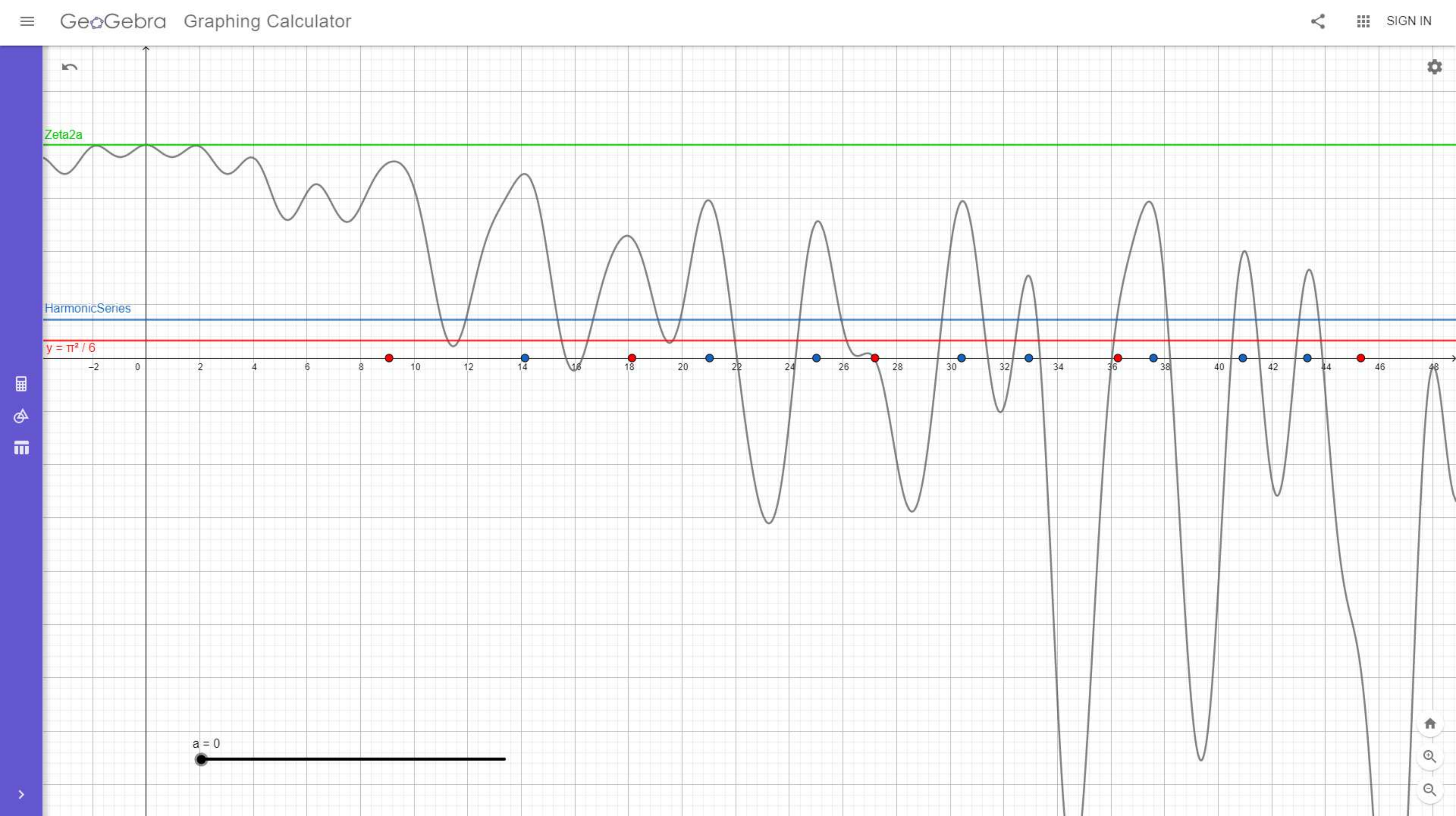


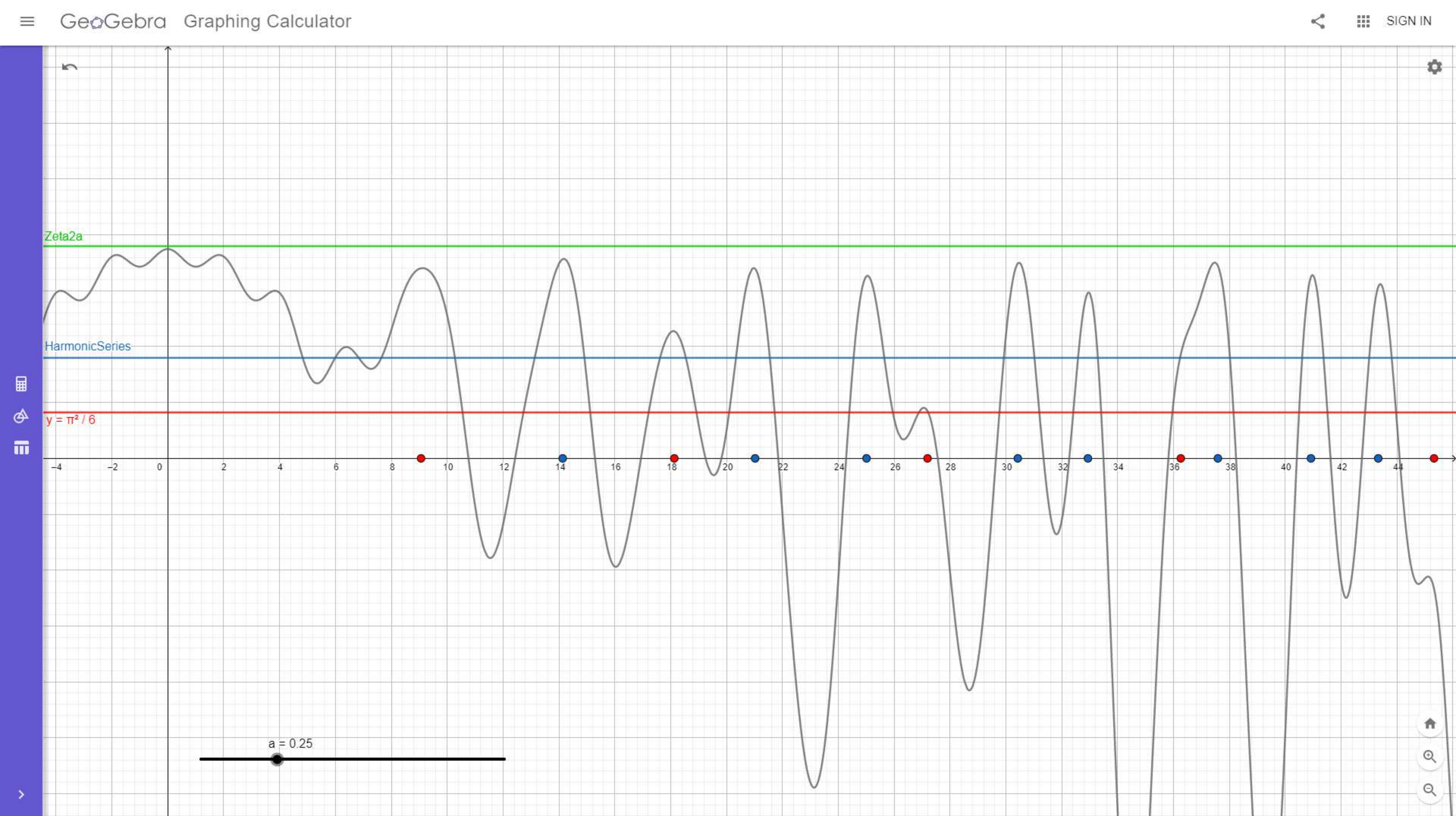
$$f(x)=\frac{\cos(x\ln 1)}{1^a}-\frac{\cos(x\ln 2)}{2^a}+\frac{\cos(x\ln 3)}{3^a}-\frac{\cos(x\ln 4)}{4^a}+....$$

$$g(x)=\frac{\sin(x\ln 1)}{1^a}-\frac{\sin(x\ln 2)}{2^a}+\frac{\sin(x\ln 3)}{3^a}-\frac{\sin(x\ln 4)}{4^a}+....$$

