SEM-Images indicate that Water Clusters or Ice-Crystals are the cause of Phyllotaxis

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Abstract:

With my study I want to advance the hypothesis that a lattice (network) of Ice-Crystals or large Water Clusters together with special proteins, so-called Ice-binding proteins, is causing Phyllotaxis in plants. SEM-Images seem to proof this hypothesis. The (E)SEM-Image of the remains of an evaporated water droplet clearly shows a complex phyllotactic pattern! This phyllotactic pattern formed during the evaporation of the water in the vacuum chamber of the SEM. The shown (E)SEM-Image is a first proof that Water must be the fundamental source of Phyllotaxis!

--> (E)SEM = (Environmental) Scanning Electron Microscope

Responsible for the formation of the phyllotactic pattern seems to be a large central Water Cluster, that may have an icosahedral MacKay-geometry, consisting of large icosahedral Sub-Clusters formed by the stable icosahedral water clusters (H2O)100 or (H2O)280. Typical cluster numbers of Mackay-Clusters are 13 and 55 which are Fibonacci Numbers!

Additional evidence comes from SEM-Images of the generative zone of the Sunflower Capitulum. These images indicate that new primordia in the generative zone are caused by rhombic crystals, which seem to be either ice-crystals or large water cluster crystals that formed with the help of ice-binding proteins or water-cluster binding proteins.

Additional proof for a physical cause of Phyllotaxis is provided by a study about variations in the Fibonacci-spiral patterns of twigs of the three “Pinus Mugo” which shows that the Fibonacci-pattern variation depends from the altitude (temperature / radiation conditions) where the three grows. With the results of this study I developed an infinite Fibonacci-Number-Sequences-Table that contains all existing Fibonacci-Sequences and all natural numbers. Finally I present a mathematical discovery regarding constant Phi. All natural numbers and their square roots, as well as constant Pi (π), can be expressed by only using constant Phi (1.618…) and the base unit 1.

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Part III of the book “Symmetry in Plants” emphasizes the thesis that the resolution of an important part of the mystery of Phyllotaxis can be found in the basic laws of nature responsible for the homologies of structure, and that a global and synthetic view on nature can give important keys for the understanding of Phyllotaxis.
Appendix : A.) : MOVIES of : Standing-Wave-Patterns and Acoustic-Resonance in Water
B.) : Infinite Fibonacci-Number Sequence-Table : Sequences No. 1 to 33 shown ( F1 – F33 )
C.) : Fibonacci- Sequence-Tables F1 ; F2 ; F6 ; F8
Introduction:

I am confident that my study will help to uncover and solve the mystery of Phyllotaxis. And I hope it will help to find new genetical approaches and measures, to increase the crop yield of cereal plants and other useful plants, by effectively increasing the number of seeds in seed heads (infructescence), in order to safely feed the growing world population. In botany the term “Phyllotaxis” describes the arrangement of leaves on spiral paths on the stem of a plant. Phyllotactic spirals form a distinctive class of patterns in nature. But the true cause of these phyllotactic (Fibonacci) spiral patterns, which appear in most plants, still isn’t found yet!

The current believe is that phyllotactic spiral patterns which can be explained and described by Fibonacci Number Sequences, is controlled by the plant hormone Auxin. But this can’t be correct as a number of studies clearly indicate!

The presented SEM-Images point towards a fundamental physical and inorganic cause of Phyllotaxis, that seems to be connected to the molecular properties of Water, in particular Water’s property to form large Water Clusters.

A number of studies indicate that temperature & IR-radiation (black-body radiation) also seem to be crucial influential factors for phyllotactic pattern formation, probably because of their impact on the Water-Cluster-formation process.

The presented (E)SEM-Image of an evaporated water droplet, which clearly shows a phyllotactic pattern, is a first proof that Water itself must be the fundamental source of Phyllotaxis! (∴ SEM = Scanning Electron Microscope)

A central large Water Cluster seems to be responsible for the phyllotactic pattern formation and for the First (Fibonacci) Spiral Number of the phyllotactic pattern. And the geometry of the water cluster network which is surrounding this central Water Cluster seems to be responsible for the Second (Fibonacci) Spiral Number of the (Fibonacci) spiral pattern.

Therefore I want to ask all scientists with access to a SEM (Scanning Electron Microscope) or ESEM (Environmental Scanning Electron Microscope) to produce similar images of evaporating water droplets, to confirm my hypothesis!

To achieve usable (E)SEM-Images, probably a fast examination of the evaporating water droplet is necessary, as soon as a sufficient vacuum in the (E)SEM is reached. Because the visible remains of the evaporating water droplet are probably small ice-crystals which sublimate away quickly. An ESEM may be more suitable for this task than a SEM because there is still a bit air-pressure left in the sample-chamber, which will slowdown the evaporation process. Probably it is also necessary to use a (E)SEM with a cold cathode. And it may be required to use a support plate for the water droplet (Ø = 0.1 to 1 mm) which has a high thermal insulation factor, in order to keep possible microscopic ice-crystals frozen!

Another possible proof for a physical cause of Phyllotaxis comes from new generated primordia in the generative zone of the Sunflower Capitulum, that seem to be caused by rhombic crystals which may be either Ice-Crystals or large Water-Cluster-Crystals. These crystals probably formed with the help of ice-binding- or Water-Cluster-binding proteins. H₂O - evaporation in the generative zone may play an important role in the formation of these rhombic crystal-structures.

It would also be a good idea to produce more SEM-images of these rhombic shaped sunflower primordia, and to do diffraction analyses of these rhombic shaped crystals to find out their true crystal structure!

More proof for a fundamental physical origin of Phyllotaxis comes from study 6 about variations in the phyllotactic patterns of “Pinus mugo” threes which are clearly linked to the different altitudes where this three grows, and from study 9 which shows that far-red & infrared radiation with wave-lengths > 750 nm seems to trigger floret bud-induction and phyllotactic-pattern formation in short-day-Strawberrys and other plants.

As a consequence of all these proof, biological processes linked to Phyllotaxis, for example the generation of Auxin-gradients, can only be secondary effects resulting from a primary behavior (property) of Water!

Or in other words, plants adapted to the property of Water, to form phyllotactic patterns, and they used it as a reference for growth-processes and as reference for their own structure, in symbiosis with the pattern-forming property of Water!

Finally I present a mathematical discovery regarding constant Phi which defines the numbers of the Fibonacci-Sequences and the geometry of the Platonic Solids (e.g. Icosahedron and Dodecahedron). All natural numbers and their square roots, and the transcendental constant Pi, can be expressed mathematically by only using constant Phi and base unit 1

This fact indicates the importance of constant Phi for the structure of matter and for the physical world (for Nature)!
1 Primordia in the generative zone of the Sunflower capitulum show crystalline structures

Detailed SEM-images of the disc-region of the sunflower capitulum show that the new appearing floret primordia in the generative zone have features (outlines) which are comparable with features of crystals. Precisely linear edges are visible and a rhombical structure seems to form the base of each new primordia which appears in the generative zone. Therefore I want to advance the hypothesis that Phyllotaxis is based on the formation of crystals within a complex crystal-lattice structure that is in symbiosis with the tissue-structure of the generative zone of the capitulum.

Note: The following scanning electron microscopy (SEM) images which I used for my analysis are from this study:

- "Transductions for the Expression of Structural Pattern Analysis in Sunflower" by Luis F. Hernandez & Paul B. Green

![Fig. 1](image1) Fibonacci-array of floret primordia in the Sunflower-capitulum. The generative zone is marked as „Crystalline area“

![Fig. 2](image2) Inside of this „Crystalline Area“ the new appearing primordia have a sharp arrow-shape!

Detail SEM-images of the generative zone show crystal-like outlines of the new primordia:

On the image on the right I have drawn in some lines in blue color to indicate edges and outlines of the new appeared primordia which show crystal-like structures that seem to be hidden under the epidermal layer. The sharp arrow-shaped tips of the new primordia point towards the circumference of the capitulum. The angle of the arrow-shaped tips is in the range of 50-60°. The enlarged detail below shows some of the precise linear edges in more detail. In this image it is also visible that the rhombical crystal-like structures under the epidermal layer break in two pieces in the further growth process (arrow).

![Fig. 4](image4) Detail of Fig. 3-B showing some of the precisely straight crystal-like outlines which are hidden under the epidermal layer

![Fig. 3-A/B](image3) A.) Detail image of the generative zone with some of the crystal-like outlines (primordia) marked in blue  B.) unmarked image

Black Bar = 100 µm (bottom right)  ; Fl = floret primordia
1.1 Crystals with rhombic (bi-pyramidal) shape seem to form the base of new generated primordia

A close examination of the new generated primordia which appear from the featureless tissue of the capitulum shows that **rhombic (or bi-pyramidal) "crystal-like structures" form the base of each new generated floret primordium.** If we assume that these sharp geometrical structures are real crystals, then we look at precise rhombical (bi-pyramidal) crystals which initiate (trigger) the existence of each new primordium. However these rhombic crystals seem to have only a short lifetime. Shortly after their appearance these “rhombic crystals“ break in half, in the further growth-process of the primordia, and the two halves of the original crystal then form the bases of a disc-floret (DF) and a floret bract (FB).

![Image](image1.png)

**Fig. 5-A**: Section of the generative zone (GZ) or “crystalline area“ showing in detail new generated primordia. Blue lines indicate rhombical-shaped „crystals“ which form the base-structure of the new primordia. Some of them are „broken“ in two pieces (red lines).

**Fig. 6**: A rhombic primordia broken-apart develops into a bract (FB) and a flower (DF).

1.2 The rhombic crystals that form the base of primordia may grow on Ice-binding proteins:

If we compare the shape and size of the rhombic-shaped primordia, visible in the SEM-images, with ice-crystals grown on Ice-binding Proteins, then we can see a close similarity! The image below shows a rhombic (bi-pyramidal) Ice-crystal grown in an A-20 Ice-binding-Protein solution. The shape and size of this ice-crystal is very similar to the size and shape of the rhombic primordia visible in the SEM-image (see Fig. 5-B). The rhombic crystals underneath the epidermal layer seem to have an inclined orientation (→section A-A) The red lines in Fig. 5-A indicate break-lines of the rhombic crystals, as they break in two pieces during the growth process. Some break-lines in Fig. 5-A have a slight inclination towards the right side. This trend may have decided over the rotary direction (red arrow) of the flat CW-spiral pattern.

**Fig. 7**: This detail view of Fig 5-A shows one new generated primordia. The rhombic shape is indicated by the blue outline. The schematic section A-A on the right shows the assumed inclination of the crystals.

**Fig. 8**: A rhombic ice-crystal grown in an A20 Ice-binding-Protein solution.
2 Water/Ice-Crystals grown on Ice-binding Proteins are the probable cause of Phyllotactic Patterns
Primordia in plants seem to be caused (initiated) by rhombic crystals, probably made of water-Ice or water-clusters. These rhombic (bi-pyramidal) crystals probably start growing on special Ice-binding proteins in the plant cells, and they seem to be responsible for the phyllotactic pattern in the generative zone of the plant with the help of a polypentagonal water network, or a network of water clusters (for example \((H_2O)_{12}\) or \((H_2O)_{100}\)) which extends through the meristem.

Study 1 : “Ice-Binding Proteins in Plants”
- by Melissa Bredow & Virginia K. Walker, Queen’s University, CA - weblink to study: Ice-Binding Proteins in Plants

Abstract: Sub-zero temperatures put plants at risk of damage associated with the formation of ice crystals in the apoplast. Some freeze-tolerant plants mitigate this risk by expressing Ice-binding Proteins (IBPs), that adsorb to ice crystals and modify their growth. IBPs are found across several biological kingdoms, with their ice-binding activity and function uniquely suited to the lifestyle they have evolved to protect, be it in fishes, insects or plants. While IBPs from freeze-avoidant species significantly depress the freezing point, plant IBPs typically have a reduced ability to lower the freezing temperature. Nevertheless, they have a superior ability to inhibit the recrystallization of formed ice.

