Essay written for the Gravity Research Foundation 2020 Awards for Essays on Gravitation

On a possible logarithmic connection between Einstein's constant and the fine-structure constant, in relation to a zero-energy hypothesis

*

Andrei-Lucian Drăgoi[1,2]

DOI: **10.13140/RG.2.2.15894.98886** (**URL-RG**)

*

Abstract

This paper brings into attention *a possible logarithmic connection between Einstein's constant and the fine-structure constant*, based on a hypothetical electro-gravitational resistivity of vacuum: we also propose *a zero-energy hypothesis* which predicts a general formula for all the rest masses of all elementary particles from Standard model, also indicating an unexpected profound bijective connection between the three types of neutrinos and the massless bosons (gluon, photon and the hypothetical graviton).

***

1st observation. Each of all known electromagnetically-charged elementary particles (**cEP**) in the Standard model has a non-zero rest energy which, in turn, is always associated with non-zero spacetime curvature (gravity) as implied by General relativity. Furthermore, because the electron (with elementary electromagnetic charge $-|e|$, rest mass m_p and rest energy $E_e = m_e c^2$) is the lightest known cEP with the largest known (absolute)charge-to-(rest)energy ratio in nature $\phi_{\max} = |e| / E_e$, thus *electromagnetic charge appears to cannot exist (and thus cannot manifest) without a minimum degree of spacetime curvature indirectly measured by almost infinitesimal* $\kappa E_e^2 \left(\cong 10^{-69} Nm^2\right)$, with $\kappa = 8\pi G / c^4$ being the Einstein's constant.

*

2nd observation. There is a simple logarithmic function which appears to relate both κ and $\phi_{\max}$ to the fine-structure constant at rest $\alpha_0 = k_e q_e^2 / (\hbar c) \left(\cong 137^{-1}\right)$ which is the *asymptotical minimum* at rest of the electromagnetic running coupling constant $\alpha(E) = \alpha_0 / \left(1 - \alpha_0 f(\mathrm{E})\right)$ [3]:

$$\alpha_0 \overset{99.92\%}{\cong} \left[\log_2\left(\kappa^{-1} k_e \phi_{\max}{}^2\right)\right]^{-1} \left(\cong 136.93^{-1}\right) \quad \textbf{(1)}$$

α_0 may be directly related to $\left[\log_2\left(\kappa^{-1} k_e \phi_{\max}{}^2\right)\right]^{-1}$ with the following numbered arguments and explanations:

(**i**) If the very *large dimensionless physical constants* (**DPCs**) (which are gravity-related in general, like $\kappa^{-1} k_e \phi_{\max} \cong 10^{41}$ for example) are deeply related with the small DPCs (usually close to 1 and related to quantum mechanics, like α_0 for example), by any (yet unknown) mathematical function, then a *logarithmic function* (**LF**) would be the simplest (and thus the most natural) candidate solution of connecting these large and small DPCs, as other authors also considered in the past [**1**,**2**]. Furthermore, even if it is not the case of such a logarithmical connection, possible LFs (connecting those DPCs) would still have to be ruled out first.

(**ii**) A direct logarithmic relation between an electromagnetic minimum of nature $\left(\alpha_0\right)$ and an "electro-gravitational" maximum of nature $\phi_{\max}$ is quite intuitive;

(**iii**) $\kappa^{-1}\left(\cong 10^{42} N\right)$ (which is relatively close to the Planck force $F_{Pl} = c^4 / G \cong 10^{44} N$) may be interpreted as a *global average "tension" of the spacetime fabric (as also interpreted by other authors*[**3**]*) which strongly opposes to any spacetime curvature (**SC**) induced by any source of energy (including electromagnetic and/or gravitational energy tensors)*: because of this resistance to any induced SC (by any rest energy and/or movement of any bosonic or fermionic EP), κ^{-1} is identified with the approximate value at rest of an (energy/length-)scale-dependent *electro-gravitational resistivity of vacuum* (**EGRV**) $R(E)$ with an asymptotic maximum value at rest $R_0 = 2^{1/\alpha_0} / k_e \phi_{\max}\left(\cong 10^{43} N\right)$ estimated to exactly correspond to the asymptotic minimum α_0, so that $\alpha_0 = \left[\log_2\left(R_0 k_e \phi_{\max}{}^2\right)\right]^{-1}$. EGRV (measured by $R(E)$ and R_0 at rest) may be considered a truly fundamental parameter of spacetime with both c and G being actually determined by $R(E)$ and thus being indirect measures of EGRV. Another argument for α_0 measuring EGRV (which α_0 is alternatively defined as the probability of a real electron to emit or absorb a real photon) is that EGRV actually opposes to the photon emission process, in the sense that, for any real EP to emit a real photon, that photon first needs to overcome EGRV.

