Abstract: The velocity $v$ of a Transverse Electromagnetic (TEM) Wave in the vacuum of space is said to be the speed of light $c$ [6]. The derivation for the velocity of the TEM wave in a vacuum is shown to include the presence of charge $Q$. In the pure vacuum of space, no charge or matter present, it is shown that the TEM Wave is free to propagate with velocity determined by its internal relative permittivity $\varepsilon$ and permeability $\mu$.

Introduction
The concepts for permittivity $\varepsilon$ and permeability $\mu$ are introduced using Coulomb’s law and the Biot-Savart law respectively. Next, the velocity of the TEM wave is shown to be related to the measured speed of light $c$ in the vacuum of space. A unique conclusion is drawn that relates the velocity of the TEM wave to its physical internal relative permittivity and permeability in the vacuum of space free of charge.

Permittivity $\varepsilon$

The force $F$ shown in figure 1 measured on charge $Q_2$ a distance $R$ from the source charge $Q_1$ is measured using Coulomb’s law (1)[1]. The unit vector $r$ points in the direction from $Q_1$ to $Q_2$. The force $F$ is a vector with both magnitude $F$ and direction $r$ on $Q_2$. The permittivity $\varepsilon$ is determined from the medium in which the charge is situated. The permittivity in the vacuum of space $\varepsilon_0$ is $8.85 \times 10^{-12}$ Fm$^{-1}$ (Farads per meter).

$$F = r \frac{Q_1 Q_2}{4\pi \varepsilon R^2}$$

(1)

Figure 1. Point charge $Q_1$ with arrows showing $E$ field direction and Force on charge $Q_2$ at distance $R$.

A dielectric has a dimensionless permittivity $\varepsilon_r$ to that of a vacuum $\varepsilon_0$ equal to $\varepsilon$ (2). The relative permittivity of vacuum $\varepsilon_0$ is 1 by definition, for paraffin 2.1 and lead glass 6 [2].

$$\varepsilon = \varepsilon_r \varepsilon_0$$

(2)

The electric field intensity $E$ (3) [1] in a vacuum is defined as the force per unit charge on $Q_2$ at point $P$.

$$E = \frac{F}{Q_2} = r \frac{Q_1}{4\pi \varepsilon_0 R^2}$$

(3)

Notice that the permittivity of a vacuum $\varepsilon_0$ is derived in the presence of charge $Q_1$. 
Permeability constant $\mu$

The magnetic field $\mathbf{B}$ at a point $\mathbf{P}$ due to a current $\mathbf{I}$ is measured using physical parameters of the Biot-Savart Law (4)[3][4]. The current element $\mathbf{I} \, dl$ a distance $r$ to point $\mathbf{P}$ with angle $\theta$ produces a differential magnetic field $dB$, figure 2.

$$dB = \frac{\mu}{4\pi} \frac{I \, dl \sin \theta}{r^2} \tag{4}$$

Figure 2. Biot-Savart law.

The angle $\theta$ is measured clockwise from the positive direction of current $\mathbf{I}$ along $dl$ to direction of radius vector $r$ extending from $dl$ to $\mathbf{P}$. The permeability $\mu$ (5) is determined by the characteristics of the medium. Magnetic materials have a dimensionless relative permeability $\mu_r$ to that of the vacuum $\mu_0$ [5].

$$\mu = \mu_r \mu_0 \tag{5}$$

The permeability of vacuum $\mu_0$ is $4\pi \times 10^{-7}$ Hm$^{-1}$ (Henry per meter). The relative permeability can range from slightly less than 1 in Bismuth (0.99983) to 1,000,000 in Supermalloy [5].

Notice that the permeability of vacuum $\mu_0$ is defined in the presence of a source current.

Electromagnetic wave equation

A Transverse Electromagnetic Wave (TEM) traveling in empty space with velocity $v$ in the $x$ direction with time-varying electric $\mathbf{E}_y$ and magnetic $\mathbf{B}_z$ field orientations is shown figure 3.

Figure 3. Field components $\mathbf{E}$ and $\mathbf{B}$ in relation to space coordinates.

The wave equation in (6)[6] relates the space and time variation of the scalar magnitude $E_y$ of the electric field intensity. The velocity $v$ of wave propagation in a lossless medium is (7)[6].

$$\frac{\partial^2 E_y}{\partial t^2} = v^2 \frac{\partial^2 E_y}{\partial x^2} \tag{6}$$

$$v^2 = \frac{1}{\varepsilon \mu} \tag{7}$$

Per [7] the velocity of the speed of light $c$ in the vacuum of space is (8) with the measured value of $c$ equal to 299.79x10$^6$ (m/s).

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{8}$$

Now by definition the permeability of vacuum $\mu_0$ is $4\pi \times 10^{-7}$ Hm$^{-1}$ combined with the measured value for the velocity of the speed of light $c$ from (8) we get the calculated value of the permittivity in the vacuum of space $\varepsilon_0$ is 8.85 x 10$^{-12}$ Fm$^{-1}$ [7].
Notice again that any measurement of $c$ in the laboratory would require the presence of matter even if the circuit for light travel is the vacuum of space.

The relative phase velocity $p$ is defined as the ratio of the wave velocity $v$ to the speed of light (9)[8] where $\mu$ is the relative permeability of the medium and $\varepsilon$ is the relative permittivity of the medium.

$$p = \frac{v}{c} = \frac{\sqrt{\varepsilon_0 \mu_0}}{\sqrt{\varepsilon \mu}} = \frac{1}{\sqrt{\varepsilon_r \mu_r}} \quad (9)$$

The index of refraction $\eta$ in optics is defined as the reciprocal of the relative phase velocity $p$ (10)[8].

$$\eta = \frac{1}{p} = \frac{1}{v/c} = \frac{c}{v} = \sqrt{\varepsilon_r \mu_r} \quad (10)$$

From (10) $v$ is (11).

$$v = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{c}{\eta} \quad (11)$$

When $\varepsilon \mu$ is greater than 1 the TEM wave velocity $v$ is less than the speed of light. Now when $\varepsilon \mu$ in (11) is 1 in a vacuum the wave velocity is equal to the speed of light.

Notice when the TEM wave is traveling with velocity $v$ in a vacuum, this vacuum contains no matter or charge. The vacuum does contain energy in the form of a TEM wave traveling with velocity $v$ defined in (6).

**Observations**

The derivation of permittivity $\varepsilon_0$ and permeability $\mu_0$ for a vacuum was not performed in the absence of matter or charge $Q$ it was derived in the presence of charge. The TEM wave velocity $c$ in a vacuum assumes that the permittivity $\varepsilon$ and permeability $\mu$ are equal to $\varepsilon_0$ and $\mu_0$ which were not derived in a vacuum.

**Conclusion**

The initial or primordial TEM wave velocity $v$ could be equal to or greater than the speed of light when the product of $\varepsilon \cdot \mu$ is $\leq 1$, see (11). In (11) the relative permittivity and permeability are now associated physically with the waves internal properties since its environment in the context of this work is the vacuum of space free of charge or matter.

**References**