Study 2 : “Polypentagonal Ice-like Water Networks emerge in an activity-improved Ice-binding Protein”
- by Daichi Fukamia, Sheikh Mahatabuddina, Tatsuya Araia and others - weblink: Weblink to study

Abstract: Polypentagonal water networks were recently observed in a protein capable of binding to ice crystals, or Ice-binding Protein (IBP). To examine such water networks and clarify their role in icebinding, we determined X-ray crystal structures of a 65-residue defective isoform of a Zoarcidae-derived IBP (wild type, WT) and five single mutants (A20L, A20G,...). Inclusion of a symmetrical water cluster in the polypentagonal network showed a perfect complementarity to the waters constructing the (2021) pyramidal ice plane. The order of ice-binding strength was A20L < A20G < WT < A20V...

Ice-binding Proteins (IBPs) (or Antifreeze-proteins), and their function in plants and animals:
Different species have developed Ice-binding Proteins to reduce the risk of cell-damage associated with the formation of ice-crystals in the apoplast. Ice-binding proteins can control and modify the growth of ice crystals within the cell structure.

Extracts from the Studies 1 and 2:
Ice-Binding Proteins (IBPs) found in cold-hardy animals (e.g. fishes, insects and microbes) and in freeze-tolerant plants are unique macromolecules that are capable of binding to one or more ice-planes and creating a convex ice front on the plane between the bound IBPs through a Gibbs–Thomson effect. The ice-binding ability of an IBP also depresses the freezing point of water. Some freeze-tolerant plants have ice-binding proteins (IBPs) that adsorb to ice crystals and modify their growth. IBPs serve to control the growth of ice crystals and reduce freezing damage. Plant IBP’s also have a superior ability to inhibit the recrystallization of formed ice.

A polypentagonal water network consisting of Pentamers \((H_2O)_5\) and probably other \(H_2O\)-clusters, for example \((H_2O)_{12}\), \((H_2O)_{100}\) etc., connects the Ice-crytal to the IBP.

Fig. 1: Rhomboidal shape of a primordia Shape & size as A20

Fig. 2: A20 (wild type) Ice crystal grown in a A20 - Protein Solution

Fig. 3: Modification and growth of single ice crystals into bi-pyramids (within 5 min.) in solutions with the Ice-binding-Proteins A20L, A20G & WT (wild-type)

Fig. 4: Comparison of ice-growth ability of the Ice-binding Proteins WT (wild-type), A20L, A20G

Fig. 5a: Overlay of the backbone structures of the WT-, A20L & A20G-Ice-binding-proteins (IBPs)
The two squares separated by a broken line indicate the location of two ice-binding sites (IBS) One is the first prism plane (prism IB5 yellow) and another the pyramidal plane (pyramidal IBS cyan)

Fig. 5b: Structure of the A20I-IBP with =50 semiclathrate waters (water-clusters) which form the polypentagonal network where the pyramidal water cluster (bipiramid-crystal) is connecting to.

Fig. 5c: water-cluster with a space-match to pyramid crystal planes

Fig. 5d: Function of the polypentagonal waters (water-clusters)
The water clusters first merge with a disordered quasi-liquid layer (I) which is connected to the ice-crystal-lattice (II). The water-clusters then connect to the pyramidal Ice-Crystal (bi-pyramid)
3  Crystalline Structures seem to form the base of the Apical Meristem

Primordia seem to be initiated by crystal-structures that grow under the epidermal layer. These crystal-like structures are indicated by linear features visible in the deeper layers of the meristem. The base of the meristem seems to be a large primary crystal-structure from which smaller secondary crystal-structures grow out, that then initiate the primordia

Below the epidermal layer linear features are visible that may be formed by a Water-Cluster Crystal

The following images show central sections through the apical meristem of Arabidopsis. The first image shows the meristem with no new initiated primordia. The second image shows a newly initiated primordium in the peripheral zone of the meristem. Both images show linear “crystal-like” structures under the outer epidermal layer. These linear “crystal-like” structures are indicated by cell material of the inner epidermal layers. The crystalline structures itself seem to be transparent and they seem to extend through the complete apical meristem, an apparent “open-cell structure”. These crystal-like structures seem to be formed by a large “water-cluster crystal” that may have a gel-like (clotted) consistency. The images are from Study 4 (see also Chapter 12): “Auxin influx carriers stabilize phyllotactic patterning” see: study 4

Image processing:
The original images were enhanced in color, contrast and brightness. In order to bring out the details a bit clearer the original images were inverted in colors (blue images) Linear features are indicated by lines aside these features

Fig 1: Details of the Central section through the apical meristem showing clear linear features; black Bar = 25 µm

Fig 2: A newly initiated primordium in the peripheral zone of the meristem (see black box). This primordium seems to grow on top of a secondary pyramidal crystal-structure (yellow) extending from a primary crystal structure (green)
4 The rhombic Sunflower Primordia may be caused by Ice-Crystals or by Water-Cluster-Crystals

There are two principle scenarios possible:

In the First scenario the rhombic or bi-pyramidal structures are indeed formed by water-ice crystals grown on ice-binding proteins. This is the most obvious scenario. But it can’t explain how primordia would be initiated far above the freezing point. In the Second scenario a rhombic water-cluster lattice formed with support of similar proteins, initiates the growth of primordia. This is possible up to a temperature of \( \approx +25^\circ C \)

**First Scenario**: The rhombic crystals are indeed made of Water-Ice

This means the rhombic or bi-pyramidal crystal would have a hexagonal crystalline structure internally, denoted as ice \( \text{Ih} \). The three-dimensional crystal structure of \( \text{H}_2\text{O} \)-ice is composed of bases of \( \text{H}_2\text{O} \) ice molecules located on lattice points within a two-dimensional hexagonal space lattice (\( \Rightarrow \) left image). The bi-pyramidal crystal is then formed by hexagonal ice-plates which grow on top of each other, fixed in place by ice-binding-proteins.

Ice-binding proteins are secreted into the environments around the host cells or are anchored on their cell membranes. Ice-binding proteins (IBPs) that perform a variety of biological roles have been isolated and characterized from different organisms, including fishes, insects, plants, bacteria, fungi, yeasts & algae. IBPs control growth & shape of ice crystals to cope with subzero temperatures in psychrophilic and freeze-tolerant organisms. IBPs in polar fishes block further growth of internalized ice and inhibit ice recrystallization of accumulated internal crystals. Algae use IBPs to structure ice.

**Second Scenario**: The rhombic crystal-like structures which form the primordia are caused by a „Water-Cluster-(Crystal)-lattice“.

Here we must consider the most stable water-clusters as base units for a large „Water-Cluster-Crystal” with a multi-level structure as indicated in the images below. The small Pentamer \( (\text{H}_2\text{O})_5 \) and the pancake-shaped \( (\text{H}_2\text{O})_{25} \) water-clusters, which are both stable even in the vacuum and up to \( \approx 25^\circ C \), are the base elements of the much larger Icosahedral Water-Clusters \( (\text{H}_2\text{O})_{100} \) and \( (\text{H}_2\text{O})_{280} \). These stable Icosahedral-Water Clusters seem to be able to form thousand-times larger „Icosahedral Super-Clusters“, which then may form a „Rhombic-Super-Water-Cluster Crystal“ as shown below. The image on the bottom right shows a \( 6 \mu m \) cluster of polystyrene particles which probably was formed by \( (\text{H}_2\text{O})_{100} \) or \( (\text{H}_2\text{O})_{280} \) Clusters.

**Fig. 2**: The rhombic shape and size of primordia in sunflower meristem is similar to bi-pyramid water-ice-crystals grown in Ice-binding protein solutions.

**Fig. 3**: An icosahedral Cluster made of spherical Polystyrene particles (\( \Phi 240nm \)), formed during the evaporation of a Water / PS-particle suspension at \( \approx 5^\circ C \) (\( \Rightarrow \) Chapter 7 & 11).
The SEM-Image of an evaporated (dried) Water Droplet shows that Water is causing Phyllotaxis!

Water seems to cause Phyllotaxis! An accidentally taken (E)SEM-image of a dried water droplet indicates that phyllotactic patterns seem to be caused by Water! (→ SEM= scanning electron microscope) Note: Water makes up >80% of the meristem (plant) mass! Someone put this interesting SEM-image of an evaporated water-droplet in an internet forum (→ see weblink: SEM-image water drop) The little dots which are the remains of the evaporated water drop must be either salt (lime) deposits or ice-crystals, because the vacuum of a (E)SEM would quickly evaporate & freeze a water drop!

Following Image-details are interesting: 1. The pattern clearly has its origin in a cluster-shaped center 2. The remains of the Central Cluster (salt/ice) indicate a „Super Water Cluster Crystal“ as the source of the Phyllotactic Pattern.

SEM-image shows the remains of an evaporated Water Drop Phyllotactic pattern with a ≈90 clockwise & ≈92 anti-clockwise parastichy-pair (spiral-pattern) visible. Compare with Diatom!

Evaporated Water Drop
Ø = 100 – 1000 µm 
(estimated)

Diatom with phyllotactic pattern. 60 anti-clockwise and 61 clockwise parastichy-pair (spiral-pattern) visible on *Azpeitia nodulifera* Diatoms are micro-algae with SiO₂ cellwalls

Detail View of Central Cluster (from Detail 4):

Central Cluster of Detail 4 (unmarked) Structure of Central Cluster indicated

To the Central Cluster: The roughly pentagonal shaped outline visible in the center of the SEM-image seems to show the remains (salt-/icecrystals) of a „Dodecahedral Super Water Cluster-Crystal“ as indicated on the image shown on the lefthand side. This Dodecahedral Super Water Cluster-Crystal” probably consisted of ≥6 „Icosahedral Super Water Clusters“ made of thousands of (H₂O)₁₀₀ or (H₂O)₂₈₀ Water Cluster → see Ch. 6
5.1 A phyllotactic pattern is a water-cluster-lattice formed by a pulsating large central water cluster. The SEM-image of the evaporated water drop indicates that phyllotactic patterns seem to be caused by a large pulsating (or rotating) water-cluster-(crystal) at the center of the pattern, as the wave-groups marked in Fig. 2 show. The rim-area of the water-drop probably evaporated first. Then in the final stage the water in the center evaporated, leaving behind a frozen 2D-projection (outline) of the central “Super Water Cluster Crystal” in the form of small ice-crystals. Each of the original “Super Water Clusters” (Ø 1-10 µm) is represented by a small white dot on the SEM-image, which in my opinion are small hexagonal ice-crystals as Fig. 4 indicates. (→ website that shows evaporation/freezing in a vacuum: Weblink)

The phyllotactic pattern of this diatom (micro-algae) is nearly identical to the pattern of the evaporated water drop. The pattern is influenced by a pulsating central cluster.

The SEM-image of central area of an evaporated water drop. There is a clear connection visible between the pattern and the central cluster. The pattern is influenced by a pulsating central cluster. The rim-area of the water drop probably evaporated first. Then in the final stage the water in the center evaporated, leaving behind a frozen 2D-projection (outline) of the central “Super Water Cluster Crystal” in the form of small ice-crystals. Each of the original “Super Water Clusters” (Ø 1-10 µm) is represented by a small white dot on the SEM-image, which in my opinion are small hexagonal ice-crystals as Fig. 4 indicates. (→ website that shows evaporation/freezing in a vacuum: Weblink)

Fig 1: SEM-Image of central area of an evaporated water drop. There is a clear connection visible between the pattern and the central cluster. The pattern is influenced by a pulsating central cluster.

Fig 2: Each corner of the pentagonal central area seems to be the starting point of one (or more) v-shaped wave-group (or shock-front). The images indicate an elasticity of >30% of the spacing.

Fig 3: Two same-size areas of the salt- or ice-crystal-pattern that represents the evaporated water drop. The crystals in area 1 & 2 are equally spaced but density is different. The “Super-Water-Clusters” which first occupied these positions must have formed a crystal-lattice.