(**iv**) EGRV is very plausibly determined by the short-lived *virtual particle-antiparticle pairs* (**VPAPs**) emerging from the vacuum, which VPAPs interact with both photons and gravitational waves plausibly limiting their speed to a common maximum speed-limit for both speed of gravity and speed of light in vacuum. Charged EPs (composing charged VPAPs) interact much more strongly with photons than neutral EPs (composing neutral VPAPs) so that $R(E)$ may actually depend on (and vary with) the ratio between the volumic concentrations of charged and neutral virtual EPs at various length scales of vacuum.

[**1**] **Email**: dr.dragoi@yahoo.com

[**2**] **Mailing address**: Aleea Arinii Dornei, nr. 11, bloc i9, scara A, et.1, ap. 4, Bucuresti (Bucharest), sector 6 (6th sector), Romania; cod postal (zip code): 060797

[3] the leading log approximation of $\alpha(E)$, which is only valid for large energy scales $E >> E_e$, with $f(\mathrm{E}) = \ln\left[\left(E / E_e\right)^{2/(3\pi)}\right]$

(**v**) By replacing $k_e \phi_{\max}{}^2$ with its equivalent $\alpha_0 \hbar c / E_e{}^2$, α_0 and R_0 become related by a special type of exponential equation such as:

$$\left(1/\alpha_0\right) 2^{1/\alpha_0} = R_0 \hbar c / E_e{}^2 \quad \textbf{(2)}$$

(**vi**) Based on the previous equality, α_0 may be also considered an indirect measure of EGRV and inversely redefined as the unique positive solution w of the exponential equation $\left(1/\mathrm{w}\right) 2^{1/\mathrm{w}} = C$, with $C = R_0 \hbar c / E_e{}^2 \cong \kappa^{-1} \hbar c / E_e{}^2$. This equation can be solved by using the *Lambert function* only after converting it to its natural-base (e) variant $\left(\ln(2)/\mathrm{w}\right) e^{\ln(2)/\mathrm{w}} = C \ln(2)$ so that:

$$\alpha_0 = \ln(2) / W\left(C \ln(2)\right) \quad \textbf{(3)}$$

(**vii**) By considering $\hbar$, E_e and c to all be scale-invariant, $R(E)$ *can be generalized and* $\alpha(E)$ *can be redefined as a function of this generalized* $R(E)$ *such as:*

$$R(E) = \left[R_0 - R_0 f(\mathrm{E}) / \log_2(C)\right] / 2^{\mathrm{f(E)}} \quad \textbf{(4a)}$$

$$\alpha(E) = \ln(2) / W\left(\ln(2) R(\mathrm{E}) \hbar c / E_e{}^2\right) \cong \cong \alpha_0 / \left(1 - \alpha_0 f(\mathrm{E})\right) \quad \textbf{(4b)}$$

(**viii**) A predicted quantum big G $G_q(E)$ (which also varies with energy scale E) can be also derived from the same $R(E)$, also implying that big G may be actually a function of both the speed of gravity v_g ($v_g{}^4$ to be more specifically) and EGRV, such as:

$$G_q(E) = \frac{c^4}{8\pi R(E)} = \frac{v_g{}^4}{8\pi R(E)} \quad \textbf{(5)}$$

From the previous relation, one may easily note that any subtle variation of v_g and/or $R(E)$ may produce a slight variation of big G numerical value: this fact may actually explain the apparently paradoxal divergence (with deviations up to $\pm 1\%$) of big G experimental values despite the technical advances in the design of the modern experiments.

**

A zero-energy hypothesis (ZEH). We also propose *a zero-energy hypothesis* (**ZEH**) applied on any virtual particle-antiparticle pair (**VPAP**) popping out from the quantum vacuum at hypothetical length scales comparable to Planck scale. ZEH can be regarded as an extension of the notorious *zero-energy universe hypothesis* first proposed by the theoretical physicist Pascual Jordan. Presuming the gravitational and electrostatic inverse-square laws to be valid down to Planck scales and considering a VPAP composed from two electromagnetically-charged EPs (**cEPs**) each with non-zero rest mass m_{EP} and energy $E_{EP} = m_{EP} c^2$, electromagnetic charge q_{EP} and negative energies of attraction $E_g = -G m_{EP}{}^2 / r$ and $E_q = -k_e \left|q_{EP}\right|^2 / r$, *ZEH specifically states that*:

$$2E_{EP} + E_g + E_q = 0 \quad \textbf{(6a)}$$

Defining the ratios $\phi_g = G / r$ and $\phi_e = k_e / r$ the previous equation is equivalent to the following simple quadratic equation with unknown $x\left(= m_{EP}\right)$:

$$\phi_g x^2 - \left(2c^2\right) x + \phi_e q_{EP}{}^2 = 0 \quad \textbf{(6b)}$$

The previous equation is easily solvable and has two possible solutions which are both positive reals if $c^4 \geq \phi_g \phi_e q_{EP}{}^2 \geq 0$:

$$m_{EP} = \frac{c^2 \pm \sqrt{c^4 - \phi_g \phi_e q_{EP}{}^2}}{\phi_g} \quad \textbf{(6c)}$$