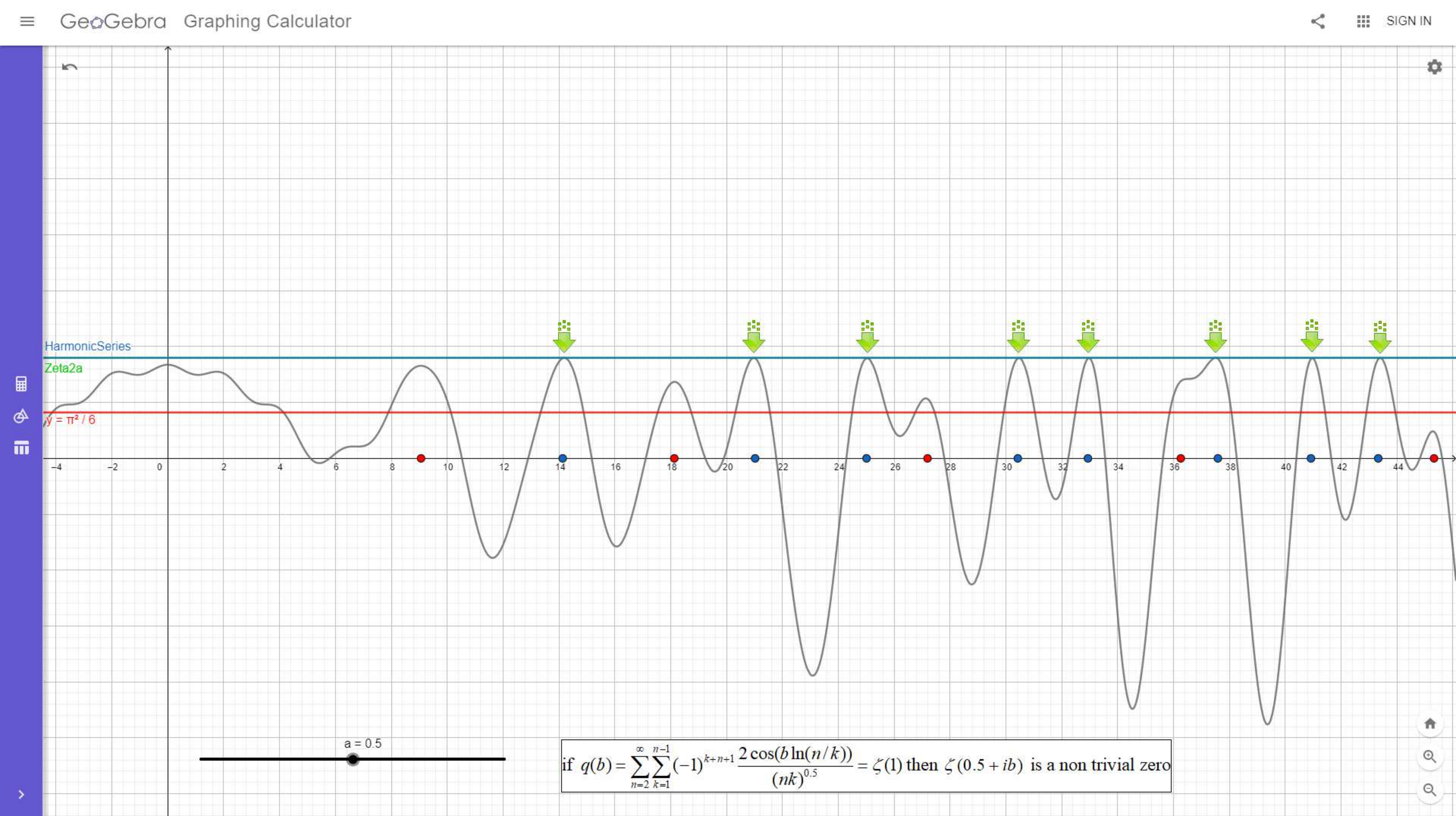
$$h(x)=f(x)f(x)+g(x)g(x)\geq 0$$

a = 0.5

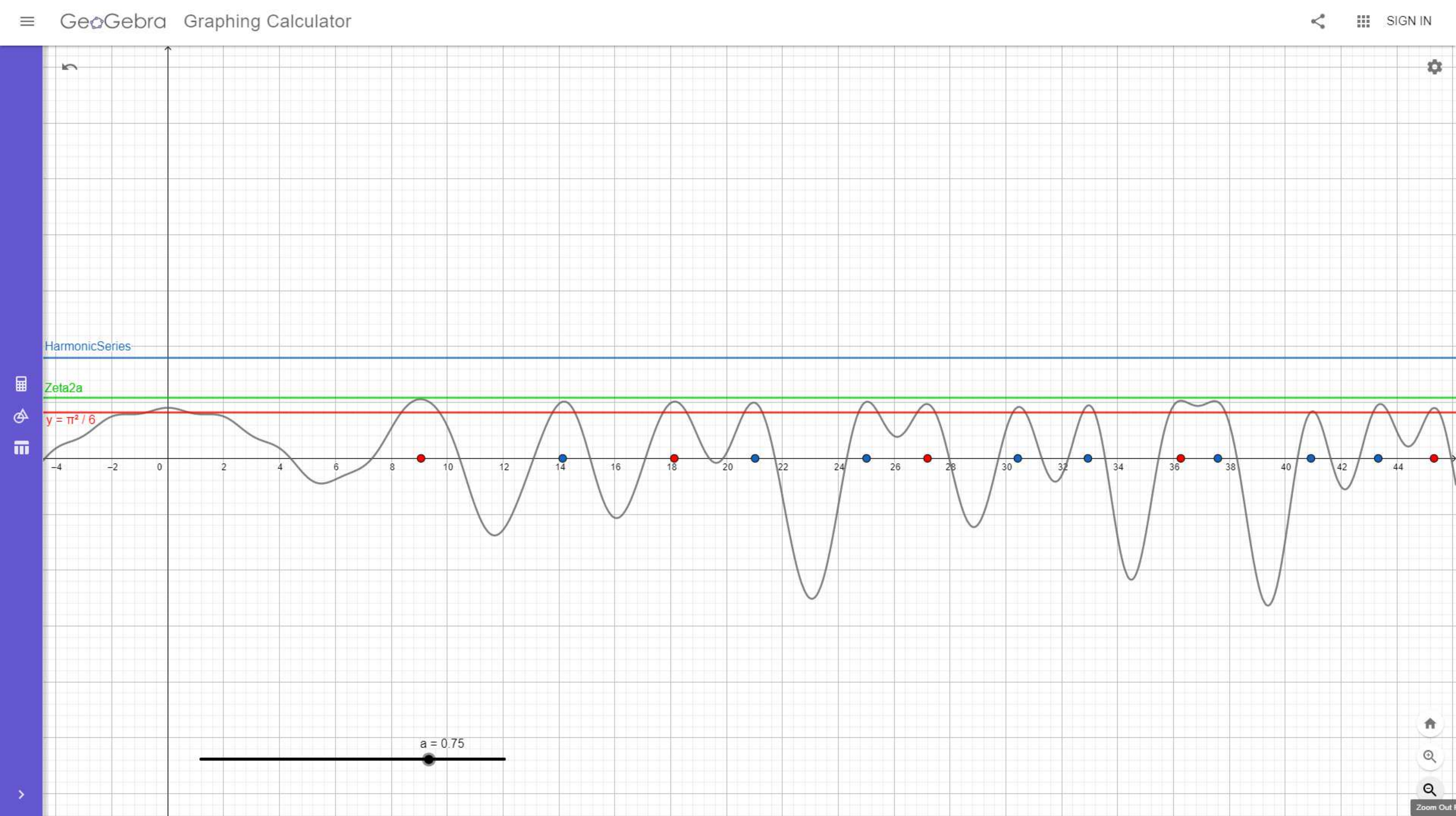

GeoGebra Graphing Calculator
SIGN IN
Zeta2a
HarmonicSeries
y = π² / 6
–2
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
46
48
a = 0