The contrast- and brightness enhanced detail area 3 seems to show hexagonal shaped crystals with ≈ 1.5 to 15 µm diameter. These crystals which probably represent frozen “super water clusters” form the phyllotactic pattern.

The phyllotactic pattern formed by the evaporated water drop is similar to the pattern formed by the floret primordia on the sunflower capitulum. The difference is that the phyllotactic pattern of the sunflower shows more uniformity and it normally forms a Fibonacci-array (parastichy-pair) defined by two successive Fibonacci-Numbers from the Fibonacci-Main Sequence F1. These are usually Fibonacci-Number pairs like 21, 34 or 34, 55, out of the F1-Fibonacci-Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,... or in rare cases out of other Fibonacci-Sequences like F2, F7, F8 .... And in very rare cases also Non-Fibonacci-number-pairs as 48 & 50 are possible (see image). What mechanism is responsible for Fibonacci-Number-pairs is not clear! But the principle cause of Phyllotactic-Patterns seems to be a large pulsating central “Super Water Cluster Crystal” that generates an unusual parastichy pair of 50(blue)/48(red) - see Ch. 13.4 a “Reference Pattern” for the plant.

Fig 4: The contrast- and brightness enhanced detail area 3 seems to show hexagonal shaped crystals with ≈ 1.5 to 15 µm diameter. These crystals which probably represent frozen “super water clusters” form the phyllotactic pattern.

Fig 5: Phyllotactic Pattern (Fibonacci-array) of the floret primordia in the Sunflower capitulum. This pattern is very similar but more uniform than Fig. 1.

Fig 6: Sunflower capitulum with an unusual parastichy pair of 50(blue)/48(red) - see Ch. 13.4 a “Reference Pattern” for the plant.
6 Fibonacci Numbers are defined by a large central Water-Cluster with MacKay-Cluster Geometry

There is a clear physical cause for phyllotactic Patterns, and for Fibonaccispiral patterns that appear in the apical meristem of plants, the site of organ formation. As the SEM-Image of the evaporated water drop shows (→ see right image & last two pages) a large central “Super Water Cluster Crystal” seems to cause the phyllotactic pattern. From each “Super Water Cluster” (→ white dots) of this “Cluster Crystal” a “chain of Super Water Clusters” is leading nearly radially outward on a slightly curved spiral-path (→ yellow lines). This set of “Super Water Cluster Chains” defines the first (1) Fibonacci-Number! This number is precisely linked to the number of “Super Water Clusters” in the central “Super Water Cluster Crystal” which is mostly defined by the icosahedral "MacKay'-Cluster geometry.

A MacKay Cluster is a very stable Nanoparticle/Cluster due to its electron configuration. It is important to note that the first two MacKay clusters are defined by the Fibonacci-Number 13 & 55! And an extreme stable variation of the third MacKay-Cluster (147) has the Fibonacci-Number 144! The 2. Fibonacci-Number in the Fibonacci-pattern is defined by the „Lattice of the Cluster Chains“ (→ see next page!).

Model of the Evaporated Water Drop:
There is a “Super Water Cluster Crystal” in the center with either a dodecahedral shape or an icosahedral MacKay-Cluster. Each “Super Water Cluster” in this central “Cluster Crystal” is the starting point of a “Super Cluster Chain” which forms with other such chains the complete pattern.

Model of the phyllotactic pattern source in the Sunflower meristem:
There is also a “Super Water Cluster Crystal” located in the center of the Sunflower meristem (capitulum). In all probability this cluster has an icosahedral MacKay-Cluster-geometry. And each “Super Water Cluster” in this central Cluster Crystal is the starting point of a “Super Water Cluster Chain” which forms the phyllotactic pattern, together with all other such Cluster Chains. But here the central cluster and the cluster-chains are probably stabilized by a specialized cell-tissue which uses special proteins (like IBP’s) that help to stabilize the position and size of the central cluster and the super water cluster chains. From each super water cluster of the phyllotactic pattern later a rhombic Super Water Cluster Crystal (ice-crystal) evolves, with the help of IBP’s, that defines the position of a floret primordia.

The central Super Water Cluster Crystal may consist of a dodecahedral shell with 92 icosahedral Super Water Clusters. This would fit to the number of cluster chains of = 90 – 92. → see 3D-animation of Cluster: weblink

Model of Water Cluster Chains and Central Cluster Crystal:
The cluster chains are connected by a (H2O)xxx network. All Super Water Clusters are connected by a matrix of (H2O)xxx-Clusters. Super Water Clusters located in specialized cells later develop into rhombic water-cluster crystals caused by IBP’s.

The central Super Water Cluster Crystal is a MacKay Cluster which probably is stabilized by a specialized cell-tissue surrounding it.
7. Fibonacci Spiral Patterns seem to be the result of a circular crystal-like lattice of Water Clusters

To generate a precise Fibonacci-Spiral Pattern the plant is using a crystal-lattice as reference! This crystal-lattice is made of “Super Water Clusters” which in principle have an icosahedral geometry. These Super Water Clusters consist either of \((\text{H}_2\text{O})_{100}\) or \((\text{H}_2\text{O})_{280}\) icosahedral water Clusters, and they may reach diameters of up to 6 \(\mu\)m. This means cell-size! Proof that such “Super Water Clusters” can form comes from an experimental study (see below)

The Pentagon and the Icosahedron are the only two geometrical objects where the Golden Ratio, the constant which defines Fibonacci-Numbers, is directly built into their geometrical structure! Water molecules can form water clusters with icosahedral shape (large clusters) and water clusters with dodecahedral shape, made of pentagons (small clusters)

The following image describes in principle how a Fibonacci-Spiral Pattern can develop, based on a circular crystal lattice that either consists of icosahedral or pentagonal Water Clusters. The 1. Fibonacci-Number is defined by the number of Super Water Clusters in the Central Cluster. And the 2. Fibonacci-Number is defined by the “Water Cluster Lattice” which is surrounding the central cluster. In the image every pentagon or icosahedron represents a cluster chain.

**Experimental Study:**

PS-Clusters with a precise geometry which form during the evaporation of Water

In this study small spherical particles made out of Poly-Styrene (PS), with a diameter of 0.25 \(\mu\)m (the little white spheres) were suspended in water. This mixture of water and PS-particles was then mixed with Oil to get an emulsion. Then Emulsion-droplets (water droplets + PS-particles) were collected & stored in 1.5 ml glass vials. These vials were then kept at different temperatures (5°C, 25°C & 85°C) for the controlled Water evaporation. At 5°C PS-Clusters (6 \(\mu\)m) with precise defined shell structures formed during water evaporation (see Fig. 4). These Clusters have a Rhombicosidodecahedron-geometry, a mixture of an Icosahedron and a Dodecahedron. A \((\text{H}_2\text{O})_{100}\) Water-Cluster has precisely the same geometry!! The authors of the study (summary of Study see in Chapter 11) say that especially capillary forces & entropy maximization are driving factors for the geometry. However this can’t explain the formation of the quite complex five-fold rotational symmetry of the PS-particle balls (dried droplets)!! PS-Clusters dried at 25°C show the birth of the five-fold symmetry axis which more likely is the result of molecular forces caused by e.g. \((\text{H}_2\text{O})_{100}\) Water Clusters!

---

**Fig 1:** shows how the Golden Ratio \(\Phi\) is built into the geometry of the Pentagon and Icosahedron

**Fig 2:** A Fibonacci Spiral Pattern precisely defined by a circular crystal lattice that either consists of icosahedral- or pentagonal Water Clusters. The 1. Fibonacci-Number is defined by the number of Super Water Clusters in the central Cluster. The 2. Fibonacci-Number is defined by the “Cluster Lattice-Geometry” surrounding the central Cluster. The Fibonacci-Numbers 13, 55, 144 represent full Mackay-Clusters & 21, 89 represent full or half outer shells.

**Fig 3:** Emulsion droplets evaporated at 25°C only show the birth of the five-fold symmetry axes on the surface

**Fig 4:** Emulsion droplets evaporated at 5°C produce a precise geometry which is nearly identical to the \((\text{H}_2\text{O})_{100}\) Cluster geometry

**Fig 5:** Schematic of emulsion-droplet production and evaporation: dried PS-particle balls show internal icosahedral Mackay-Cluster geometry and five-fold symmetry
The asymptotic ratio of successive Fibonacci numbers leads to the Golden Ratio constant $\varphi$ (or $\Phi$).

The Fibonacci Sequences describe morphological patterns in a wide range of living organisms. This is the most remarkable organizing principles in nature, mathematically describing natural and manmade phenomena.

If we want to understand where the Fibonacci Numbers in Phyllotaxis come from, we must have a look were else in nature, in the physical world, the Golden Ratio Constant ($\Phi = \varphi$ or $\Phi$) appears. Because the Fibonacci Numbers and Phyllotaxis are defined by this constant! In nature the pure constant $\Phi$ appears in crystals where it defines the crystal-lattice geometry. It appears especially in Icosahedral and in Dodecahedral Crystals.

**Note:** The shape of crystals in the macro-scale mirrors structure & arrangement of molecules in the micro-scale.

The Dodecahedron in cartesian coordinates:

The vertices of the Dodecahedron obtained from the cube and three orthogonal Golden Rectangles with the side relationship $1 / \varphi^2$ ($= 2 / \varphi : 2 \varphi$)

The Icosahedron in cartesian coordinates:

The vertices of the Icosahedron constructed with three orthogonal Golden Rectangles with the side relationship $1 / \varphi$ ($= 2 / 2 \varphi$)

The Golden Ratio constant:

$$\varphi = \frac{1 + \sqrt{5}}{2} = 1.618034...$$

The Pentagon and its correlation with $\varphi$ ($\Phi$):

(edge-length = 1)

The Fibonacci numbers defined by $\varphi$:

$$
\begin{align*}
1/1 &= 1 \\
2/1 &= 2 \\
3/2 &= 1.5 \\
5/3 &= 1.667 \\
8/5 &= 1.6 \\
13/8 &= 1.625 \\
21/13 &= 1.615 \\
34/21 &= 1.619 \\
55/34 &= 1.618
\end{align*}
$$

**weblink:** Dodecahedron

**weblink:** Icosahedron

**weblink:** Phi-sacred-solids

The Sunflower (Helianthus)
9 Icosahedral- and Dodecahedral Forms can be found in Crystals and they appear in many Organisms

Polyhedral forms (e.g. Icosahedra) do not only appear in crystals, they also occur at different length scales in lifeforms, from marine organisms (like diatoms & radiolaria) to protein nanocontainers of viruses (e.g. with icosahedral symmetry)

Water Clusters seem to play a major role in the growth-process of diatoms, radiolaria, bacteria, viruses and of course in phyllotactic patterns in plants! The mentioned lifeforms and plants seem to use the structure of water clusters and water-cluster-lattices as reference for their own structure. They grow in symbiosis with the inorganic Water Clusters!

Diatoms are a major group of algae, specifically microalgae found in the oceans & waterways of the world. Living diatoms makeup a significant portion of the Earth’s biomass: they generate about 20 - 50 percent of the oxygen produced each year, take in over 6.7 billion metric tons of material found in the oceans and contribute half of the organic material found in the oceans

Radiolaria are tiny protozoa (0.1 – 0.2mm) that produce intricate silica (mineral) skeletons. They are found as zooplankton and exist solitary and in colonies. The single-celled Radiolaria are complex, sophisticated organisms. The body is divided into a central capsule containing the endoplasm & nucleus and the extracapsulum which contains peripheral cytoplasm composed of a frothy bubble-like envelope of alveoli & a corona of ray-like axopodia and rhizopodia.

