# Transformation of 4-dimensional Rindler spacetime

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**ABSTRACT**

In special relativity theory, we discover 4-dimensional transformation of general Rindler space-time from 4-dimensional Lorentz transformation in inertial frames.

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## I.Introduction

In special relativity theory, we discover 4-dimensional transformation of general Rindler space-time from 4-dimensional Lorentz transformation in inertial frames.

At first, 2-dimensional-Lorentz transformation is in inertial frame,

$$ct = \gamma(ct' + \frac{v_0}{c} \cdot x'),$$

$$x = \gamma(x' + \vec{v}_0 t'), y = y', z = z' \quad \gamma = 1/\sqrt{1 - \frac{v_0{}^2}{c^2}} \tag{1}$$

2-dimensional transformation is in Rindler spacetime,

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0 \xi^0}{c}) \quad , x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0 \xi^0}{c}) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3 \tag{2}$$

## 2. 4-dimensional transformation in Rindler spacetime

4-dimensional-Lorentz transformation is in inertial frame,

$$ct = \gamma(ct' + \frac{\vec{v}_0}{c} \cdot \vec{x}')$$

$$\vec{x} = \vec{x}' + \gamma \vec{v}_0 t' - (1-\gamma)\frac{\vec{v}_0 \cdot \vec{x}'}{v_0{}^2}\vec{v}_0 \quad , \gamma = 1/\sqrt{1 - \frac{v_0{}^2}{c^2}} \tag{3}$$

4-dimensional-differential Lorentz transformation is in inertial frame,

$$cdt = \gamma(cdt' + \frac{\vec{v}_0}{c} \cdot d\vec{x}')$$

$$d\vec{x} = d\vec{x}' + \gamma \vec{v}_0 dt' - (1-\gamma)\frac{\vec{v}_0 \cdot d\vec{x}'}{v_0{}^2}\vec{v}_0 \quad , \gamma = 1/\sqrt{1 - \frac{v_0{}^2}{c^2}} \tag{4}$$

Hence, the proper time is

$$d\tau^2 = dt^2 - \frac{1}{c^2} d\vec{x} \cdot d\vec{x} = dt^2 - \frac{1}{c^2}(dx^2 + dy^2 + dz^2)$$

$$= dt'^2 - \frac{1}{c^2}(dx'^2 + dy'^2 + dz'^2) \tag{5}$$

If we suggest 4-dimensional transformation in Rindler spacetime,

$$ct = \sinh(\frac{a_0\xi^0}{c})\frac{c^2}{a_0}(1+\frac{\vec{a}_0}{c^2}\cdot\vec{\xi})$$

$$\vec{x} = \vec{\xi} + \frac{c^2}{a_0{}^2}\cosh(\frac{a_0\xi^0}{c})\vec{a}_0 - (1-\cosh(\frac{a_0\xi^0}{c}))\frac{\vec{a}_0\cdot\vec{\xi}}{a_0{}^2}\vec{a}_0 - \frac{c^2}{a_0{}^2}\vec{a}_0 \qquad (6)$$

Therefore, 4-dimensional-differential transformation is in Rindler spacetim

$$cdt = \cosh(\frac{a_0\xi^0}{c})cd\xi^0(1+\frac{\vec{a}_0}{c^2}\cdot\vec{\xi}) + \sinh(\frac{a_0\xi^0}{c})\frac{\vec{a}_0}{a_0}d\vec{\xi}$$

$$d\vec{x} = d\vec{\xi} + \sinh(\frac{a_0\xi^0}{c})\frac{c}{a_0}\vec{a}_0 d\xi^0 + \sinh(\frac{a_0\xi^0}{c})\frac{d\xi^0}{c}\frac{\vec{a}_0\cdot\vec{\xi}}{a_0}\vec{a}_0$$

$$-(1-\cosh(\frac{a_0\xi^0}{c}))\frac{\vec{a}_0\cdot d\vec{\xi}}{a_0{}^2}\vec{a}_0 \qquad (7)$$

Hence, the proper time is[8]

$$d\tau^2 = dt^2 - \frac{1}{c^2}d\vec{x}\cdot d\vec{x} = dt^2 - \frac{1}{c^2}(dx^2+dy^2+dz^2)$$
$$= (1+\frac{\vec{a}_0\cdot\vec{\xi}}{c^2})^2(d\xi^0)^2 - \frac{1}{c^2}((d\xi^1)^2+(d\xi^2)^2+(d\xi^3)^2) \qquad (8)$$

## 3. Conclusion

We know general Rindler coordinate transformation from 4-dimensional Lorentz transformation.

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