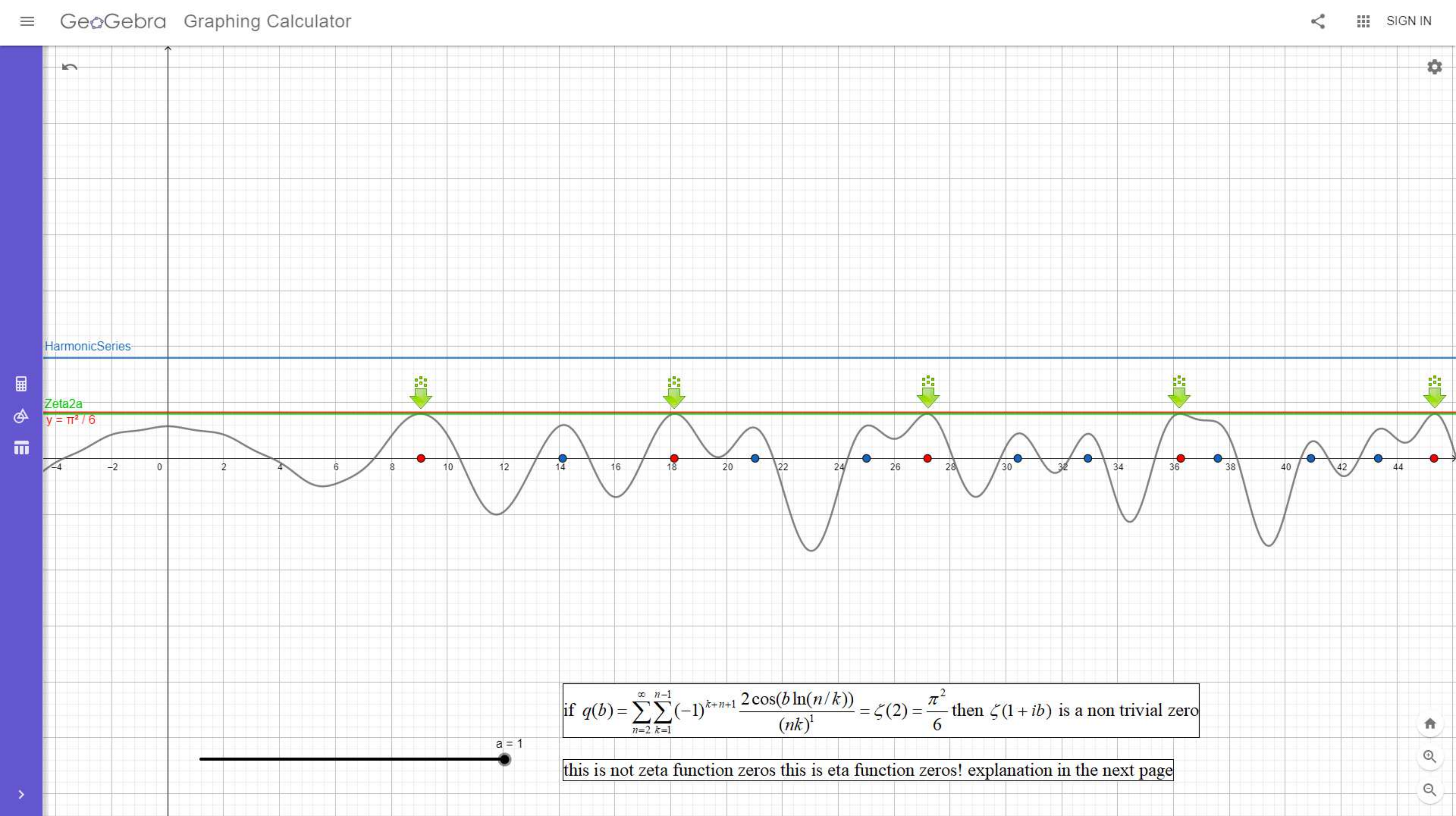

GeoGebra
Graphing Calculator
SIGN IN
Zeta2a
HarmonicSeries
y = π² / 6
−4
−2
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44

a = 0.25



if $q(b)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^{0.5}}=\zeta(1)$ then $\zeta(0.5+ib)$ is a non trivial zero


GeoGebra Graphing Calculator
SIGN IN
HarmonicSeries
Zeta2a
y = π² / 6
–4
–2
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
a = 0.75
Zoom Out



if $q(b)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^{1}}=\zeta(2)=\frac{\pi^{2}}{6}$ then $\zeta(1+ib)$ is a non trivial zero

this is not zeta function zeros this is eta function zeros! explanation in the next page

(For $1 < a$ there are no non trivial zeros this is a known fact so I am not showing why)

i used eta function summation to get $h(x)$ and because $\boxed{\left(1-\frac{2}{2^s}\right)\zeta(s)=\eta(s)}$ then for $b=\frac{2\pi t}{\ln 2}$ when $a=1$ we get

$$q(b)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^1}=\zeta(2)$$

$$\boxed{\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos\left(2\pi t\frac{\ln(n/k)}{\ln 2}\right)}{(nk)^1}=\frac{\pi^2}{6}}$$