In the higher-dimensional space we can describe a quasiperiodic structure as a periodic one. The actual quasispecrystal structure in the 3D-physical space can then be obtained by appropriate projection/section techniques. Thus it is enough to define a unit cell of the nD-structure. The contents of that nD-unit cell consists of “hyperatoms” (occupation domains, ..) in analogy to the atoms in a normal unit cell. This enables us to describe the whole quasicrystal structure with a finite set of parameters. If we described it in 3D-space only, we needed thousands of atoms to obtain a representative volume segment of the whole structure as well as all parameters that go with it (e.g. thousands of positions). 

Diatoms (Pseudoglobulus footballi)

Structural puzzles in virology solved with an overarching Icosahedral design principle by Reidun Twarock & Antoni Luque - Weblink to the study : Weblink 1; PDF-document

Extract from the study: Viruses have evolved protein containers with a wide spectrum of Icosahedral architectures. The geometric constraints defining these container designs are still open problems in virology. We show that there is an overarching design principle for icosahedral, as well as octahedral, architectures that can be formulated in terms of the Archimedean lattices and their duals. This design principle also applies to other Icosahedral Structures in nature, and it offers alternative designs for man-made materials and nanocontainers in bio-nanotechnology.

b.) Shows the Construction of Archimedean solids via replacement of the 12 hexagons by pentagons in analogy to the Caspar-Klug construction (see also Fig. 1B).

c.) The polyhedral shapes corresponding to the examples shown in b. They each correspond to the smallest polyhedron in an infinite series of polyhedra for the given lattice type.

A Virus and a Capsid with Icosahedral Shape

Weblink: https://imgur.com/gallery/wMKMv
10 Water Clusters

Water Clusters can have properties like a Liquid or a Solid, depending on cluster size and temperature.

The most stable and long-lived Water Clusters are the \((\text{H}_2\text{O})_{20}\); \((\text{H}_2\text{O})_{100}\) and \((\text{H}_2\text{O})_{280}\) Clusters. The onset of an ice-like structure occurs at a cluster-size of approximately \(n = 275 \pm 25\) molecules. For cluster sizes \(n \geq 475 \pm 25\) the band of crystalline ice (3200 cm\(^{-1}\)) dominates the OH-stretching region. \(\rightarrow\) Large Water Clusters can behave like solid ice. But the crystallization of water clusters strongly depends on the ambient temperature (\(\rightarrow\) black-body-(IR)-radiation). At higher temperatures \(>4°C\), pulsating water clusters can produce standing-wave-patterns with wave-lengths equal to around 2\(x\) their diameter, which may be very similar to the macro-scale standing wave patterns of Water \(\rightarrow\) For more info on density temperature dependence, and they Experimental datas indicate that w Water can form very larger clusters have been found experimentally in various forms of water; in ice, in crystal lattices and in bulk liquid water. Water can form very larger clusters. Li Shu et al. reported images of large water clusters of up to 100 \(\mu\)m (0.1nm) size ! Experimental datas indicate that when water has its highest density at 4°C, water clusters reach maximum sizes and stability (durability). Water Clusters may help to explain many anomalous water characteristics such as its highly unusual density temperature dependence, and they may be responsible in the stabilization of certain supramolecular structures. Support is growing behind the idea that Water Clusters play key roles in operations ranging from molecular binding to turning on and off basic cell processes. \(\rightarrow\) see „The Scientist“: Structured-water-is-changing-models For more information \(\rightarrow\) see: Cluster_Overview; (H2O)100 Cluster; Proof for (H2O)280 Cluster; Icosahedral_Clusters

**Dodecahedron**

\((\text{H}_2\text{O})_{20}\) \((\text{H}_2\text{O})_{100}\) \((\text{H}_2\text{O})_{280}\)

**Pentamer (pentagonal) coordinated water network:** (a) cyclic pentamer consisting out of 5 water molecules; (b) Dodecahedral water-cluster \((\text{H}_2\text{O})_{20}\) consisting out of 20 water-molecules; (c) homological Icosahedral water-cluster \((\text{H}_2\text{O})_{100}\) \(\rightarrow\) This cluster can break down in 5 dodecahedral clusters and; (d) The Icosahedral \((\text{H}_2\text{O})_{280}\) cluster, can break down in 14 dodecahedral clusters

**Structure of the \((\text{H}_2\text{O})_{20}\) Dodecahedral water-cluster:** The red sticks represent oxygen atoms, white sticks represent hydrogen atoms, and the black dashed lines represent hydrogen bonds

**Matrix (lattice) of the \((\text{H}_2\text{O})_{100}\) water-cluster**

Computed vibrational spectra for the \((\text{H}_2\text{O})_{20}\) & \((\text{H}_2\text{O})_{100}\) water-cluster: The absorption bands are in the frequency range of 2700-3700 cm\(^{-1}\) This corresponds to infrared radiation in the wave-length range of approx. 4 to 6 \(\mu\)m. \(\rightarrow\) see diagram below

A few words to water and water ice:

Water is described as the "solvent of life". It is the most abundant substance on Earth's surface. The shape and geometry of a Water-molecule

**Note:** the H-O-H bond angle of \(\text{H}_2\text{O}\) is 104.48° which is close to the corner angle 108° of a Pentagon ! Pentamers (Pentagons) & Dodecahedra are the perfect Geometry for \(\text{H}_2\text{O}\) All the Water-Ice on Earth’s surface is of a hexagonal crystalline structure denoted as ice Ih. The three-dimensional crystal structure of \(\text{H}_2\text{O}\) ice: ice Ih is composed of bases of \(\text{H}_2\text{O}\) ice molecules located on lattice points within the two-dimensional hexagonal spacialattice. Ice Ih is remarkable in that the oxygen ions (O2-) form an ordered lattice, while the protons (H+) lack any kind of long-range order — in flat contradiction with the usual paradigm for solids. Like water, ice absorbs light at the red end of the spectrum preferentially as the result of an overtone of an oxygen–hydrogen (O–H) bond stretch.

Further information to spectra of water-clusters:

IR vibrational spectra of \((\text{H}_2\text{O})_{20}\) and \((\text{H}_2\text{O})_{100}\) clusters: Fingerprints in IR OH spectra of H2O clusters Spectra of \((\text{H}_2\text{O})_{20}\) cluster: Spectra of (H2O)20 cluster \(\rightarrow\) To the Structure and Stability of Water-Clusters

Water clusters in plants: Water clusters in plants.pdf
11 Water-Clusters seem to cause the formation of PS-Particle-Clusters with icosahedral Geometry

An experimental study seems to provide evidence for the existence of “Super Water Clusters” with up to 6 µm diameter. At a temperature of 5°C Polystyrene (PS) Particle Clusters with a precise geometry develop out of evaporating water-droplets that contain PS-particles. The geometry of the final Clusters, which is nearly identical to the geometry of (H₂O)₁₀₀ Clusters, indicates that in all probability Water Clusters are responsible for the formation of this crystal-like icosahedral structures! Low temperature & black-body (IR)-radiation with wave-lengths >5 µm is also required for this process!

Study 3: “Magic number colloidal clusters as minimum free energy structures”

- by Junwei Wang, Chrameh Fru Mbah & others - weblink to study: Magic-number-colloidal-clusters

Abstract: Clusters in systems as diverse as metal atoms, virus proteins, noble gases, and nucleons have properties that depend sensitively on the number of constituent particles. Certain numbers are termed ‘magic’ because they grant the system with closed shells and exceptional stability. To this point, magic number clusters have been exclusively found with attractive interactions as present between atoms. Here we show that magic number clusters exist in a confined soft matter system with negligible interactions. Colloidal particles in an emulsion droplet spontaneously organize into a series of clusters with precisely defined shell structures. Crucially, free energy calculations demonstrate that colloidal clusters with magic numbers possess higher thermodynamic stability than those without magic numbers......

The (H₂O)₁₀₀ icosahedral Water Cluster has the same geometrical structure as the colloidal Clusters described in this study! The colloidal clusters seem to be only a representation of the Water Clusters which form these Clusters! (comment by Harry K. Hahn)

Fig. 1: Icosahedral Water Cluster (H₂O)₁₀₀ and underlying geometrical structure!

Particle synthesis: The 0.25 µm spherical PS-Particles (→ the little white spheres) were made out of Styrene, acrylic acid, and ammonium peroxodisulfate. These small PS-colloidal (spherical) particles were synthesized in a surfactant (tensid)-free emulsion polymerization.

Colloidal cluster assembly: The 0.25 µm spherical PS-Particles, of 1wt%, were suspended in water and loaded into 1mL syringes. Then a special 0.1wt % surfactant (Tensid) was dissolved in perfluorinated carbon oil. The syringes were connected to microfluidics Syringe pumps by PE/2 tubings (0.38mm /1.09mm). The syringes were connected to microfluidics Syringe pumps by PE/2 tubings (0.38mm /1.09mm). The Vials were then kept at different temperatures, in the oven at 85°C, at room temperature at 25°C and in the fridge at 5°C for water evaporation.

Colloidal Clusters in systems as diverse as metal atoms, virus proteins, noble gases, and nucleons have properties that depend sensitively on the number of constituent particles. Certain numbers are termed ‘magic’ because they grant the system with closed shells and exceptional stability. To this point, magic number clusters have been exclusively found with attractive interactions as present between atoms. Here we show that magic number clusters exist in a confined soft matter system with negligible interactions. Colloidal particles in an emulsion droplet spontaneously organize into a series of clusters with precisely defined shell structures. Crucially, free energy calculations demonstrate that colloidal clusters with magic numbers possess higher thermodynamic stability than those without magic numbers......

Fig. 2: Thousands of 0.25 µm spherical PS-particles suspended in water formed icosahedral clusters during water-evaporation

Fig. 3: Colloidal clusters from confined self-assembly in water-in-oil emulsion droplets. Four distinct cluster morphologies with increasing degree of ordering are observed: a) buckled clusters partially collapse upon evaporation into non-spherical shape; b) spherical clusters exhibit only local order; c) partial icosahedral clusters show one or more five-fold symmetry axes and incomplete faceting (dotted blue boxes); d) icosahedral clusters have well-defined facets, edges, and vertices and complete icosahedral symmetry. e, f) Low-magnification scanning electron microscopy (SEM) images highlight the uniformity in size and structure of the prepared clusters. Spherical and icosahedral clusters dominate in the limit of fast (e) and slow (f) evaporation, respectively. g, h) Show the statistical evaluation of the observed morphologies as a function of the evaporation rate (g) and as an evolution over time for the slowest evaporation rate (h) showing the progression from spherical to icosahedral (h). Scale bars = 2 µm
12 A physical mechanism (trigger) must be the fundamental cause of Phyllotaxis!

The following extracts from two studies indicate that plant hormones like Auxin, PIN1 etc. can’t cause Phyllotaxis alone. The studies provide evidence that a yet unknown "physical mechanism" must be the fundamental cause (trigger) of Phyllotactic Patterns (Phyllotaxis)! Study 4: “Auxin influx carriers stabilize phyllotactic patterning”

Weblink: https://www.researchgate.net/publication/5505575_Auxin_influx_carriers_stabilize_phyllotactic_patterning

Discussion: “The rapid generation of dynamic auxin gradients at the shoot apical meristem is essential for regular primordium initiation and spacing. Previous studies have focused on the role of the PIN1 auxin efflux carrier. Here, we show that in addition to PIN1, the AUX1 LAX family of auxin influx carriers is essential for stabilizing phyllotactic patterning. This finding indicates the existence of a previously uncharacterized level of complexity in the regulation of auxin distribution in the shoot apical meristem. The continuous generation of new primordia around the circumference of the meristem requires the rapid and dynamic formation of auxin peaks. Simulation models for auxin-mediated phyllotaxis propose that PIN1 orients toward a neighboring cell with a higher auxin concentration. AUX1 LAX could be part of the mechanism that orients PIN1 toward cells with highest auxin concentration. The underlying molecular mechanism is, however, still unknown!”