The realness condition $c^4 \geq \phi_g \phi_e q_{EP}{}^2 \geq 0$ implies the existence of a minimum distance between any two EPs (composing the same VPAP) $r_{\min} = q_{EP} \sqrt{G k_e} / c^2 \cong 10^{-1} l_{Pl}$ (for $q_{EP} \cong e$ and with l_{Pl} being the Planck length): obviously, for distances lower than $r_{\min}$ the previous equation has only imaginary solutions $x\left(= m_{EP}\right)$ for any charged EP; by this fact, ZEH offers a *new interpretation of the Planck length, as being the approximate distance under which charged EPs cannot have rest masses/energies valued with real numbers*; because k_e is actually variable with the energy/length scale and currently defined as a function of $\alpha(E)$ such as $k_e(E) = \alpha(E) \hbar c / e^2$, $r_{\min}$ can be generalized as $r_{\min}(E) = \left(q_{EP} / \mathrm{e}\right) \sqrt{G \alpha(E) \hbar c} / c^2$ (and can slightly vary as such). Note that $r_{\min}$ can be additionally corrected to include the strong force (implying color charge) and/or weak force (implying weak charge) between any quark (or gluon and/or leptons coupling with the weak field) and its antiparticle (composing the same VPAP): however, these potential corrections are estimated to only slightly modify $r_{\min}(E)$ values so that they're not detailed this paper.

Both generic $m_{EP}(= x)$ solutions of the previous equation **6b** indicate that, *because* m_{EP} *has discrete values only,* ϕ_G *(and* E_g *implicitly) and* ϕ_e *(and* E_q *implicitly) should all have discrete values only*. More interestingly, for neutral EPs (**nEPs**) with $q_{EP} = 0$ (which implies $\phi_g \phi_e q_{EP}{}^2 = 0$) and $r \geq r_{\min}(> 0m)$, $m_{EP}(= x)$ solutions may take *both*: (**1**) non-zero positive values $m_{EP} = 2c^2/\phi_g\,(>0)$ (like in the case of all three types of neutrinos, the Z boson and the Higgs boson) AND (**2**) zero values $m_{EP} = \left(c^2 - \sqrt{c^4}\right)/\phi_g = 0$ (like in the case of the gluon and the photon which both have zero rest mass $m_{EP}\,(=0kg)$ and are assigned only relativistic mass/energy by the Standard model).

In a first step and defining the unit of measure of $\phi_g\left(=2c^2/m_{nEP}\right)$ as $u = m^2kg^{-1}s^{-2}$, ZEH directly estimates ϕ_g for the Z boson (**Zb**) and Higgs boson (**Hb**) (with both Zb and Hb having non-zero rest energies) such as $\phi_{g(Zb)} \cong 10^{42}u$ and $\phi_{g(Hb)} \cong 8\times10^{41}u$. Based on the previously defined $r_{\min}\left(\cong 10^{-1}l_{Pl}\right)$, we then obtain $G_{Zb(\min)}\left(=\phi_{g(Zb)}r_{\min}\right) \cong G_{Hb(\min)}\left(=\phi_{g(Hb)}r_{\min}\right) \cong 2\times10^{16}G$: these huge predicted lower bounds for big G values at Planck scales indicate that E_g *may reach the same magnitude as* E_q $\left(E_g \cong E_q \Leftrightarrow \phi_g m_{EP}{}^2 \cong \phi_e q_{EP}{}^2\right)$ *at Planck scales* and also suggest that $R(E)$ *(thus* $G_q(E)$ *and* $\alpha(E)$*) may actually take discrete values only*.

In a second step, ZEH estimates the lower bounds of ϕ_g for all known three neutrinos, as deducted from the currently estimated upper bounds of the non-zero rest energies of all three known types of neutrino: the electron neutrino (**en**) with $E_{en} < 1eV$, the muon neutrino (**mn**) with $E_{mn} < 0.17MeV$ and the tau neutrino (**tn**) with $m_{mn} < 18.2MeV$: $\phi_{g(en)} >_{\cong} 10^{53}u$, $\phi_{g(mn)} >_{\cong} 6\times10^{47}u$ and $\phi_{g(tn)} >_{\cong} 6\times10^{45}u$, with $\phi_{g(en)}$ being assigned a very large big G lower bound $G_{en(\min)}\left(=\phi_{g(en)}r_{\min}\right) \cong 2\times10^{28}G$ thus strengthening the previously introduced (sub-)hypothesis $\phi_g m_{EP}{}^2 \cong \phi_e q_{EP}{}^2$ at Planck scales.