**side note:**

if $\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^1}=\zeta(1)$ then there were zeros on the $\zeta(1)$ line

but $\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^1}=\zeta(2)=\frac{\pi^2}{6}<\zeta(1)$ so no zeros on the $\zeta(1)$ line ☺

# Critical Strip

When $q(b)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^a}=\zeta(2a)$ then $\zeta(a+ib)$ is a non trivial zero

**Case #1**

for the range $0.5<a<1$ we can multiply by $\left(1-\frac{2}{2^{2a}}\right)\neq 0$

$$\left(1-\frac{2}{2^{2a}}\right)\zeta(2a)=\eta(2a) \qquad \Rightarrow \qquad \left(1-\frac{2}{2^{2a}}\right)\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(x\ln(n/k))}{(nk)^a}=\left(1-\frac{2}{2^{2a}}\right)\zeta(2a)=\eta(2a)$$

the right $\eta(2a)$ side is converging in the range $0.5<a<1$

meaning the function $f(x)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\left(1-\frac{2}{2^{2a}}\right)\frac{2\cos(x\ln(n/k))}{(nk)^a}$ has a sup value of $\eta(2a)<\zeta(1)$ which is a fixed value (a real number!)

and because of that the function (theoretically) can have values of x that will result $f(x)=0$

**Case #2**

for the range $0<a<0.5$ we can multiply by $\left(1-\frac{2}{2^{2a}}\right)\neq 0$

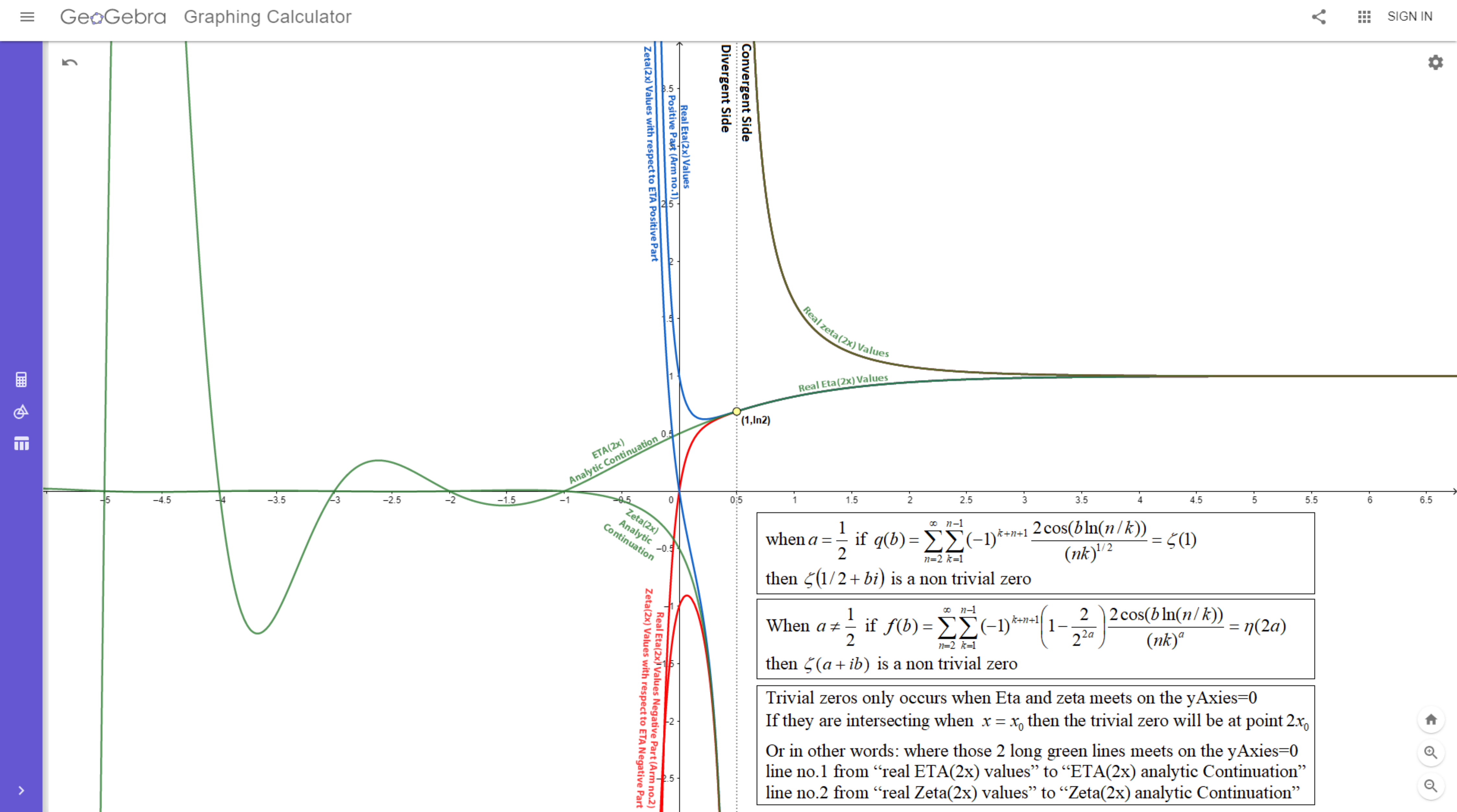
$$\left(1-\frac{2}{2^{2a}}\right)\zeta(2a)=\eta(2a) \qquad \Rightarrow \qquad \left(1-\frac{2}{2^{2a}}\right)\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(x\ln(n/k))}{(nk)^a}=\left(1-\frac{2}{2^{2a}}\right)\zeta(2a)=\eta(2a)$$