Study 5: “A plausible model of phyllotaxis”

Weblink: A_plausible_model_of_phyllotaxis; alternativ: https://www.pnas.org/content/103/5/1301

Model of Phyllotactic Patterning. We initially hypothesized that phyllotaxis in Arabidopsis is determined directly by the transport-based patterning mechanism, operating on the growing surface of the apical meristem. In simulations, however, we were not able to obtain sustained spiral phyllotactic patterns by using that mechanism alone, although patterns of irregularly spaced primordia could easily be generated. This observation was upheld by many simulations, in which we used diverse parameter values and different formulas for polarizing PIN1. "We thus conclude that additional factors play an important role in generating of phyllotactic patterns in Arabidopsis”

Data Set 1: Phyllotactic Patterning Occurs in the Outer Layer of the Shoot Meristem (L1). The PIN1 protein is located primarily, although not exclusively, in the external L1 layer (figure 1A and C in ref. 10). "This localization suggests that phyllotactic patterns may be formed essentially on the surface of the shoot apical meristem!”

Our computer model suggests that "phyllotaxis is not governed by a single mechanism, but represents a combined effect of several factors". This complexity may be needed in nature to generate phyllotactic patterns in the presence of noise. We were not able to recreate spiral phyllotactic patterns under these conditions and assumed a uniform production throughout the L1 in the peripheral zone instead, with an additional boost in the primordia. Also, our model postulates localization of PIN1 toward the neighboring cells with the highest auxin concentration but leaves open the question of what molecular mechanism may produce this localization. The answers to these questions may lead to the integration of the model of phyllotaxis with a model of vasculature formation in the leaf and stem. Although both processes are mediated by auxin, the proposed mechanism of PIN1 polarization involved in phyllotaxis is almost opposite to the canalization mechanism proposed for veins. It is thus interesting how these different mechanisms may be reconciled in the growing plant
Proof for a fundamental physical cause of Phyllotaxis that depends on Temperature / Radiation

**Study 6**: Extracts from a study produced by Dr. Iliya Iv. Vakarelov, University of Forestry, Bulgaria (1982-1994)

**Title**: “Changes in phyllotactic pattern structure (Fibonacci Sequences) in Pinus mugo due to changes in altitude” from the book “Symmetry in Plants” by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada (Part I – Chapter 9, pages 213 – 229), weblinks: Weblink 1 (Google Books), Weblink 2

**Research Site and methods**: *Pinus Mugo* grows in high mountainous parts at altitudes up to 2500m forming vast communities. The vertical profile of the research sites for *Pinus mugo* was situated along the northern slopes of the eastern part of the Rila mountain, and test specimens were collected from the following altitudes: **1900, 2200 and 2500 m**. Test specimens were also collected from the city of Sofia (at **550 m** where *Pinus mugo* is grown as decorative plant.

The research was carried out over a period of **12 years** (except of altitude 550m here research was carried out only around 6 years). The initiation of leaf primordia in the bud (meristem) occurs at the end of the growing period. The apical meristem of *Pinus mugo* starts this process around the beginning of mid of August and ends in autumn when the air temperature goes below a certain point.

### The interesting results of the study:

1. **With the increase of altitude from 1900m to 2500m** the phyllotactic pattern structure of “*Pinus mugo*” twigs changes considerably, the number of patterns (different Fibonacci Sequences) grows from 3 to 12, and the relative frequency of the main sequence decreases from 88 % to 38 %.

At the upper boundary of *Pinus mugo* natural distribution – at about 2500m, the variation of phyllotactic twig pattern structure (entropy) becomes cyclic, with six year duration of the cycles.

2. **The changes in temperature during the period of phyllotactic pattern formation of Pinus mugo twigs determine about 48 % of the changes in pattern structure**, the latter lagging behind with one or two years.

It is obvious that when the altitude increases, the number of phyllotactic patterns (Fibonacci-sequences) of the vegetative organs of *Pinus mugo* also increases above a given altitude. → see Table below!

### Table 1: Data on the frequency and relative frequency of the different phyllotactic patterns for *Pinus mugo* twigs at different altitudes.

<table>
<thead>
<tr>
<th>Sequence No.</th>
<th>FIBONACCI-Sequences present in given altitude</th>
<th>Altitude in (m)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Relative Frequency</td>
<td>Frequency</td>
</tr>
<tr>
<td>F1</td>
<td>(1,2,3,5,13,...)</td>
<td>231</td>
<td>0.902</td>
</tr>
<tr>
<td>F3</td>
<td>(2,1,3,5,8,13,...)</td>
<td>16</td>
<td>0.063</td>
</tr>
<tr>
<td>F2</td>
<td>(1,3,4,7,11,18,...)</td>
<td>3</td>
<td>0.012</td>
</tr>
<tr>
<td>F4</td>
<td>(1,2,3,5,11,18,...)</td>
<td>6</td>
<td>0.023</td>
</tr>
<tr>
<td>F8</td>
<td>(2,5,7,12,19,31,...)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>F11</td>
<td>(3,7,10,17,27,44,...)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>F6</td>
<td>(1,4,5,9,14,23,...)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>F9</td>
<td>(1,3,4,7,11,18,...)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>F6</td>
<td>(1,7,8,15,23,38,...)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>F5</td>
<td>(1,2,3,5,8,13,...)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>F13</td>
<td>(1,6,7,13,20,33,...)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>F10</td>
<td>(2,7,9,16,25,41,...)</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

*Note: The number of Fibonacci-Sequences is increasing with altitude*

**Fig 1**: *Pinus mugo*
13.1 Different Temperatures at different altitudes caused changes in Phyllotactic-pattern-variation

Different temperatures at the research sites at different altitudes (550 – 2500 m), during the period of phyllotactic pattern formation, caused the changes in variability of the found phyllotactic patterns. The number of found patterns (different Fibonacci Sequences) increased with altitude. But because „temperature at different altitudes“ is a complex subject, we need to understand „temperature at different altitudes“ more precisely in order to understand the causes of phyllotactic pattern variability!

Some fundamental facts about „Temperature“:
The temperature (thermal energy) of a solid body (e.g. a plant) is associated primarily with the vibrations of its molecules. Heat transfer to the plant happens through thermal conduction or thermal radiation. Here especially heat transfer through thermal radiation to the plant must be examined more closely! This is the transfer of energy by means of electromagnetic waves (photons). Especially Infrared-Radiation is important for the heat transfer to the plant.

Infrared radiation lies energetically in the area of the rotation niveaus of small molecules and in the area of the oscillation niveaus of molecule bindings. That means the absorption of infrared light (infrared radiation) leads to an vibration excitation of the molecule bindings and of the matter in the plant in general, or in other words to an increase of the heat energy (temperature) of the plant. The energetic Near-Infrared-Radiation (IR-A/B), with approximately 0.7 to 3 µm wavelength can excite overtone or harmonic vibrations in matter (in the plant molecules/plant structure).

13.2 Radiation is different at different altitudes

The temperature (thermal energy) of the plant increases or decreases by absorbing (see Spectroscopy) or by emitting radiation, or through thermal conduction. Especially Near-Infrared-Radiation with wave-lengths of 0.7 to 3 µm is absorbed by the water molecules of the plant and is responsible for the temperature of the plant. The distribution of Infrared-Radiation in the atmosphere is different in different altitudes, as the diagram on the right clearly shows. The sun’s IR-A/B-radiation with 1 to 3µm wave-length is absorbed by H₂O, CO₂, and other atmospheric gas, more and more on it’s way from 10 km altitude to sealevel. But also IR-C and Far-IR radiation with 3-50µm gets absorbed more & more.

Another important result of Dr. Vakarelov’s study:

Additional Dr. Vakarelov’s study showed that the phyllotactic pattern variability (Fibonacci Sequence-variability) changed over the years! The study also showed that the variability of the phyllotactic patterns in high altitude changed cyclic, with six year duration of the cycles.

Figure 3: The diagram on the right-hand side shows the variability of entropy (variability of Fibonacci Sequences) with respect to altitude for „Pinus Mugo“ twigs. It is obvious that at 2500 m the curve shows a clear cyclic process, while at 2200 m the cyclic process is less significant, and at 1900 m nonexistent. The cyclic process has a period of “6 Years.”

13.3 Phyllotactic-pattern-variability seems to vary with the sunspot-cycle

Figure 4: The next diagram on the right shows how sunspot-numbers, cosmic ray flux, X-ray’s and proton flux changes with the 11 to 12 year sunspot-cycle. A weak correlation between phyllotactic-pattern-variability and cosmic ray flux is noticable.

How does the radiation in the atmosphere change with the sunspot-cycle?: Solar-X-ray radiation and Ultraviolet radiation (especially extreme UV (EUV) with 10 to 124 nm wavelength varies markedly over the sunspot-cycle (UV-B at 300 nm (by up to 400% !)). This radiation has a big impact on Earth’s upper atmosphere. Increased X-ray & UV-radiation leads to heating of the Ionosphere. The ionisation of the Ionosphere also affects the propagation of radio-waves. Especially the HF-radio spectrum (3-30 MHz), but also the MF- & VHF-radio-spectrum is effected (MF=300kHz-3MHz & VHF=30-100 MHz). 30 MHz corresponds to 10 m wave-length.

Fig. 3: Distribution of radiation in the atmosphere, at 11 km altitude and at sealevel. It is obvious that at higher altitude the variation of radiation with different wave lengths is higher than at sea level.

Fig. 4: see: Sun-Climate-Connections
13.4 Two more quantitative studies to phyllotactic-pattern-variations in Pine-Cones & Sunflowers

The results of the two quantitative studies do not refer to research sites at different altitudes. But the results of Study 7 are interesting in reference to Dr. Vakarelov’s study. And the results of the quantitative Study 8 to Sunflower-seedheads: shows interesting Fibonacci-Pattern-Variations in the Sunflower-seedhead.

Study 7: „Abortive phyllotactic patterns in cones of some conifers: a quantitative study“ by Veronika Fierz, Weblink: “Abortive phyllotactic patterns in cones of some conifers”

Pinus nigra

Fig. 2 – a cone with the very rare spiral pattern 9:13 (Fibonacci-sequence 4, 9, 13, 22, 35, 57, ...)

<table>
<thead>
<tr>
<th>Fibonacci-type sequence</th>
<th>Spiral pattern NaN</th>
<th>Number N of Pinus nigra cones</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3, 5, 8, 13, ...</td>
<td>F1</td>
<td>8:13 normal 5838 (97%)</td>
</tr>
<tr>
<td>2, 4, 6, 10, 16, ...</td>
<td>F2</td>
<td>10:16 69</td>
</tr>
<tr>
<td>1, 2, 3, 5, 8, 13, ...</td>
<td>F3</td>
<td>7:11 20</td>
</tr>
<tr>
<td>3, 6, 9, 15, ...</td>
<td>F4</td>
<td>9:15 9</td>
</tr>
<tr>
<td>1, 2, 3, 4, 5, 9, 14, 20, 29, ...</td>
<td>F5</td>
<td>9:14 3</td>
</tr>
<tr>
<td>2, 5, 7, 12, ...</td>
<td>F6</td>
<td>7:12 3</td>
</tr>
<tr>
<td>7, 12, ...</td>
<td>F7</td>
<td>8:12 5</td>
</tr>
<tr>
<td>1, 5, 6, 11, 17, ...</td>
<td>F8</td>
<td>6:11 1</td>
</tr>
<tr>
<td>2, 3, 5, 8, 13, ...</td>
<td>F9</td>
<td>9:13 2</td>
</tr>
<tr>
<td>3, 7, 10, 17, ...</td>
<td>F11</td>
<td>7:19 1</td>
</tr>
</tbody>
</table>

Vakarelov’s study 6: The ~270 Pinus Mugo sunflowers examined by Vakarelov at 550m altitude show similar phyllotactic-pattern-variability

Study 8: „Novel Fibonacci and non-Fibonacci structure in the Sunflower“ by J. Swinton, E. Ochu & Others

Weblink: Novel_Fibonacci_and_non-Fibonacci_structure_in_the_sunflower

Fibonacci-structures in the spirals of Sunflower seedheads were evaluated. We collected data on 657 sunflowers. In our most reliable data subset, we evaluated 768 clockwise or anticlockwise parastichy numbers of which 565 were Fibonacci numbers, and a further 67 had Fibonacci structure of a predefined type. We also found more complex Fibonacci structures not previously reported in sunflowers.