ZEH cannot directly estimate the values of $\phi_{g(nEP)}$ for the massless photon (**ph**) $\phi_{g(ph)}$ and the gluon (**gl**) $\phi_{g(gl)}$ due to the division-by-zero error/paradox. However, ZEH additionally states that $\phi_{g(ph)}$ and $\phi_{g(gl)}$ may have very large values coinciding with $\phi_{g(en)}$, $\phi_{g(mn)}$ and $\phi_{g(tn)}$. More specifically, ZEH speculatively predicts that $\phi_{g(ph)} > \phi_{g(gl)}$ and that there also exists a massless graviton (**gr**) defined by $\phi_{g(gr)} > \phi_{g(ph)}\left(>\phi_{g(gl)}\right)$ so that: $\boxed{\phi_{g(gr)} = \phi_{g(en)}}$, $\boxed{\phi_{g(ph)} = \phi_{g(mn)}}$ and $\boxed{\phi_{g(gl)} = \phi_{g(tn)}}$. ZEH thus explains *the non-zero rest masses of 8 known or hypothetical nEPs (Zb, Hb, en, mn, tn, gl, ph and gr) plus their antiparticles by only five discrete ratios*: $\phi_{g(Zb)}$, $\phi_{g(Hb)}$, $\phi_{g(gr)}\left(=\phi_{g(en)}\right)$, $\phi_{g(ph)}\left(=\phi_{g(mn)}\right)$ and $\phi_{g(gl)}\left(=\phi_{g(tn)}\right)$.

The discrete values of ϕ_g for all the other (charged) EPs can also be easily determined by using the additional sub-hypothesis of ZEH $\left(\phi_g m_{EP}{}^2 \cong \phi_e q_{EP}{}^2\right)$ which simplifies the initial equation 6b and allows the estimation of ϕ_g as approximately $\phi_g \cong c^2/m_{EP}$ for all known charged leptons, with slight variations in the case of quarks (depending on the exact fractional charge of those quarks): $\phi_g \cong 2c^2/\left(\tfrac{13}{9}m_{EP}\right)$ (in the case of $\tfrac{2}{3}|e|$-quarks) and $\phi_g \cong c^2/\left(\tfrac{5}{9}m_{EP}\right)$ in the case of $\tfrac{1}{3}|e|$-quarks).

**

Final conclusions. The energy/length scale-dependent *electro-gravitational resistivity of vacuum* $R(E)$ may determine both a variable $G_q(E)$ and $\alpha(E)$ bringing General relativity to quantum field theory more closer to one another: the same with the zero-energy hypothesis proposed in this paper which predicts a general formula for all the rest masses of all elementary particles from Standard model, indicating an unexpected profound bijective connection between the three types of neutrinos and the massless bosons (gluon, photon and the hypothetical graviton).

***

II. References

[**1**] E. Teller (1948). "On the Change of Physical Constants". Physical Review (April 1948) Vol. 73, Issue 7, page 801. URL1 (https://journals.aps.org/pr/abstract/10.1103/PhysRev.73.801), URL2 (https://journals.aps.org/pr/issues/73/7). See also Barrow J.D. (1981) (which also cites Teller's paper). "The lore of large numbers: some historical background to the anthropic principle", Q. Jl. R. Astr. Soc. 22 (1981) 388-420. This extract can also be found at page 397: http://articles.adsabs.harvard.edu//full/1981QJRAS..22..388B/0000397.000.html

[**2**] Sirag S-P. (1980, 1983). "Physical constants as cosmological constraints" (Received on November 22, 1980), International Journal of Theoretical Physics, 1983, vol. 22, no. 12, pages 1067–1089. DOI: 10.1007/BF02080315. Bibcode: 1983IJTP...22.1067S. URL1 (https://link.springer.com/article/10.1007/BF02080315) and URL2 (https://ui.adsabs.harvard.edu/abs/1983IJTP...22.1067S/abstract)

[**3**] Estakhr, Ahmad Reza (2016). "Planck Force is Tension of Spacetime (General Relativity & Estakhr's expression of Einstein Field Equation)". 2016 Annual Meeting of the Far West Section of the American Physical Society (APS), Volume 61, Number 17, Abstract ID: BAPS.2016.FWS.G1.31, Abstract: G1.00031, Friday–Saturday, October 28–29, 2016; Davis, California, Session G1: Poster Session 4:00 PM, Friday, October 28, 2016, Conference Center Room: Lobby. Retrieved on 20.10.2019 from URL http://meetings.aps.org/link/BAPS.2016.FWS.G1.31 which redirects to http://meetings.aps.org/Meeting/FWS16/Event/286265