the right side $\eta(2a)$ is diverging in the range $0<a<0.5$

meaning the function $f(x)=\sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\left(1-\frac{2}{2^{2a}}\right)\frac{2\cos(x\ln(n/k))}{(nk)^a}$ has no (fixed) sup value!

The sup value should have been $\eta(2a)$ but this is not a fixed value in the range $0<a<0.5$ and because of that
the function changing all the time as n gets biger and biger making the values of x to changed on the cos function summation.
the x value cant diverge when $n\to\infty$ it need to be a fixed value!

That is why there are no zeros in the range of $0<a<0.5$



when $a = \frac{1}{2}$ if $q(b) = \sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(b\ln(n/k))}{(nk)^{1/2}} = \zeta(1)$

then $\zeta(1/2+bi)$ is a non trivial zero

When $a \neq \frac{1}{2}$ if $f(b) = \sum_{n=2}^{\infty}\sum_{k=1}^{n-1}(-1)^{k+n+1}\left(1-\frac{2}{2^{2a}}\right)\frac{2\cos(b\ln(n/k))}{(nk)^{a}} = \eta(2a)$

then $\zeta(a+ib)$ is a non trivial zero

Trivial zeros only occurs when Eta and zeta meets on the yAxies=0
If they are intersecting when $x = x_0$ then the trivial zero will be at point $2x_0$

Or in other words: where those 2 long green lines meets on the yAxies=0
line no.1 from “real ETA(2x) values” to “ETA(2x) analytic Continuation”
line no.2 from “real Zeta(2x) values” to “Zeta(2x) analytic Continuation”

(This time I am using something that already been proven) (Functional equation)

$$\boxed{\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)}$$

Because in the range $0 < a < 0.5$ the function has no zeros that means that in the range $0.5 < a < 1$ there are no zeros as well!

**Case #3**

when $a = 0.5$ the function $q(x) = \zeta(1)$ is devergent **but** the eta value is a fixed value equal to $\ln 2$
meaning no values with negitive part of eta only one way to go and its up
making the devergent part only to reflect the going to infinity part in the formula

$$\boxed{\lim_{M\to\infty}\sum_{n=2}^{M}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(x\ln(n/k))}{(nk)^a} \le \lim_{M\to\infty}\left[\frac{1}{1^{2a}}+\frac{1}{2^{2a}}+\frac{1}{3^{2a}}+\ldots+\frac{1}{M^{2a}}\right] = \frac{\eta(2a)}{\left(1-\frac{2}{2^{2a}}\right)}}$$

(the dividing by 0 part on the right side is for illustration purposes only)

$$\boxed{\lim_{M\to\infty}\sum_{n=2}^{M}\sum_{k=1}^{n-1}(-1)^{k+n+1}\frac{2\cos(x_0\ln(n/k))}{(nk)^{1/2}} = \lim_{M\to\infty}\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{M}\right] = \lim_{M\to\infty}\sum_{n=1}^{M}\frac{1}{k} = \zeta(1)}$$

and we already know there are infintly many zeroes on the critical line