Important Results of this study:

Different parastichy-(Fibonacci)-numbers visible in one seedhead are always smaller at the center, and larger at the outer rim of the seedhead (see sample images: Fig 9, 11, 13), and there are up to 4 different parastichy-numbers visible per seedhead (from the same Fibonacci-Sequence). Beside the standard F1-Fibonacci-Numbers: 21, 34, 55, 89 normally visible in Sunflowers, there are also parastichy-numbers of other Fibonacci-Sequences like F2 (Lucas), F7, F8, F27 visible in a small number of samples.

The parastichy-number count (spiral-pattern) normally is more orderly in one direction than in the other. The unusual parastichy-pair 48, 50 (Fig. 18) is a special case!

Methods & Results: In search for aberrant (usual) patterns in „European black pine“, 6000 cones from one single tree have been examined, almost its whole cone production of about two years. This tree was planted more than 60 years ago in a garden in Küsnacht near Zürich, Switzerland at 560 m altitude.

Apart from the usual pattern 8:13, nine different types of unusual patterns have been found. The bijugy pattern 10:16 (69 cones), the first accessory pattern 7:11 (20 cones) and the trijagy pattern 9:15 (9 cones) were the most frequent. The patterns 8:12, 9:14 and 7:12 followed with 3–5 cones, and the rarest patterns were: (F14) 9:13 (2 cones), (F7) 6:11 (one cone) and (F11) 7:10 (one cone).
14 Electromagnetic-Radiation from specific wave-length-ranges can change Phyllotactic Patterns

As described in Chapter 13 the number of different phyllotactic patterns (Fibonacci Sequences) visible in the twig-patterns of “Pinus mugo” threes, increases with increasing altitude. The main reason for this increase of phyllotactic-pattern variation, at higher altitudes, are different environmental conditions. Especially changes in temperature, or more precise, changes in the „radiation-mix“ (or radiation composition) seem to be responsible for the increase of phyllotactic-pattern variation, at higher altitudes. At higher altitudes there are more wave-length ranges present, that can affect (change) the phyllotactic (Fibonacci) pattern formation.

In the following we have a closer look at the electromagnetic Spectrum at sealevel and at a higher altitude to identify the wave-length ranges which may be responsible for the increase in phyllotactic pattern formation.

<table>
<thead>
<tr>
<th>Wave-length-range</th>
<th>Property of Radiation</th>
<th>Effect on the plant</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 nm to 2000 nm and ( &gt; 120 nm )</td>
<td>Has a penetration depth &gt; 1mm in water. - See diagram 5</td>
<td>This radiation from the sun can reach the Apical Meristem, and so it can change and affect phyllotactic pattern formation</td>
</tr>
<tr>
<td>280 nm to 450 nm</td>
<td>Ultraviolet radiation and strong Blue Light (&gt;30 Wm²) This is the most energetic radiation from the Sun It has a damaging effect on plant cells (Chloroplasts)</td>
<td>Causes strong Chloroplast Movements (see following studies: Study 10; Study-II) can disturb Water Cluster formation</td>
</tr>
<tr>
<td>380-480 &amp; 620-688 nm</td>
<td>main Absorptions bands of Chlorophyll a/b (visible light)</td>
<td>→ radiation absorbed by Chlorophyll</td>
</tr>
<tr>
<td>700 nm to 2000 nm (0.7 to 2 µm)</td>
<td>red light and higher energy Infrared-Radiation (IR-A/B) can excite overtone and harmonic vibrations in matter many absorption lines of Water in this radiation range</td>
<td>This radiation is absorbed by different molecules in the plant cells. It seems to trigger floral bud induction see Study 9</td>
</tr>
<tr>
<td>= 6 - 20 µm (peak at 10 µm)</td>
<td>Black-body Radiation of a body at 20°C (293 K) temperature</td>
<td>→ radiation that matter at 20°C emits</td>
</tr>
</tbody>
</table>

Diagram 3: The small diagram on the right (top) shows Water-Cluster absorption spectra. The diagram (bottom) shows the Cellulose absorption spectra. → In high altitude there is more infrared radiation available in the spectral range of 4.5 to 8 µm which can cause increased excitation in this spectral range.

Water Cluster formation can be affected by the following radiation:
The high energetic wave-length-range 280 to 450 nm causes strong Chloroplast motion in cells that can disturb Water Cluster formation. The energetic Infrared wave-length-range 700 to 2000 nm can excite strong vibrations in molecules (H₂O) in the Apical-Meristem. And the 4500 - 8000 nm range can excite large Water-Clusters.

Diagram 4: Absorption Spectrum of Chlorophyll a & b

Diagram 5: Penetration-depth of electromagnetic radiation in Water in Millimeter, from the Ultraviolet- to the Radio-wavelength-Range. Blue Light has the maximum penetration with ≈ 60m water-depth. The blue marked areas indicate a water penetration-depth > 1mm.
15 Phyllotactic-pattern (bud induction) caused by far-red & infrared light: 750 nm to ≈2000 nm

This study No 9 shows that far-red and infrared radiation with wavelengths > 750 nm must be the trigger for phyllotactic-pattern formation & bud-induction, in short-day-strawberry plants (Fragaria ananassa) examined in this study. Radiation with wavelengths < 750 nm didn’t reach the apical meristem in this test setup, because it was filtered (eliminated) by heavy leaf-cover above the apical meristem. But the true source of the IR-radiation > 780 nm isn’t clear because triphosphor-fluorescent-lamps don’t produce this radiation!! The source of the IR-radiation probably were additional strong infrared lamps or maybe a heater in the growth chamber, which caused strong IR-radiation in the range of 780 - 2000 nm which could penetrate the leaf cover.

Study 9: „Red Light Affects Flowering under long days in a Short-day Strawberry Cultivar“

by Fumiomi Takeda & D. Michael Glenn – USDA-ARS, Appalachian Fruit Research Station, WV 25430

Abstract. July-plugged transplants of short-day cv. Strawberry Festival (Fragaria xananassa) flowered in October and November although they were grown under long photoperiods and warm temperatures (greater than 21°C) in July and August. These unexpected results were attributed to a high plant density (320 transplants/m²) that provided continuous and heavy leaf cover, which eliminated red light (less than 700 nm) from reaching the crowns. This hypothesis was tested by illuminating crowns of transplants growing in 50-cell packs for 16 h·d⁻¹ with red light-emitting diode lamps (maximum wavelength at 639 nm and 80% of output between 617 and 655 nm). Red light treatment caused a significant reduction in fall flowering. It is proposed that a high ratio of far-red light to visible light reaching the crown will play a role in floral bud induction, possibly as early as mid-August. Transplants of some short-day cultivars started as plug plants in early July have the capacity toflower and fruit in the fall and the next spring, enabling growers in the mid-Atlantic coast region to obtain two harvests within 1 year from a single planting. Weblinks to study: Weblink 1, Weblink 2

Summary of the experiment and important results of this study:

Short-day strawberry cultivars have been induced to flower in the fall (autumn) without exposing the plants to the normally required cold temperatures or short-day conditions, needed for bud induction. July-plugged plants grown in a greenhouse at high plant density, under long days, and at temperatures >21°C during day and night flowered in the fall (autumn). Early in August 4 trays of “short-day”strawberry plants (320 transplants/m²) were placed in an EGC M-36 growth chamber. On two trays the crowns (SAM) of the plants were illuminated with red-LED’s (80% of LED output was in the 617 – 655nm range).

On the control plants (the other two trays), the crowns (the SAM) were not illuminated with the red LEDs.

Spectroradiometric measurements on the control plants, in mid-August (in the growth chamber), showed no transmission of red light and shorter wave-length light through the leaves to the crowns, but only far-red & near-infrared light was reaching the crowns (SAM) - see Fig.3 The crowns (SAM) of the plants were under a dense (heavy) leaf cover (leaf canopy). The light reaching the crowns (SAM) of the plants was depleted of wavelengths less than 700 nm because of the heavy leaf cover (canopy) above the crowns. Only far-red and infrared light >700 nm reached the crowns (SAM) in the control plants. → See Fig. 3

The final results showed that red light from the LED-lamps directed at the crown actually delayed the flower bud induction. Flower bud emergence was observed in only 17% of plants, which were illuminated with the red LED, compared with 38% of the control plants which were not illuminated with red LED’S. By late November 95% of the non illuminated control plants had open flowers, compared with only 54% of those illuminated with the red LED’S.

The high ratio of infrared light to visible light, which reached the crowns (Shoot Apical Meristem) played a significant role in floral bud induction. Light is absorbed by photo-receptors, which promote the expression of genes that change the fate of the shoot apical meristem (SAM) from vegetative growth to reproductive development.

In this experiment selective filtering by the heavy leaf cover, resulted in the illumination of the apical meristem with only light with wavelengths >700 nm. This shift in spectral composition of the received light was biologically significant, regarding the floral bud induction and the reproductive development of the flowers. It is possible that transition to flowering in the SAM, can be achieved by a high share of far-red and infrared light >750 nm, and only a small share or none visible light < 750 nm present → see spectra „crown level” in Fig. 3.
From the Fibonacci-Sequences shown by *Pinus mugo* at 2500m an infinite Fibonacci-Table was developed:

There are clear spatial interdependencies noticable between the different Fibonacci-Sequences, which are connected by the golden ratio \( \phi \). There is a complex network visible between the numbers of all Sequences. This table of Fibonacci-Number Sequences can be extended towards infinity and all natural numbers are contained in the lower half only once!

For 3 numbers A, B and C in the below shown arrangement, which belong to the same 3 ( or 2 ) different Fibonacci-Sequences, the following rule is true :

The ratio of the difference ( C-A ) indicated by a "red line", to the difference ( B-C ) indicated by a "black line" is approaching the golden ratio \( \phi \) for the further progressing Fibonacci-Number Sequences towards infinity (downwards in the table).

"Main Bow-Structures" are also linked by the "golden ratio" \( \phi \)!

**FIBONACCI – Number Sequences No. 1 to 14 ( F1 - F14 ) → see extended table in the Appendix !**

<table>
<thead>
<tr>
<th>Row No.</th>
<th>F1 Fibonacci-Base-Sequence</th>
<th>F2 Lucas-Sequence</th>
<th>F3 Fibonacci-Sequence (x 2)</th>
<th>F4 Fibonacci-Sequence (x 3)</th>
<th>F5 Fibonacci-Sequence (x 4)</th>
<th>F6 Lucas-Sequence (x 2)</th>
<th>F7 Lucas-Sequence (x 3)</th>
<th>F8 Lucas-Sequence (x 4)</th>
<th>F9 Lucas-Sequence (x 5)</th>
<th>F10 Lucas-Sequence (x 6)</th>
<th>F11 Lucas-Sequence (x 7)</th>
<th>F12 Lucas-Sequence (x 8)</th>
<th>F13 Lucas-Sequence (x 9)</th>
<th>F14 Lucas-Sequence (x 10)</th>
</tr>
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<tr>
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<td>89</td>
<td>144</td>
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<td>46368</td>
<td>75401</td>
<td>125864</td>
<td>209011</td>
<td>334953</td>
</tr>
</tbody>
</table>

Note: Below this line all natural numbers are contained in the Fibonacci Sequences just once !
17  A general rule exists which connects numbers of different Fibonacci-Sequences by the golden ratio $\phi$

The following two examples explain the rule which was described in general on the previous page:

The examples show how the quotient of the differences between the numbers of designated Fibonacci-Sequences (indicated by red- and black-lines in the table), is approaching the golden ratio for the number sequences progressing towards infinity.

For the examples we look at the Fibonacci Sequences $F_1$, $F_2$ and $F_3$ (→ $F_2$ is the Lucas-Sequence, $F_3 = F_1 \times 2$)

$$\begin{align*}
F_1 & \quad \text{Note: The differences correspond to numbers of the } F_1-\text{main sequence} \\
13 & \quad 18 - 13 = \frac{5}{3} = 1,666 \\
21 & \quad 29 - 21 = \frac{8}{5} = 1,6 \\
34 & \quad 42 - 34 = \frac{8}{5} = 1,6
\end{align*}$$

$$\phi$$

17.1 Interesting properties of the Fibonacci-F1 Sequence (and other Fibonacci-Sequences):

- The numbers of the Fibonacci F1 – Number Sequence seem to contain all prime numbers as prime factors!
- This is not the case for all other Fibonacci-Sequences where certain prime factors are missing! (see Appendix)
- And all prime factors appear periodic in defined “number-distances” in the sequence (see left side of table)
- This is the case for all Fibonacci-Sequences! (→ These mentioned properties must be analysed in more detail!)

Table 2: Periodicity of the prime factors of the Fibonacci F1 - Number Sequence:

<table>
<thead>
<tr>
<th>some prime factors shown in table form</th>
<th>in prime factors factorized Fibonacci-Numbers</th>
<th>Sluvé-Sums</th>
</tr>
</thead>
<tbody>
<tr>
<td>41 37 29 23 19 17 11 7 5 3 2</td>
<td>repeating products new products</td>
<td></td>
</tr>
</tbody>
</table>

$\begin{align*}
F & \quad F' & \quad F^\ast & \quad \text{Nr.} \\
1 & 1 & 1 & 1 \\
2 & 1 & 1 & 2 \\
3 & 2 & 1 & 3 \\
5 & 5 & 2 & 15 \\
8 & 8 & 3 & 16 \\
13 & 13 & 5 & 18 \\
21 & 21 & 8 & 20 \\
34 & 34 & 13 & 22 \\
55 & 55 & 15 & 24 \\
89 & 89 & 17 & 26 \\
144 & 144 & 19 & 28 \\
233 & 233 & 21 & 30 \\
377 & 377 & 23 & 32 \\
610 & 610 & 25 & 34 \\
987 & 987 & 27 & 36 \\
1597 & 1597 & 29 & 38 \\
2584 & 2584 & 31 & 40 \\
4181 & 4181 & 33 & 42 \\
6765 & 6765 & 35 & 44
\end{align*}$

$\Rightarrow$ See some selected Fibonacci-Sequences in more detail in the Appendix!
17.2 Constant $\varphi$ (Φ) defines all Fibonacci-Sequences and the square roots of all natural numbers

The asymptotic ratio of successive Fibonacci numbers leads to the Golden Ratio constant $\varphi$ (or $\Phi$), and the Fibonacci Sequences describe morphological patterns in a wide range of living organisms. It is one of the most remarkable organizing principles mathematically describing natural and manmade phenomena.

The constant $\varphi$ is the positive solution of the following quadratic equation:

$$x + 1 = x^2$$

$$\Rightarrow \varphi = \frac{1 + \sqrt{5}}{2} = 1.618034...$$

Because the value of constant $\varphi$ is close to the square root of 2 and the square root of 3, I draw $\varphi$ into the start section of the Square Root Spiral:

17.3 To the discovery of an important algebraic equation regarding Constant $\varphi$ (Phi)

This discovery indicates that constant $\varphi$ and the base unit 1 form the base of mathematics and geometry.

And the distribution and structure of matter (energy) in space, is fundamentally based on constant Phi and 1.

The start of the Square Root Spiral is shown with the constant $\varphi$ drawn in:

Now we see what we can do with this arrangement of right triangles, and with the help of the Pythagorean theorem.

From the right triangle $\varphi$, square root of 2 & $u$ follows:

$$\varphi^2 = (\sqrt{2})^2 + u^2$$

Application of the Pythagorean theorem

$$\Rightarrow u = \sqrt{\varphi^2 - 2} = 0,786151377.....$$

We can calculate this value of $u$ with the calculator.

I did research with Google, and I found a study where the constant $u$ was expressed with an algebraic term!

With the help of this algebraic term it was possible to find interesting new properties of constant $\varphi$!

See next page!
From Equation (4.10) from the study shown on the righthand side I have found the algebraic term which describes the calculated value of \( u \):

\[
\frac{\sqrt{2\sqrt{5} - 2}}{2} = 0.786151377... \quad = u
\]

From this algebraic term it follows:

\[
\sqrt{\varphi^2 - 2} = \frac{\sqrt{2\sqrt{5} - 2}}{2}
\]

\[\rightarrow 4\varphi^2 - 8 = 2\sqrt{5} - 2 \quad ; \quad \text{we square both sides and transform}
\]

\[
\varphi^2 = \frac{\sqrt{5} + 3}{2} \quad ; \quad (1) \quad \text{we solve for } \varphi^2
\]

\[
\sqrt{5} = 2\varphi^2 - 3 \quad ; \quad (2) \quad \text{we solve for } \sqrt{5}
\]

Now we go back to the square root spiral and use the following right triangle:

\[
(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem}
\]

\[
6 = (2\varphi^2 - 3)^2 + 1 \quad ; \quad \text{we replace } \sqrt{5} \text{ by equation (2) and transform}
\]

\[
\rightarrow 3 = \frac{\varphi^4 + 1}{\varphi^2} \quad (3) \rightarrow \sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} \quad (4) \quad ; \quad \text{square root 3 expressed by } \varphi \text{ and 1 !}
\]

Now we use the following right triangle:

\[
(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem & inserting equation (3)}
\]

\[
\rightarrow 2 = \frac{\varphi^4 + 1}{\varphi^2} - 1 \rightarrow 2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \quad (5) \quad \text{and } \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (6)
\]

Now we insert equation (3) in equation (2):

\[
\rightarrow \sqrt{5} = 2\varphi^2 - \frac{\varphi^4 + 1}{\varphi^2} \rightarrow \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2} \quad ; \quad (7) \quad ; \quad \text{square root 5 expressed by } \varphi \text{ and 1}
\]
Now we use the following right triangle:

\[
(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem & inserting equation (7)}
\]

\[
6 = \left(\frac{\varphi^4 - 1}{\varphi^2}\right)^2 + 1 \quad \Rightarrow \quad 6 = \frac{\varphi^8 - \varphi^4 + 1}{\varphi^4} \quad (8) \quad \text{and} \quad \sqrt{6} = \frac{\varphi^8 - \varphi^4 + 1}{\varphi^4} \quad (9)
\]

We can now continue and use the following right triangles of the square root spiral:

\[
(\sqrt{7})^2 = (\sqrt{6})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem & inserting equation (8)}
\]

\[
7 = \frac{\varphi^8 + 1}{\varphi^4} \quad (10) \quad \Rightarrow \quad \sqrt{7} = \frac{\varphi^8 + 1}{\varphi^4} \quad (11)
\]

In the same way we can now calculate all square roots of all natural numbers with the next right triangles:

\[
8 = \frac{\varphi^8 + \varphi^4 + 1}{\varphi^4} \quad (12) \quad \text{and} \quad \sqrt{8} = \frac{\varphi^8 + \varphi^4 + 1}{\varphi^4} \quad (13)
\]

\[
10 = \frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4} \quad (14) \quad \text{and} \quad \sqrt{10} = \frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4} \quad (15)
\]

\[
11 = \frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4} \quad (16) \quad \text{and} \quad \sqrt{11} = \frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4} \quad (17)
\]

\[
12 = \frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4} \quad (18) \quad \text{and} \quad \sqrt{12} = \frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4} \quad (19)
\]

From the above shown formulas (equations) I have read a general rule for all natural numbers $> 10$:

Note: $\Rightarrow$ The expression $(3+n)$ in the rule can be replaced by products and/or sums of the equations (3) to (13)

\[
(10+n) = \frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4} \quad (20) \quad \text{and} \quad \sqrt{10+n} = \frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4} \quad (30)
\]

With this general formula we can express all natural numbers $\geq 10$ and their square roots only with $\varphi$ and $1$!

This is a quite interesting discovery!
17.5 Constant Pi (π) can also be expressed by only using constant φ and 1 !

\[ \pi = \lim_{k \to \infty} \frac{\sqrt{\phi^4 - \phi^2 + 1}}{\phi^2} \]

It is also possible to derive from Viète’s formula a related formula for π that still involves nested square roots of two, but uses only one multiplication:

\[ \pi = \frac{2}{\sqrt{2}} \cdot \frac{2}{\sqrt{2 + \sqrt{2}}} \cdot \frac{2}{\sqrt{2 + \sqrt{2 + \sqrt{2}}}} \ldots \]

If we replace the number 2 in the above shown formulas by the found equation (5) where number 2 can be expressed by constant φ and 1, then we can express the constant Pi (π) also by only using the constant φ and 1 !

Replace Number 2 in the above shown formulas with this term.

\[ 2 = \frac{\phi^4 + 1}{\phi^2} - 1 \]

(5) and \[ \sqrt{2} = \sqrt{\frac{\phi^4 - \phi^2 + 1}{\phi^2}} \] (6)

It becomes clear that the irrationality of Pi (π) is also only based on the constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 ! Numbers don’t exist! Only φ & 1 exist!

Constant Pi (π) can now be expressed in this way, by only using constant φ and 1:

\[ \pi = \lim_{k \to \infty} \left[ \frac{\phi^4 - \phi^2 + 1}{\phi^2} \right]^k \]

It becomes clear that the irrationality of Pi (π) is only based on constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 !

Natural Numbers, their square roots and irrational and transcendental constants like Pi (π) can be expressed (calculated) by only using constant φ and 1! This is also valid for all rationals (fractions) and their square roots.

Numbers and number-systems don’t seem to exist! They are manmade and therefore can be eliminated.

This is an interesting discovery because it allows to define most (maybe all) geometrical objects only with φ & 1!

The result of this discovery may lead to a new base of number theory. Not numbers like 1, 2, 3,...... and constants like Pi (π) etc. are the base of Number Theory! Only the constant φ and the base unit 1 (which shouldn’t be considered as a number) form the base of mathematics and geometry. This will certainly also have an impact on Physics!

Constant φ and the base unit 1 must be considered as the fundamental „space structure constants“ of the real physical world!

In the physical world the geometries of all possible crystal-lattice-structures are fundamentally based on Phi and 1.

There probably isn’t something like a base unit if we consider a „wave model“ as the base of physics and if we see the universe as one oscillating unit. In the universe everything is connected with everything. see: Quantum Entanglement

⇒ Please also read my 12 Conjectures on the next page (Chapter 17.6)
Referring to my discovery regarding constant $\varphi$ (Phi), I want to define the following 12 Conjectures:

**Here the 12 conjectures:**  ( $\rightarrow$ you can call them Harry K. Hahn's conjectures )

1.) All Natural Numbers and their square roots can be expressed (calculated) by only using the mathematical constant Phi (golden mean = 1.618...) and number 1. This statement is also valid for all rationals (fractions) and their square roots.

2.) All existing irrational numbers seem to be constructions out of Phi and 1. For example the irrational transcendental constant Pi (3.1415926...) can also be expressed by only using Phi and 1!

3.) Phi and 1 are the base units of Mathematics! Numbers and number-systems don’t exist! They are manmade and therefore can be eliminated. In principle Mathematical Science can be carried out by only using Phi and 1, as base units.

4.) All geometrical objects, including the Platonic Solids can be described by only using constant Phi and 1. Because all natural numbers, their square roots, rationals (fractions) and probably all irrational and all transcendental numbers too, can be expressed by only using Phi and 1.

5.) Point 4.) leads me to the conclusion that in the physical world the geometries of all possible crystal-lattice-structures are fundamentally based on Phi and 1. The more fundamental the lattice the simpler it can be expressed by Phi and 1.

6.) Point 4.) 5.) & 7.) leads me to the conclusion that on the molecular level (and probably on the atomic level too), as well as on the macroscopic level (8.) the distribution and structure of matter (=energy) in space, is fundamentally based on constant Phi and 1. It seems to be a fundamental physical „Space Structure Constant“

Together with Point 7.) this indicates that the curvature of spacetime at the molecular level (crystals) and at the atomic level, as well as on the macroscopic level is defined only by the “Space Structure Constant Phi” and the base unit 1. This idea will help to unify General Relativity with Quantum Mechanics! If the gravitational singularity in M87 indeed has a dodecahedral structure then gravitation, which is the geometric property of spacetime, can be described in Quantum Mechanics and at the cosmic level by the same constant duo: Phi and base unit 1!

7.) The structure of the M87 black hole ( $\rightarrow$ EHT2017 ) indicates a dodecahedral structure. The distribution of matter in gravitational singularities therefore seems to be defined essentially by constant Phi and base unit 1! The largescale distribution of matter in the universe seems to be predominantly based on an order-5 Poincare-Dodecahedral-Space.

$\rightarrow$ weblink to my study, ( or alternatively here: http://vixra.org/abs/1907.0348 )

Title: “EHT2017 may provide evidence for a Poincare Dodecahedral Space Universe”

8.) The natural numbers can be assigned to a defined infinite set of Fibonacci-Number Sequences.

9.) This infinite set of Fibonacci-Number Sequences, and the numbers contained in these sequences, are connected to each other by a complex precisely defined spatial network based on constant Phi. ( $\rightarrow$ see table in Appendix B ) For the progressing Fibonacci-Sequences towards infinity, the connections between the numbers approach constant Phi.

$\rightarrow$ see Chapter 16 and 17 and Appendix B

10.) Constant Phi (golden mean = 1.618..) must be a fundamental constant of the final equation(s) of the universal mathematical and physical theory. ( $\rightarrow$ It may be the only irrational constant that appears in the(se) equation(s) )

11.) The number-5-oscillation ( $\rightarrow$ the numbers divisible by 5 ) in the two number sequences 6n+5 (Sequence 1) and 6n+1 (Sequence 2), with n=(0,1,2,3,...) , defines the distribution of the prime numbers and non-prime-numbers. The number-5-oscillation defines the starting point and the wave length of defined non-prime-number-oscillations in these Sequences 1+2 (SQ1 & SQ2). ( Note : the combination of the two sequences SQ1 & SQ2 is considered here )

$\rightarrow$ weblink to my study: https://arxiv.org/abs/0801.4049 ( or alternatively here: http://vixra.org/abs/1907.0355 ) For a quick overview please see pages 15 to 18 in this study: weblink to the study: “EHT2017 may provide evidence....”

12.) The importance of the number-5-oscillation for the distribution of primes and non-primes is a further indication for the conjecture that the large-scale structure of the universe seems to be predominantly (mainly) based on an order-5 Poincare-Dodecahedral-Space structure. The space structure of the universe seems to be based essentially on the Platonic Solid: the Dodecahedron ( $\rightarrow$ consisting of 12 regular pentagonal faces, three faces meeting at each vertex )

The time will show if my Conjectures are correct!
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Entropy-driven formation of large icosahedral colloidal clusters - by Bart de Nijs, S. Dussi, F. Smallenburg, ...
https://www.nature.com/articles/nmat4072?proof=trueIn%EF%BB%BF
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Study 6: “Changes in phyllotactic pattern structure in Pinus mugo due to changes in altitude” Study to Fibonacci pattern variation in Pinus Mugo by Dr. Iliya Iv. Vakarelov, University of Forestry, Bulgaria (1982-1994) From the book “Symmetry in Plants” by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada (Part I. Chapter 9, pages 213-229), ISBN: 981-02-2621-7, Weblinks: Weblink 1 (Google Books); Weblink 2

Study 7: „Aberrant phyllotactic patterns in cones of some conifers: a quantitative study“ - by Veronika Fier Weblink: Aberrant phyllotactic patterns in cones of some conifers (researchgate.net)


To the importance of constant Phi for the physical world: → see Chapter 17.2 to 17.6


The golden ratio Phi (ϕ) in Platonic Solids: http://www.sacred-geometry.es/?q=en/content/phi-sacred-solids


Appendix: Movie - Weblinks:

Movie 1 (short_3min): Amazing Resonance Experiment with Chladni-plates - see also: Movie-1_(long_8min + tone) https://www.youtube.com/watch?v=wVjAgrUBF4w & https://www.youtube.com/watch?v=1yaqUl4b974

Movie 2: Cymatic Music (in Water) - https://www.youtube.com/watch?v=sThS9OfnM1s

Movie 3: Amazing Resonance Experiment with Metal and Water (Cymatics) → see from 3:35 Min https://www.youtube.com/watch?v=_OFWp3Gxl9g

Movie 4: Sound monolith resonance patterns - Water - https://www.youtube.com/watch?v=F95Oowf4gPA

Movie 5: The beauty of twelve piano notes made visible on CymaScope in Water https://www.youtube.com/watch?v=9al397N6Tzs

Movie 6: Cymatics / Cimatica - Experiment 16 (432 Hz) - experiment with sound water & light - Start at 1:45 min https://www.youtube.com/watch?v=iD6XUSyF_N_A

Movie 7: Sound Frequencies in Water: A=440Hz & A=432Hz Using Sine, Square & Sawtooth Waves https://www.youtube.com/watch?v=UyXxGK-hwh4

Movie 8: Cymatik 2 - Vibrationen auf dem Wasser https://www.youtube.com/watch?v=Zfv6ilMF5eIg

Movie 9: Sound Resonance of Water Droplet https://www.youtube.com/watch?v=NGeGPt0QJtk

Movie 11: Cornstarch and vibrations https://www.youtube.com/watch?v=UU7iuJ98fRQ
Standing-Wave-Patterns and Acoustic-Resonance in Water

The following movies show water which was excited with defined frequencies by using a tone-generator, or a loud-speaker, or a special scientific instrument which is called a CymaScope. Please have a look at these YouTube-movies!

In the apical meristem of plants the water probably gets excited by a large pulsating central water cluster (crystal) which may produce similar patterns in the micro-scale as the following shown macro-scale patterns.

There is a similarity between some of the standing-wave patterns in the movies and phyllotactic patterns in plants! If we consider that plant cells in the apical meristem consist to 80% of water, then it is easy to imagine that phyllotactic patterns in plants may have a similar physical cause!

**CymaScope:**
The CymaScope is a scientific instrument that makes sound visible. Great detail could be obtained by imprinting sonic vibrations on the surface of ultra pure water. The surface tension of water has high flexibility and fast response to imposed vibrations, even with transients as short-lived as a few milliseconds. Water is able to translate many sinusoidal periodicities, of a given sound sample, into physical sinusoidal structures on the water's surface. Current limits to imprinting sound on water occur in the higher harmonics and are due mainly to there being insufficient energy available in this area of the audio spectrum to cause excursions of the surface tension membrane.
Appendix B.:

Infinite Fibonacci - Number - Sequence - Table:

Sequences No. 1 to 33 shown (F1 – F33):

<table>
<thead>
<tr>
<th>Row No.</th>
<th>Fibonacci-Sequences</th>
<th>Lucas-Sequences</th>
<th>Fibonacci-Sequences</th>
<th>Lucas-Sequences</th>
<th>Fibonacci-Sequences</th>
<th>Lucas-Sequences</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>F1</td>
<td>F2</td>
<td>F3</td>
<td>F4</td>
<td>F5</td>
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<td>F8</td>
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<td>F21</td>
<td>F22</td>
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<td>F27</td>
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<tr>
<td>6</td>
<td>F31</td>
<td>F32</td>
<td>F33</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Meaning of the line colors:

For 3 numbers A, B and C in the shown arrangement the following is true:

\[
\frac{C - A}{B - C} \rightarrow \phi
\]

for A, B, C

The ratio of the difference (C - A) indicated by a "red line" to the difference (B - C) indicated by a "black line" is approaching the golden ratio \( \phi \) for the further progressing number sequences (which contain these numbers) towards infinity (→ downwards).
Table 2: Periodicity of some of the prime factors of the numbers of the Fibonacci F1 - Number Sequence:

<table>
<thead>
<tr>
<th>some prime factors shown in table form</th>
<th>in prime factors factorized Fibonacci-Numbers</th>
<th>Fibonacci-Sequence F1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nr. 41 37 31 29 23 19 17 13 11 7 5 3 2</td>
<td>repeating products new products</td>
<td>F F* Nr.</td>
</tr>
<tr>
<td>41 37 31 29 23 19 17 13 11 7 5 3 2</td>
<td>2^3 2x2x2</td>
<td>1 1 1</td>
</tr>
<tr>
<td>29 13 5 2 7 x3x7</td>
<td>2x17</td>
<td>2 1 3</td>
</tr>
<tr>
<td>37 19 17 2^3 2x17 2x2x2</td>
<td>2x3x3</td>
<td>3 1 4</td>
</tr>
<tr>
<td>41 11 5 3 2</td>
<td>3x7x11</td>
<td>5 2 1</td>
</tr>
<tr>
<td>29 13 5 2 7 x3x7</td>
<td>2x17</td>
<td>8 3 1</td>
</tr>
<tr>
<td>37 19 17 2^3 2x17 2x2x2</td>
<td>2x3x3</td>
<td>11 5 2</td>
</tr>
<tr>
<td>21 7 3 2^2 2^4 2x2x2 2x3x3 2x7x23</td>
<td>47</td>
<td>14 8 3</td>
</tr>
<tr>
<td>5^2 5x5x3001</td>
<td>2207</td>
<td>17 144</td>
</tr>
<tr>
<td>37 233x521</td>
<td>121393</td>
<td>19 17711</td>
</tr>
<tr>
<td>21 7 3 2^2 2^4 2x2x2 2x3x3 2x7x23</td>
<td>47</td>
<td>21 196418</td>
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<tr>
<td>37 233x521</td>
<td>121393</td>
<td>23 17711</td>
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<tr>
<td>41 11 5 3 2</td>
<td>3x3x3x3x3x3</td>
<td>25 196418</td>
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<td>21 7 3 2^2 2^4 2x2x2 2x3x3 2x7x23</td>
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<td>27 196418</td>
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<td>37 233x521</td>
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<td>41 11 5 3 2</td>
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<td>21 7 3 2^2 2^4 2x2x2 2x3x3 2x7x23</td>
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<td>33 196418</td>
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<tr>
<td>37 233x521</td>
<td>121393</td>
<td>35 196418</td>
</tr>
<tr>
<td>41 11 5 3 2</td>
<td>3x3x3x3x3</td>
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<tr>
<td>21 7 3 2^2 2^4 2x2x2 2x3x3 2x7x23</td>
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<td>39 196418</td>
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<td>37 233x521</td>
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<td>41 11 5 3 2</td>
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<td>43 196418</td>
</tr>
<tr>
<td>21 7 3 2^2 2^4 2x2x2 2x3x3 2x7x23</td>
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<td>45 196418</td>
</tr>
<tr>
<td>37 233x521</td>
<td>121393</td>
<td>47 196418</td>
</tr>
</tbody>
</table>

Note: all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Note: The numbers of the Fibonacci F1 - Number Sequence seem to contain all prime numbers as prime factors! and all prime factors appear periodic in defined “number-distances” in the sequence (see left side of table)
Table 3: Periodicity of some of the prime factors of the numbers of the Fibonacci F2 (Lucas) - Number Sequence:

| Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime 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Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Prime factor is missing | Primary prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange.
Table 4: Periodicity of some of the prime factors of the numbers of the Fibonacci F6 - Number Sequence:

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<th>Fibonacci-F6 Sequence</th>
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Note: all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange.
### Table 5: Periodicity of some of the prime factors of the numbers of the Fibonacci F8-Number Sequence:

#### Periodicity of the prime factors 2 - 41 shown in table form

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**Note:** all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange.