# The difference between a theory, a calculation and an explanation 

Jean Louis Van Belle, Drs, MAEc, BAEc, BPhil<br>20 April 2020<br>Email: jeanlouisvanbelle@outlook.com


#### Abstract

Summary In this paper, we try to show where and why quantum mechanics went wrong - and why and when the job of both the academic physicist as well as the would-be student of quantum mechanics turned into calculating rather than explaining what might or might not be happening. Modern quantum physicists effectively resemble econometrists modeling input-output relations: if they are lucky, they will get some kind of mathematical description of what goes in and what goes out, but the math doesn't tell them how stuff actually happens.

To show what an actual explanation might look like, we bring the Zitterbewegung electron model and our photon model together to provide a classical explanation of Compton scattering of photons by electrons so as to show what electron-photon interference might actually be: two electromagnetic oscillations interfering (classically) with each other. While developing the model, we also offer some reflections on the nature of the Uncertainty Principle.


## Contents

The characteristics of a good theory ..... 1
Positive and negative energy states ..... 1
Positive and negative energy wells. ..... 4
The nature of antimatter .....  5
The Emperor wears no clothes: matter does not dissipate ..... 8
The (Un)Certainty Principle ..... 12
The meaning of probability amplitudes and state vectors ..... 14
The meaning of spin in quantum mechanics ..... 15
The meaning of conjugating amplitudes, wavefunctions and matrices ..... 20
A classical explanation of Compton scattering ..... 23
Energy and momentum conservation ..... 23
What might actually be happening ..... 25
The Compton wavelength as a circumference ..... 25
Probing electrons with photons ..... 28
The electron-photon excitation: a temporary spin-2 particle? ..... 30
Planck's quantum of action as a vector ..... 31
Conclusions ..... 35

## The difference between a theory, a calculation and an explanation

## The characteristics of a good theory

The science of physics probes reality physically as well as logically. Physical exploration involves experiments, measurements and discoveries. Logical progress occurs when a new theory or explanation replaces previously held beliefs and convictions. Such theory or explanation should be logically and mathematically consistent. The mathematical description, for example, should respect Occam's Razor-the lex parsimoniae: one should not multiply concepts without necessity. The need for new concepts or new principles - such as the conservation of strangeness, or postulating the existence of a new force or a new potential ${ }^{1}$ - should, therefore, be continuously questioned.

When postulating the existence of the positron in 1928 - which directed experimental research to a search for it and which, about five years later, was effectively found to exist - Paul Dirac unknowingly added another condition for a good theory: all of the degrees of freedom in the mathematical description should map to a physical reality.

The mention of Dirac's hole theory makes us think we could, perhaps, usefully present the idea of energy states in a rather intuitive way before requesting the reader to wrestle his or her way through the nitty-gritty of this paper.

## Positive and negative energy states

Dirac's positron hypothesis is often said to be based on a hole theory. Dirac himself, however, phrased his theory of antimatter in terms of negative energy states. It is probably useful to quote his original brief writing on it: "The wave equation for the electron admits of twice as many solutions as it ought to, half of them referring to states with negative values for the kinetic energy." ${ }^{2}$

The concept of negative energy is, of course, nothing special in physics-but negative energy will usually

[^0]be associated with potential energy rather than with kinetic energy. ${ }^{3}$ The question, therefore, becomes: how can kinetic energy be negative? Dirac was very much puzzled by it. Indeed, he refers to his wave equations as equations of motion and the question, therefore, becomes: how can motion be negative?

The answer is this: motion cannot be negative but its direction is that of the force, so its direction can be this or that which, using our right-hand rule or Minkowski's metric signature convention, we call positive or negative. We will explain this in more detail later but, ultimately, the reader should think of energy as a force over some distance. The motion may be linear but, as Dirac noted in his Nobel Lecture, "an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us." ${ }^{4}$

Dirac is talking about the spin property here, and spin is either in this or in the opposite direction. This explains Dirac's negative kinetic energy. It is not negative, really. It just appears because of a deep conceptual flaw in the mainstream interpretation of quantum mechanics. We will come back to this later, but the gist of the matter is this:

The mainstream interpretation of quantum mechanics does not integrate the concept of particle spin from the outset because it thinks of the + or - sign in front of the imaginary unit (i) in the elementary wavefunction ( $a \cdot e^{-i \cdot \theta}$ or $a \cdot e^{+i \cdot \theta}$ ) as a mathematical convention only.

Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i \cdot \theta}=a \cdot e^{-i \cdot(\omega t-k x)}$ and $a \cdot e^{+i \cdot \theta}=a \cdot e^{+i \cdot(\omega t-k x)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). One would then expect that the professors would proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different but, no! That is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention." ${ }^{5}$

We are totally dumbfounded by this. All of the great physicists knew all particles - elementary or not have spin. ${ }^{6}$ All of them also knew it is spin that generates the magnetic moment. They should, therefore,

[^1]have integrated the idea of spin in their mathematical description from the outset. The mistake is illustrated below.


Figure 1: The meaning of $+i$ and $-i$
It is a very subtle but fundamental blunder. Spin-zero particles do not exist. ${ }^{7}$ All real particles electrons, photons, anything - have spin, and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the magnitude of the spin that differs. It is, therefore, completely odd that the plus $(+)$ or the minus ( - ) sign of the imaginary unit (i) in the $a \cdot e^{ \pm i \theta}$ function was not used to include the spin direction in the mathematical description from the start.

We do not hesitate to call it a blunder or a mistake rather than just an oddity because it is much more than just odd: it's plain wrong, indeed. Why? Because it has a lot of sad consequences. For example, this non-used degree of freedom in the mathematical description also leads to the false argument that the wavefunction of spin- $1 / 2$ particles has a 720 -degree symmetry. Indeed, physicists treat -1 as a common phase factor in the argument of the wavefunction: they think we can just multiply a set of amplitudes let's say two amplitudes, to focus our mind (think of a beam splitter or alternative paths here) - with -1 and get the same states.

We find it rather obvious that that is not necessarily the case: -1 is not necessarily a common phase factor. We should think of -1 as a complex number itself: the phase factor may be $+\pi$ or, alternatively, $-\pi$. To put it simply, when going from +1 to -1 , it matters how you get there - and vice versa - as

[^2]illustrated below. ${ }^{8}$


Figure 2: $e^{+i \pi} \neq e^{-i \pi}$
We will come back to all of this later - as part of a more detailed discussion of the meaning of quintessential concepts in quantum mechanics - so we will let this matter rest for the time being. Before we really get into the nitty-gritty, we'd just like to make another introductory remark on positive and negative energy states so as to make sure the reader has a bit of an intuitive grasp of our rather physical interpretation of what is usually presented as abstract concepts only.

## Positive and negative energy wells

The reader should note that the property of the spin being up or down is always defined with reference to an external electric or magnetic field (or some combination thereof, of course). ${ }^{9}$ The energy being positive or negative can, therefore, only be defined with reference to the direction of the magnetic field. Positrons should, therefore, not be thought of as holes. Similarly, we may note physicists always think of particles being trapped in a potential well with lower energy. However, a particle may well be trapped in a potential well with higher energy. This situation is illustrated in Feynman's very discussion of a twostate system, which we summarize by the illustrations below. ${ }^{10}$

[^3]

Figure 3: Two trivial examples of two-state systems in quantum mechanics
The illustration on the left shows the two possible energy states of an ammonia molecule $\left(\mathrm{NH}_{4}\right)$ in the context of a discussion of how an ammonia maser works - the ammonia molecule basically oscillates between two states that have an opposite electric dipole moment ${ }^{11}$ - while, on the right, we have an illustration of the splitting of energy levels in a magnetic field. We might have added an illustration of the spin of a proton can flip between an up and a down state in an MRI apparatus ${ }^{12}$ or other diagrams illustrating the basics of a two-state system in quantum mechanics, but we don't want to be too long here. Paraphrasing Feynman once again, we may say the point is this:

No matter what the physics is originally-the ammonia molecule in the ammonia maser, laser technology, the fine or hyperfine splitting of the hydrogen spectrum, proton spin flipping in MRI technology, or whatever-you can translate it into a corresponding electron problem. So if we can solve the electron problem in general, we have solved all two-state problems. And we solved the electron problem. Everything else is, therefore, very easy to understand. ${ }^{13}$

In case you wonder what we want you to remember going to the next, it is this: don't think too much about the usual hocus-pocus involved in tunneling in and out of potential wells or walls and other

[^4]quantum-mechanical blah. Once you have an intuitive understanding of what spin actually is, everything can be explained in very simple terms. ${ }^{14}$

## The nature of antimatter

The reader may or may not find the introductory remarks above intuitive and self-evident, but the question which puzzled Dirac remains, of course: what is the nature of antimatter, really? We have no definite answer to that question but we do have a few thoughts that may amount to some answer, at least.

First, you should note that the suggestion of some physicists - Richard Feynman, in particular - that anti-particles may be particles travelling back in time is plain nonsense. Time has one direction only, and we will soon explain why. ${ }^{15}$ But what is the alternative? As explained above, Dirac's suggestion that, somehow, antimatter may be defined by referring to the concept of negative energy states isn't much of an explanation.

The CODATA value ${ }^{16}$ of the magnetic moment of an electron is $\mu \approx-9.2847647043(28) \times 10^{-24} \mathrm{~J} \cdot \mathrm{~T}^{-1}$. Its theoretical value is $\mu=-q_{\mathrm{e}} \hbar / 2 \mathrm{~m} .{ }^{17}$ (its theoretical value). The minus sign is there because of the negative (elementary) charge of the electron. Indeed: $\mathrm{q}_{\mathrm{e}}$ is the elementary charge that's associated with a proton or a positron - the latter being the electron's antimatter counterpart - and so it equal to minus the electron charge.

Now, consider a particular direction of the elementary current generating the magnetic moment. [In case you wonder: that's what spin actually is: it's an elementary current. ${ }^{18}$ ] It is then quite easy to see

[^5]that the magnetic moment of an electron ( $\mu=-q_{e} \hbar / 2 \mathrm{~m}$ ) and that of a positron ( $\mu=+\mathrm{q}_{\mathrm{e}} \hbar / 2 \mathrm{~m}$ ) would be opposite. We may, therefore, associate a particular direction of rotation with an angular frequency vector $\boldsymbol{\omega}$ which - depending on the direction of the current - will be up or down with regard to the plane of rotation. ${ }^{19}$ We can, therefore, associate this with the spin property, which is also up or down. We, therefore, have four possibilities ${ }^{20}$ :

| Matter-antimatter | Spin up | Spin down |
| :---: | :---: | :---: |
| Electron | $\mu_{-\mathrm{e}}=-\mathrm{q}_{\mathrm{e}} \hbar / 2 \mathrm{~m}$ | $\mu_{-\mathrm{e}}=+\mathrm{q}_{\mathrm{e}} \hbar / 2 \mathrm{~m}$ |
| Positron | $\mu_{+\mathrm{e}}=+\mathrm{q}_{\mathrm{e}} \hbar / 2 \mathrm{~m}$ | $\mu_{+\mathrm{e}}=-\mathrm{q}_{\mathrm{e}} \hbar / 2 \mathrm{~m}$ |

Table 1: Occam's Razor: mathematical possibilities versus physical realities (1)
The table shows the ring current model also applies to antimatter. It also shows that antimatter has a different spacetime signature. Abusing Minkowski's notation, we may say the spacetime signature of an electron would be ++++ while that of a positron would be +---. ${ }^{21}$

Of course, we readily admit this cannot be the complete answer: the difference between a positive and a negative Zitterbewegung charge cannot be reduced to its direction of motion only! Why not? The answer is obvious: try to bring two electrons together, and then try to bring an electron and a positron together, and you will see two very different things happening. (). Hence, we obviously do have an unexplained intrinsic property here. Indeed, we need the idea of positive and negative charge here! Why? Because the magnetic moment alone does not allow one to distinguish between an electron with spin up and a positron with spin down.

Is this good? We think it is. Is it good enough? We are not sure. Intrinsic properties need an explanation, don't they? Yes, most probably. ${ }^{22}$ However, we are where we are right now and so we cannot add much to this discussion for the time being. Having said that, the reader will have to admit we were able to go way beyond Dirac's rather native hole theory here: in the electron with spin up, we have the same current as in the positron with spin down, but it is because we have an opposite charge (negative instead of positive) spinning in the opposite direction (up instead of down, or right versus left-whatever you want to call it). Hence, negative energy states have to do with the direction of the

[^6]force. ${ }^{23}$ Antimatter seems to have a different metric signature than matter.
That is, for the time being, all we can say about it. ${ }^{24}$ Let us get into the meat of the matter now-almost literally speaking! ()

## The Emperor wears no clothes: matter does not dissipate

We already mentioned how science progresses: it does so by probing reality physically as well as logically. Physical exploration involves experiments, measurements and discoveries. Logical progress occurs when a new theory or explanation replaces previously held beliefs and convictions.

We said such theory or explanation should be logically and mathematically consistent. We should add another requirement: it should also be applicable to a great variety of facts. Newton's force law ${ }^{25}$ and theory of gravitation, Maxwell's equations ${ }^{26}$, and Einstein's theory of relativity ${ }^{27}$ are obvious examples of such theories or explanations. However, we do not count Heisenberg's quantum mechanics among such great theories or explanations.
${ }^{2323}$ If you find this statement confusing, try to think of it like this: the current that generates the magnetic moment in an electron versus a proton must be different: one carries a negative charge, and the other carries a positive charge. The direction of the physical current (the motion of the zbw charge, positive or negative) must, therefore, be opposite. The question then becomes: what distinguishes the positive and a negative zbw charge inside the zbw electron and positron? For the latter question, we have no answer: that is why we think of the property of the charge being positive or negative as being intrinsic. In other words, we have an explanation for the direction of motion and, therefore, for its fundamental nature. However, we have no explanation for the nature of the charge itself.
${ }^{24}$ We offer some more very speculative thoughts on the nature of antimatter in our paper on The Ring Current Model for Antimatter and Other Questions, but we do not want to mention these here because we feel these ideas do not make all that much sense.
${ }^{25}$ Newton's force law is also referred to as the law of motion. The idea of a force and the idea of motion are inextricably linked: a force is that what changes the state of motion of an object, and mass is a measure of inertia or resistance to such change. A force changes the state of motion by acting on a charge. [Of course, you may think that, in many cases, a force does not necessarily change the state of motion of an object: it may also keep something together. As Feynman notes, "The Empire State Building swings less than one inch in the wind because the electrical forces hold every electron and proton more or less in its proper place." However, even this should be thought of as motion: Dirac, at least, referred to Schrödinger's equation as an equation of motion for the electrons.]

Historically, the idea of a force acting on a (positive or negative) charge came later: it is usually associated with Benjamin Franklin's experiments around 1750 and - more systematically, perhaps - the work of Charles-Augustin de Coulomb, who published three memoirs on electromagnetism in 1785. Newton's Philosophiæ Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy) were first published in 1687: scientific revolutions take time. After a hundred years or so, it seems the quantum-mechanical one hasn't ended yet.
${ }^{26}$ Maxwell's equations show the electric and magnetic forces are one electromagnetic force only.
${ }^{27}$ This is a reference to Einstein's special relativity theory, which may be said to correct Newton's law of motion so as to ensure consistency with Maxwell's equations, which show the velocity of light is absolute: it is always equal to $c$ - in the inertial as well as in the moving frame of reference. Einstein's general relativity theory explains gravity in terms of the geometry of spacetime. We do not consider gravity in this paper and, as such, we do not want to highlight this theory for the time being.

Why not? First, we think of it as an approach more than like a theory-some kind of problem-solving algorithm, if you want. Second, even as an approach or an algorithm, it would seem to be based on the wrong assumptions. Why? We will not ask you to think about some thought experiment here. No. We ask you to just think for yourself about the following common-sense remarks.

Prof. H. Pleijel, then Chairman of the Nobel Committee for Physics of the Royal Swedish Academy of Sciences, dutifully notes the following on the nature of the new 'matter waves' in the ceremonial speech for the 1933 Nobel Prize, which was awarded to Heisenberg for nothing less than "the creation of quantum mechanics" ${ }^{28}$ :
"Matter is formed or represented by a great number of this kind of waves which have somewhat different velocities of propagation and such phase that they combine at the point in question. Such a system of waves forms a crest which propagates itself with quite a different velocity from that of its component waves, this velocity being the so-called group velocity. Such a wave crest represents a material point which is thus either formed by it or connected with it, and is called a wave packet. [...] As a result of this theory on is forced to the conclusion to conceive of matter as not being durable, or that it can have definite extension in space. The waves, which form the matter, travel, in fact, with different velocity and must, therefore, sooner or later separate. Matter changes form and extent in space. The picture which has been created, of matter being composed of unchangeable particles, must be modified."

This should sound very familiar to you. If so, you should ask yourself this question: why is this everything but true? Real-life particles - electrons or atoms traveling in space - don't dissipate. Matter does not change form and extent in space! ${ }^{29}$

We trust the academic physicist will switch off and stop reading here—if he or she hasn't already. That's fine. We're not writing this paper for the academic physicist. We write it for you, so you should think about it. Show us one experiment in which matter obviously does change form and extent in space. Matter may disintegrate, of course, but that's something entirely different, and it only applies to composite non-stable particles which - for that reason - shouldn't be referred to as particles: call them resonances or transients instead!

Indeed, we refer to particles as particles because of their stability and integrity and that integrity is defined by the Planck-Einstein relation: $\mathrm{E}=h \cdot f .{ }^{30}$ Nothing more. Nothing less. It's all we need as an

[^7]explanation. A rather smart young man recently wrote this rather scathing criticism of his own profession-physics:
"QED should be the quantized version of Maxwell's laws, but it is not that at all. QED is a simple addition to quantum mechanics that attempts to justify two experimental discrepancies in the Dirac equation: the Lamb shift and the anomalous magnetic moment of the electron. ${ }^{31}$ The reality is that QED is a bunch of fudge factors, numerology, ignored infinities, hocus-pocus, manipulated calculations, illegitimate mathematics, incomprehensible theories, hidden data, biased experiments, miscalculations, suspicious coincidences, lies, arbitrary substitutions of infinite values and budgets of 600 million dollars to continue the game. Maybe it is time to consider alternative proposals. Winter is coming."32

Prof. Dr. John P. Ralston practices the same self-criticism in a recent publication: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." ${ }^{33}$

You think such rebel speech is unconvincing-sacrilegious, perhaps? Let us quote a proper Nobel Prize winner then. ${ }^{34}$ Willis E. Lamb got a Nobel Prize in 1955 for, yes, the discovery of the Lamb shift in the spectrum of the hydrogen atom. ${ }^{35}$ When he was well over 80 , he wrote a rather remarkable paper on

[^8]the (non)sense of the theoretical explanations of the phenomenon he had experimentally discovered. ${ }^{36}$ The title of the paper is, significantly, Anti-Photon, and, in this paper, he politely blames Bohr rather than Heisenberg for all of these weird theories we are still struggling to understand:
"The Complementarity Principle and the notion of wave-particle duality were introduced by N. Bohr in 1927. They reflect the fact that he mostly dealt with theoretical and philosophical concepts, and left the detailed work to postdoctoral assistants. It is very likely that Bohr never, by himself, made a significant quantum-mechanical calculation after the formulation of quantum mechanics in 1925-1926." ${ }^{37}$

In fact, it is hard to see what Heisenberg's matrix mechanics actually explain. Pretty much nothing, perhaps. Indeed, we may want to think of it as a calculation rather than as a theory. We may illustrate this by referring to Heisenberg's pioneering of the scattering matrix, which is now more generally referred to as the $S$-matrix in quantum mechanics. ${ }^{38}$ Scattering is actually a somewhat misleading term because it may refer to any interaction process. As usual, we can rely on Richard Feynman for a more vivid description of what an $S$-matrix actually is:
"The high-class theoretical physicist working in high-energy physics considers problems of the following general nature (because it's the way experiments are usually done). He starts with a couple of particles, like a proton and a proton, coming together from infinity. (In the lab, usually one particle is standing still, and the other comes from an accelerator that is practically at infinity on atomic level.) The things go crash and out come, say, two K-mesons, six r-mesons, and two neutrons in certain directions with certain momenta. What's the amplitude for this to happen? The mathematics looks like this: The $\varphi$-state specifies the spins and momenta of the incoming particles. The $\chi$ would be the question about what comes out. For instance, with what amplitude do you get the six mesons going in such-and-such directions, and the two neutrons going off in these directions, with their spins so-and-so. In other words, $\chi$ would be specified by giving all the momenta, and spins, and so on of the final products. Then the job of the theorist is to calculate the amplitude $\langle\chi| S|\varphi\rangle$."39

The reader may or may not recognize the latter admonishment. The job of the theorist - or of the student in quantum mechanics - is to calculate rather than to think about what might or might not be

[^9]${ }^{39}$ See: How states change with time, Feynman's Lectures, Vol. III, Chapter 8, section 4.

## happening. ${ }^{40}$

We will come back to this, and in much boring detail, I am afraid. Before we get into the nitty-gritty and because many readers may not want to venture that far - we will first offer some preliminary remarks on the nature of the Uncertainty Principle.

## The (Un)Certainty Principle

What is uncertainty in quantum mechanics? How does it relate to the philosophical concepts of freedom and determinism? When discussing the philosophical implications of the Uncertainty Principle in quantum mechanics, Richard Feynman notes that, "from a practical point of view", we have "indeterminability in classical mechanics as well." ${ }^{41}$ He gives the example of water splashing over a dam:
"If water falls over a dam, it splashes. If we stand nearby, every now and then a drop will land on our nose. This appears to be completely random, yet such a behavior would be predicted by purely classical laws. The exact position of all the drops depends upon the precise wigglings of the water before it goes over the dam. How? The tiniest irregularities are magnified in falling, so that we get complete randomness. Obviously, we cannot really predict the position of the drops unless we know the motion of the water absolutely exactly."

While it is, effectively, not practically possible to know the motion of the water "absolutely exactly", the deeper question is: is it theoretically possible? In classical mechanics, we assume it is. In quantum mechanics, we are supposed to assume it is not. Quoting Feynman once again, we may say that "the Uncertainty Principle protects quantum mechanics." Why should this be so?

There is no reason whatsoever to assume the uncertainty in classical mechanics is any different from the uncertainty in quantum mechanics. The uncertainty in quantum mechanics is not anything more than the Ungenauigkeit-imprecision, which is the term Heisenberg first used to describe the apparent randomness in our mathematical description of Nature - we encounter in classical mechanics as well. It is just an imprecision, indeed-as opposed to the weird metaphysical quality which Heisenberg would

[^10]${ }^{41}$ See: Feynman's Lectures, III-2-6.
later claim it to be and which, without any precise definition, physicists now refer to as 'uncertainty'. The concept of 'uncertainty' effectively mixes two different connotations:
(1) an imprecision in our measurements that is due to the wave-particle nature of electrons and other elementary particles - a duality which the ring current model ${ }^{42}$, combining the idea of charge and its motion, captures from the outset (as opposed to trying to cope with the duality in some wave equation or other dual mathematical descriptions); and
(2) a rapid magnification of what Feynman refers to as 'the tiniest irregularities' but which are, in practice, purely statistical.

This is, effectively, reflected in what Feynman refers to as the 'great equation of quantum mechanics'-the wave equation for an electron in free space:

$$
\frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}}+\frac{\partial^{2} \phi}{\partial z^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} \phi}{\partial t^{2}}=\frac{\mathrm{m}^{2} c^{2}}{h^{2}} \phi
$$

The problem is: it is not a wave equation for an electron. This wavefunction does not model an electron: it models our rapidly increasing uncertainty about where it might be at a later point in time.

Why does it increase so rapidly? It's just scale: Planck's quantum of action can (also) be written as the product of momentum over some distance ( $h=\mathrm{p} \cdot \lambda)^{43}$ and, in light of the very tiny distance scale ( $\lambda$ ), the momentum ( $p$ ) is, effectively, relatively large - which is why any imprecision in its magnitude or direction quickly translates into huge uncertainties concerning the actual position of the elementary particle that we are trying to describe.

We will revisit all of this in very much detail later. In order to do so, we must talk some more about what quantum mechanics is and isn't. Before we do so, we briefly want to think a rather weird and totally non-scientific question: what made Heisenberg change his thoughts about the Ungenauigkeit in practical experiments? Why did a simple statement about the unavoidable interaction between the measurement apparatus and that what is being measured turn into a sweeping metaphysical statement?

We are not sure. We just have this rather uncanny feeling all of these thought experiments are not about the Uncertainty Principle "protecting" quantum physics (we are quoting Feynman here): we feel it's about physicists protecting the Uncertainty Principle instead! Of course, the question then becomes: why would they do this? They are searching for the truth as well, don't they? They sure are.

We have no answer here, except this: geniuses like Heisenberg were - ultimately - religious too, and so they might have felt quantum-mechanical indeterminism (as opposed to the statistical indeterminism in classical mechanics) was the very last place where God - or the Mystery, or whatever we cannot prove but happen to believe in - can hide. ${ }^{44}$

[^11]Let us turn back to a more scientific and rational discourse. What are those amplitudes, really?

## The meaning of probability amplitudes and state vectors

The $\langle\chi| S|\varphi\rangle$ amplitude - which, according to Feynman, is all that matters in quantum physics - is a probability amplitude associated with the in-state $\varphi$ and the out-state $\chi$. These states are referred to as state vectors ${ }^{45}$ and they are described in terms of base states which are written as $\langle i|$ and $|j\rangle$ for an out- and an in-state respectively. Hence, the $S$-matrix is actually written as $\langle i| S|j\rangle$ and, assuming we know what it is, we will be able to resolve and calculate the $\langle\chi| S|\varphi\rangle$ amplitude as $\Sigma_{i j}\langle\chi \mid i\rangle\langle i| S|j\rangle\langle$ $j|\varphi\rangle$.

This may all look a bit mind-boggling and rightly so, because it actually is mind-boggling. Just google, for example the $S$-matrix for Compton scattering ${ }^{46}$ and let me know how inspiring you think these so-called explanation of a physical process actually are. For me, they are not an explanation at all. I think it is way out of whack. It is like economists modeling input-output relations: we get a mathematical description of what goes in and what goes out but it doesn't tell us how stuff actually happens. ${ }^{47}$ Furthermore, when the process involves the creation or annihilation of elementary or non-elementary particles (think of short-lived mesons here), even the math becomes ugly. Paraphrasing Dirac, one is then tempted to think that it may be "more important to have beauty in one's equations than to have them fit experiment." 48

The question, of course, runs deeper than that: we can and should not doubt that the likes of Bohr, Pauli, Heisenberg, Schrödinger, Dirac did what we are all doing, and that is to, somehow, try to make sense of it all. Hence, I am sure that even Heisenberg did not initially think of his interpretation of these relations as some kind of surrender of reason-which is, effectively, what we think it actually amounts to: stating that it is a law of Nature that even experts cannot possibly understand Nature "the way they would like to", as Richard Feynman put $\mathrm{it}^{49}$, relegates science to the realm of religious belief.

To be sure, the accounting rules and the properties of the $S$-matrix (as well as those of operator and Hamiltonian matrices) do incorporate some basic physical principles. One of them is the principle of reversibility, which is related to CPT-symmetry. We will say more about this later. As for now, the reader should note that the property of reversibility in Nature is logical but somewhat less straightforward than

[^12]it appears to be at first. If we reverse all signs - the direction of time, the directions in space ${ }^{50}$, and the sign of the charges - then, yes, Nature seems to respect the principle of reversibility. ${ }^{51}$ However, we may see what physicists gravely refer to as symmetry-breaking if we only reverse signs of, say, the directions in space and the sign of the charge: some processes violate what is referred to as CP-symmetry, which involves swapping spatial directions and charges only.

We are not worried about that: if anything, it shows physical time has one direction only. However, because we wrote at length about these things elsewhere, we do not want to bore the reader with unnecessary philosophical reflections here. ${ }^{52}$ We do want to highlight a few points here though, so as to give the reader a bit of an impression of what quantum-mechanical descriptions of physical processes by talking about amplitudes and states may actually represent.

## The meaning of spin in quantum mechanics

We must now turn back to what we referred to as a deep conceptual flaw in the mainstream interpretation of quantum mechanics: the mainstream interpretation does not integrate the concept of particle spin from the outset because it thinks of the + or - sign in front of the imaginary unit (i) in the elementary wavefunction ( $a \cdot e^{-i \cdot \theta}$ or $a \cdot e^{+i \cdot \theta}$ ) as a mathematical convention only. Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i \cdot \theta}=a \cdot e^{-i \cdot(\omega t-k x)}$ and $a \cdot e^{+i \cdot \theta}=a \cdot e^{+i \cdot(\omega t-k x)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). Hence, one would, effectively, expect that the professors - mindful of Occam's Razor for a good theory - would then proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different but, no! That is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention. ${ }^{53}$

We are really dumbfounded by this. ${ }^{54}$ All of the great physicists knew all particles - elementary or not have spin. ${ }^{55}$ All of them also knew it is spin that generates the magnetic moment and that we should,
${ }^{50}$ We are talking parity-symmetry here: think of the mirror image of things, or of turning space inside out.
${ }^{51}$ As mentioned, we will make a critical note here later.
${ }^{52}$ See, for example, our paper on the physical meaning of the quantum-mechanical wavefunction (Euler's wavefunction: the double life of -1 ) and our blog post(s) on CPT-symmetry.
${ }^{53}$ In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1x). We quote this example for the same reason as why we use Feynman's Lectures as a standard reference: it's authoritative and, more importantly, also available online so the reader can check and explore for himself.
${ }^{54}$ The reader should not think we did not ask the question in the mentioned edX course on quantum mechanics. We did. We asked many other questions as well. However, we were effectively kindly instructed to "shut up and calculate" so as to prepare ourselves for the examination. We will be honest and admit we did not do the exam. We don't mind calculations. We do mind mindless calculations, however.
${ }^{55}$ Even photons - despite their spin-one property - cannot not come in a zero-spin state. When studying Feynman's Lectures, I found that to be one of the weirdest things: Feynman first spends several chapters on explaining spin-one particles to, then, in some obscure footnote suddenly write this: "The photon is a spin-one particle which has, however, no "zero" state." The issue is related to another glaring inconsistency: in the first three chapters of his Lectures on physics, he talks about adding wavefunctions and the basic rules of quantum mechanics, and it all happens with a plus sign. However, in his chapter on the theoretical distinction between bosons and fermions, he suddenly says we should be adding the amplitudes of fermions combine with a minus
therefore, consider the idea of thinking of elementary particles as a ring current. Indeed, the ring current model is not applicable to electrons (and positrons only). Indeed, Dirac starts his derivation of his famous wave equation for free electrons in his 1933 Nobel Prize speech with the following remark ${ }^{56}$ :
"This procedure is successful in the case of electrons and positrons. It is to be hoped that in the future some such procedure will be found for the case of the other particles. I should like here to outline the method for electrons and positrons, showing how one can deduce the spin properties of the electron, and then how one can infer the existence of positrons with similar spin properties."

This should suffice to remove any doubt in regard to the importance of the concept of spin in quantum mechanics: it is the lynchpin of everything. As mentioned, the introduction above serves as an introduction to Dirac's derivation of his wave equation for an electron in free space ${ }^{57}$ - which we will not write down but which involves four operators, denoted as $\alpha_{r} p_{r}(r=1,2,3)$ and $\alpha_{0} m c$ respectively ${ }^{58}$ - and says the following about it:
"The new variables $\alpha_{r}$ which we have to introduce to get a relativistic wave equation linear in $W^{59}$, give rise to the spin of the electron. From the general principles of quantum mechanics one can easily deduce that these variables a give the electron a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton in the reverse direction to the angular momentum. These results are in agreement with experiment. They were, in fact, first obtained from the experimental evidence provided-by spectroscopy and afterwards confirmed by the theory."

He then continues by leaving us an excellent historical summary of the ring current or Zitterbewegung model of the electron ${ }^{60}$ :
"The variables $\alpha$ also give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is
sign. In any case, we cracked our own jokes on the boson-fermion theory and so we should leave it at that.
${ }^{56}$ Needless to say, all italics in the quote are ours.
${ }^{57}$ The concept of free space should not confuse the reader: it makes abstraction of any external fields or forces, such as the force between a positively charged nucleus and the electron in some orbital.
${ }^{58}$ Dirac's wave equation can be written in various equivalent ways. We refer to Dirac's Principles of Quantum Mechanics or, for the reader who can follow Dirac's very succinct summary of it, the above-mentioned Nobel Prize speech.
${ }^{59}$ As mentioned, we don't want to get into the detail of (the derivation of) Dirac's equation but the reader should note Dirac only considers the kinetic energy of the electron which is, therefore, denoted as $W$ rather than as $\mathrm{E}=$ $m c^{2}$. In our ring current model of an electron,
${ }^{60}$ The term Zitterbewegung was, effectively, coined by Erwin Schrödinger in a 1930 paper which analyzed the solutions to Dirac's equation (Dirac had derived his equation in 1928 but the whole theoretical framework that accompanied it - his Principles to Quantum Mechanics - was also published in 1930 as well.
a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

We wrote about this elsewhere ${ }^{61}$ and so we will not dwell on it much longer. The question here is: why did such great intellects fail to fully exploit the power of Euler's ubiquitous $\psi=a \cdot e^{i \theta}$ function? They knew about Occam's Razor as a criterion for a good scientific theory too, didn't they? We are not sure. Perhaps Schrödinger and Dirac were too obsessed by their wave equation - as opposed to the wavefunction that is its solution. Whatever the reason might have been, the fact is this: they did not integrate spin in their mathematical description-not from the outset, at least. The mistake is illustrated below.


Figure 4: The meaning of $+i$ and $-i$
It is a very subtle but fundamental blunder. Spin-zero particles do not exist. ${ }^{62}$ All real particles electrons, photons, anything - have spin, and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the magnitude of the spin that differs. It is, therefore, completely odd that the plus ( + ) or the minus ( - ) sign of the imaginary unit (i) in the $a \cdot e^{ \pm i \theta}$ function is not being used to include the spin direction in the mathematical description.

In fact, we call it a blunder because it is more than just odd: it's plain wrong because this non-used degree of freedom in the mathematical description leads to the false argument that the wavefunction of spin- $1 / 2$ particles have a 720 -degree symmetry. Indeed, physicists treat -1 as a common phase factor in the argument of the wavefunction: they think we can just multiply a set of amplitudes - let's say two amplitudes, to focus our mind (think of a beam splitter or alternative paths here) - with -1 and get the same states. We find it rather obvious that that is not necessarily the case: -1 is not necessarily a common phase factor. We should think of -1 as a complex number itself: the phase factor may be $+\pi$ or, alternatively, $-\pi$. To put it simply, when going from +1 to -1 , it matters how you get there - and vice versa - as illustrated below. ${ }^{63}$
${ }^{61}$ See our Explanation of the Electron and Its Wavefunction.
${ }^{62}$ We are obviously not talking about the zero-spin state of, say, a spin-one atom here. We talk about elementary particles only here.
${ }^{63}$ The quantum-mechanical argument is technical, and I did not reproduce it in this book. I encourage the reader to glance through it, though. See: Euler's Wavefunction: The Double Life of -1 . Note that the $e^{+i \pi} \neq e^{-i \pi}$ expression may look like horror to a mathematician! However, if he or she has a bit of a sense for geometry and the difference between identity and equivalence relations, there should be no surprise. If you're an amateur physicist, you should


Figure 5: $e^{+i \pi} \neq e^{-i \pi}$
The reader may be very tired by now ${ }^{64}$, so let us ask the primordial question for him or her: what's the point here? It is this: if we exploit the full descriptive power of Euler's function, then all weird symmetries disappear - and we just talk standard 360-degree symmetries in space. Also, weird mathematical conditions - such as the Hermiticity of quantum-mechanical operators - can easily be explained as embodying some common-sense physical law: energy and momentum conservation, for example, or physical reversibility: we need to be able to play the movie backwards.

Here we must note something funny with charges, however. Combining the + and - sign for the imaginary unit with the direction of travel, we get four possible structures for the wavefunction for an electron:

| Spin and direction of travel | Spin up $(J=+\hbar / \mathbf{2})$ | Spin down $(J=-\hbar / \mathbf{2})$ |
| :--- | :---: | :---: |
| Positive $\boldsymbol{x}$-direction | $\psi=\exp [i(\mathrm{kx}-\omega \mathrm{t})]$ | $\psi^{*}=\exp [-i(\mathrm{kx}-\omega \mathrm{t})]=\exp [i(\omega \mathrm{t}-\mathrm{kx})]$ |
| Negative $\boldsymbol{x}$-direction | $\chi=\exp [-i(\mathrm{kx}+\omega \mathrm{t})]=\exp [i(\omega \mathrm{t}-\mathrm{kx})]$ | $\chi^{*}=\exp [i(\mathrm{kx}+\omega \mathrm{t})]$ |

Table 2: Occam's Razor: mathematical possibilities versus physical realities (2)
However, this triggers the obvious question: how do we know it's an electron or a negative charge, as opposed to a positron, or a positive charge? Indeed, consider a particular direction of the elementary current generating the magnetic moment. It is then very easy to see that the magnetic moment of an electron ( $\mu=-q_{\mathrm{e}} \hbar / 2 \mathrm{~m}$ ) and that of a positron ( $\mu=+\mathrm{q}_{\mathrm{e}} \hbar / 2 \mathrm{~m}$ ) would be opposite. We may, therefore, associate a particular direction of rotation with an angular frequency vector $\omega$ which - depending on the direction of the current - will be up or down with regard to the plane of rotation. We associate this with the spin property, which is also up or down. We, therefore, have another set of four possibilities reflecting one another:

[^13]| Matter-antimatter | Spin up $(J=+\hbar / \mathbf{2})$ | Spin down $(J=-\hbar / \mathbf{2})$ |
| :--- | :---: | :---: |
| Electron | $\mu_{e_{-}}=-q_{e} \hbar / 2 m$ | $\mu_{e_{-}}=+q_{e} \hbar / 2 m$ |
| Positron | $\mu_{e^{+}}=+q_{e} \hbar / 2 m$ | $\mu_{e^{+}}=-q_{e} \hbar / 2 m$ |

Table 3: Electron versus positron spin
So how do we distinguish them? It is a deep philosophical question which we cannot fully answer for the time being. We can only offer a few thoughts here:

1. The idea of time reversal is a wonderful invention of our mind but we think it does not correspond to anything real. When charges are involved, you will see two like charges repel each other and, therefore, you will see them move away from each other. However, when playing the movie backward, you will know it the movie cannot be real because the same two charges now move towards each other. The funny thing is that a reversal of the sign of the charges doesn't help to fix the situation because, say, reversing the charge of two electrons now gives us two positrons moving towards each other, which doesn't make sense physically either!

The reader may think we did, perhaps, not think of P-symmetry: all left-handed things are now righthanded and vice versa, right? Yes, but that doesn't solve the problem here.
2. So what about CPT-symmetry then? Frankly, we think the concept of CPT-symmetry is not all that useful and we, therefore, wonder why physicists are so enthralled about it. We think the concept of motion itself implies that time can have one direction only. If it wouldn't, then we would not be able to describe trajectories in spacetime by a well-behaved function. In other words, the concept of motion itself would become meaningless.

The diagrams below illustrate the point. The spacetime trajectory in the diagram on the right is not kosher, because our object travels back in time in not less than three sections of the graph. Spacetime trajectories - or, to put it more simply, motions - need to be described by well-defined functions: for every value of $t$, we should have one, and only one, value of $x$. The reverse, of course, is not true: a particle can travel back to where it was (or, if there is no motion, just stay where it is). Hence, it is easy to see that the concepts of motion and time are related to us using well-behaved functions to describe reality. ${ }^{65}$


Figure 6: A well- and a not-well behaved trajectory in spacetime ${ }^{66}$

[^14]We must now come back to the matter at hand, which is the meaning of conjugates in quantum physics.

## The meaning of conjugating amplitudes, wavefunctions and matrices

The ubiquitous concept of a state in quantum mechanics is very abstract and general and, therefore, adds to the mystery of the quantum-mechanical description of things. We may start by noting that the concept of an initial and a final state - or an in- and an out-state - are very intimately linked: they correspond to the bra and the ket in Dirac's bra-ket notation which - we should not remind the reader must be read from right to left. As such, we like to think the concepts of the initial and final state simply separate the present from the (possible) the future.

In practice, we will use matrix mechanics when states are discrete, while the wavefunction approach allows us to model how a particle may evolve which, in practice, usually means what and where we expect its momentum and position to be. One of Feynman's very first examples of a state is the spin state of spin-one atoms ${ }^{67}$, which may be $+1,0$ or -1 . These are referred to as base states. Base states need to be independent, which is captured by the usual Kronecker delta rule:

$$
\langle i \mid j\rangle=\delta_{\mathrm{ij}}
$$

This basically says a base state is a base state: $\langle i \mid j\rangle=1$ if, and only if, $i=j$. Otherwise $\langle i \mid j\rangle=0$. It obviously also means a particle can be in one state only at any point in time. The evolution from one state to another involves a process or - another oft-used terms in introductory courses - an apparatus, such as a filter, which may also be referred to as polarizer. ${ }^{68}$ Now, the amplitude for a particle to go from state $\varphi$ to state $\chi$ will be $\langle\chi \mid \varphi\rangle$. An amplitude is, of course, a probability amplitude here: it is a complex number or a complex-valued wavefunction. There is a most remarkable law in quantum mechanics, and it is this: the amplitude to go from the out-state $\chi$ to the in-state $\varphi$ is the (complex) conjugate of the amplitude to go from the in-state $\varphi$ to the out-state $\boldsymbol{\chi}$ - and vice versa, of course! It is more easily written like this:

$$
\langle\varphi \mid \chi\rangle=\langle\chi \mid \varphi\rangle^{*} \text { and }\langle\chi \mid \varphi\rangle=\langle\varphi \mid \chi\rangle^{*}
$$

Feynman proves this rule for a three-state system and a simple filter apparatus ${ }^{69}$ but we find it easier to think of it as a simple consequence of this theoretical possibility of reversibility: a sign reversal of the imaginary unit $i$. Instead of $+i$ (or, conversely, $-i$ ), we now use $-i$ (or, conversely, $+i$ ) in our wavefunction.

It is probably easiest to show what this means in terms of actual probabilities - some easily understandable number between 0 and 1 , that is - rather than in terms of probability amplitudes, which

[^15]are complex-valued numbers evolving in space and in time. Indeed, we get the actual probability from taking the absolute square ${ }^{70}$ of the complex-valued amplitude.

In our realist interpretation, we understand this quantum-mechanical rule in terms of mass or energy densities: we think of the oscillating or Zitterbewegung charge as passing more time here than there and, hence, the associated energy density will be higher here than there. ${ }^{71}$ However, that interpretation does not matter here. We just want to show you that we can use this quantum-mechanical rule for calculating probabilities to derive the above-mentioned quantum-mechanical reversibility law. Indeed, the absolute square of a complex number is the product of the same number with its complex conjugate. We, therefore, get this ${ }^{72}$ :

$$
|\langle\varphi \mid \chi\rangle|^{2}=\langle\varphi \mid \chi\rangle\langle\varphi \mid \chi\rangle^{*}=\langle\chi \mid \varphi\rangle^{*}\langle\chi \mid \varphi\rangle=\langle\chi \mid \varphi\rangle^{*}\langle\chi \mid \varphi\rangle=|\langle\chi \mid \varphi\rangle|^{2}
$$

At this point, the reader may wonder what we want to say here. We are just saying that the quantummechanical $\langle\varphi \mid \chi\rangle=\langle\chi \mid \varphi\rangle^{*}$ rule only states physical processes should be reversible in space and in time, and that it amounts to stating that the probability of going from the out-state $\chi$ to the in-state $\varphi$ is the same as going from the in-state $\varphi$ to the out-state $\chi$.

The question is: does this rule or law make sense? In other words, is it true, always? In mechanics think of elastic collisions of particles or particles moving in force fields - it should be the case. However, when disintegration processes are involved, one should not expect this law to apply. Why not? Disintegration involves unstable systems moving to some kind of stable state. Energy, angular momentum, and linear momentum will be conserved, but that's about all we can say about it. Inventing new quantities to be conserved - such as strangeness ${ }^{73}$ - or invoking other consequences of so-called symmetry-breaking processes does not add much explanatory power in our view.

## [...]

Let us wrap up here - this section of this rather long and winding paper, at least! Let's summarize the basics of our discussion above: if we can go from state $\varphi$ to state $\chi$, then we must be able to go back to state $\varphi$ from state $\chi$ in space and in time. This amounts to playing a movie backwards: an exploding suitcase bomb will, in practice, not suddenly un-explode and get back into the suitcase but, theoretically

[^16](read: making abstraction of arguments involving entropy and other macroscopic considerations), such reversed processes may be imagined. ${ }^{74}$ However, instead of imaging an exploding or un-exploding bomb, let us stick to the more abstract business of states and vectors and so we should try to think of an apparatus or a process that will be operating on some state $|\psi\rangle$ to produce some other state $|\varphi\rangle$. The Great Dirac taught us to write that like this:
$$
\langle\varphi| A|\psi\rangle
$$

We can now take the complex conjugate:

$$
\langle\varphi| A|\psi\rangle^{*}=\langle\psi| A^{\dagger}|\varphi\rangle
$$

$A^{\dagger}$ is, of course, the conjugate transpose of $A$ - we write: $A t_{i j}=\left(A_{j i}\right)^{*}$ - and we will call the operator (and the matrix) Hermitian if the conjugate transpose of this operator (or the matrix) gives us the same operator matrix, so that is if $A^{\dagger}=A$. Many quantum-mechanical operators are Hermitian, and we will also often impose that condition on the S-matrix. Why? Because you should think of an operator or an Smatrix as a symmetric apparatus or a reversible process. It's as simple as that. Hence, the Hermiticity condition amounts to a simple reversibility condition too!

Needless to say, we may be mistaken, of course! We, therefore, invite the professional reader to challenge this interpretation! The professional reader may also note we forgot to specify the second of the three basic rules of quantum math, and that's the resolution of amplitudes into base states-which is written like this:

$$
\langle\chi \mid \varphi\rangle=\Sigma_{\text {all } i}\langle\chi \mid i\rangle\langle i \mid \varphi\rangle
$$

This, too, can be shown to be true based on a thought experiment ${ }^{75}$ or, else, one can use a much more abstract argument (as Dirac does as part of his formalization of the theoretical framework for working with amplitudes) and simply write:

$$
\left.\left|=\Sigma_{\text {all } i}\right| i\right\rangle\langle i|
$$

This, then allows us to do what we did, and that's to write the $\langle\chi| S|\varphi\rangle$ amplitude as $\Sigma_{i j}\langle\chi \mid i\rangle\langle i| S \mid$ $j\rangle\langle j \mid \varphi\rangle$.

Any case, this is all very abstract and, therefore, very tiring. Furthermore, it does not amount to what we think of as any real explanation for real-life physical processes: it merely describes them. As mentioned, this is more like economists modeling input-output relations: we get a mathematical description of what goes in and what goes out but it doesn't tell us how stuff actually happens. So how would or might a real explanation then look like? Let us illustrate this using the example of Compton scattering.

[^17]
## A classical explanation of Compton scattering

## Energy and momentum conservation

Compton scattering is referred to as inelastic because the frequency of the incoming and outgoing photon are different. The situation is illustrated below.


The reader will be familiar with the formulas and, most probably, also with how these formulas can be derived from two very classical laws only: (1) energy conservation and (2) momentum conservation. Let us quickly do this to remind the reader of the elegance of classical reasoning:

1. The energy conservation law tells us that the total (relativistic) energy of the electron ( $E=m_{e} c^{2}$ ) and the incoming photon must be equal to the total energy of the outgoing photon and the electron, which is now moving and, hence, includes the kinetic energy from its (linear) motion. We use a prime (') to designate variables measured after the interaction. Hence, $\mathrm{E}_{e^{\prime}}$ and $\mathrm{E}_{\gamma^{\prime}}$ are the energy of the moving electron ( $\mathrm{e}^{\prime}$ ) and the outgoing photon ( $\psi^{\prime}$ ) in the state after the event. We write:

$$
\mathrm{E}_{\mathrm{e}}+\mathrm{E}_{\gamma}=\mathrm{E}_{\mathrm{e}^{\prime}}+\mathrm{E}_{\gamma^{\prime}}
$$

We can now use (i) the mass-energy equivalence relation ( $\mathrm{E}=\mathrm{mc} c^{2}$ ), (ii) the Planck-Einstein relation for a photon ( $E=h \cdot f$ ) and (iii) the relativistically correct relation ( $E^{2}-p^{2} c^{2}=m^{2} c^{4}$ ) between energy and momentum for any particle - charged or non-charged, matter-particles or photons or whatever other distinction one would like to make ${ }^{76}$ - to re-write this as ${ }^{77}$ :

$$
\mathrm{m}_{\mathrm{e}} c^{2}+h f=\sqrt{\mathrm{p}_{\mathrm{e}}^{2} c^{2}+\mathrm{m}_{\mathrm{e}}^{2} c^{4}}+h f^{\prime} \Leftrightarrow \mathrm{p}_{\mathrm{e}^{\prime}}^{2} c^{2}=\left(h f-h f^{\prime}+\mathrm{m}_{\mathrm{e}}^{2} c^{4}\right)^{2}-\mathrm{m}_{\mathrm{e}}^{2} c^{4}(1)
$$

2. This looks rather monstrous but things will fall into place soon enough because we will now derive another equation based on the momentum conservation law. Momentum is a vector, and so we have a vector equation here ${ }^{78}$ :

[^18]$$
\overrightarrow{\mathrm{p}}_{\gamma}=\overrightarrow{\mathrm{p}}_{\gamma^{\prime}}+\overrightarrow{\mathrm{p}}_{\mathrm{e}^{\prime}} \Leftrightarrow \overrightarrow{\mathrm{p}}_{\mathrm{e}^{\prime}}=\overrightarrow{\mathrm{p}}_{\gamma}-\overrightarrow{\mathrm{p}}_{\gamma^{\prime}}
$$

For reasons that will be obvious later - it is just the usual thing: ensuring we can combine two equations into one, as we did with our formulas for the radius - we square this equation and multiply with Einstein's constant $c^{2}$ to get this ${ }^{79}$ :

$$
\begin{gathered}
\overrightarrow{\mathrm{p}}_{\mathrm{e}^{\prime}}^{2}=\overrightarrow{\mathrm{p}}_{\gamma}^{2}+\overrightarrow{\mathrm{p}}_{\gamma^{\prime}}^{2}-2 \overrightarrow{\mathrm{p}}_{\gamma} \overrightarrow{\mathrm{p}}_{\gamma^{\prime}} \Leftrightarrow \mathrm{p}_{\mathrm{e}^{\prime}}^{2} c^{2}=\mathrm{p}_{\gamma}^{2} c^{2}+\mathrm{p}_{\gamma^{\prime}}^{2} c^{2}-2\left(\mathrm{p}_{\gamma} c\right)\left(\mathrm{p}_{\gamma^{\prime}} c\right) \cdot \cos \theta \\
\Leftrightarrow \mathrm{p}_{\mathrm{e}^{\prime}}^{2} c^{2}=h^{2} f^{2}+h^{2} f^{\prime 2}-2(h f)\left(h f^{\prime}\right) \cdot \cos \theta(2)
\end{gathered}
$$

3. We can now combine equations (1) and (2):

$$
\mathrm{p}_{\mathrm{e}^{\prime}}^{2} c^{2}=(E q .1)=(E q .1)=\left(h f-h f^{\prime}+\mathrm{m}_{\mathrm{e}}^{2} c^{4}\right)^{2}-\mathrm{m}_{\mathrm{e}}^{2} c^{4}=h^{2} f^{2}+h^{2} f^{\prime 2}-2(h f)\left(h f^{\prime}\right) \cdot \cos \theta
$$

The reader will be able to do the horrible stuff of actually squaring the expression between the brackets and verifying only cross-products remain. We get:

$$
\left(h f-h f^{\prime}\right) \mathrm{m}_{\mathrm{e}} c^{2}=h\left(f-f^{\prime}\right) \mathrm{m}_{\mathrm{e}} c^{2}=h^{2} f f^{\prime}(1-\cos \theta)
$$

Multiplying both sides of the equation by the $1 / h m_{e} f f$ constant yields the formula we were looking for:

$$
\begin{gathered}
\frac{\left(f-f^{\prime}\right) \mathrm{m}_{\mathrm{e}} c}{f \cdot f^{\prime}}=\frac{f \mathrm{~m}_{\mathrm{e}} c-f^{\prime \mathrm{m}_{\mathrm{e}}} c}{f \cdot f^{\prime}}=\frac{h}{\mathrm{~m}_{\mathrm{e}} c}(1-\cos \theta) \\
\Leftrightarrow \frac{c}{f^{\prime}}-\frac{c}{f}=\lambda^{\prime}-\lambda=\Delta f=\frac{h}{\mathrm{~m}_{\mathrm{e}} c}(1-\cos \theta)
\end{gathered}
$$

The formulas allow us also to calculate the angle in which the electron is going to recoil. It is equal to:

$$
\cot \left(\frac{\theta}{2}\right)=\left(1+\frac{E_{\gamma}}{E_{\mathrm{e}}}\right) \tan \varphi
$$

The $h / \mathrm{mc}$ factor on the left-hand side of the right-hand side of the formula for the difference between the wavelengths is, effectively, a distance: about 2.426 picometer ( $10^{-12} \mathrm{~m}$ ). The $1-\cos \theta$ factor goes from 0 to 2 as $\theta$ goes from 0 to $\pi$. Hence, the maximum difference between the two wavelengths is about 4.85 pm . This corresponds, unsurprisingly, to half the (relativistic) energy of an electron. ${ }^{80}$ Hence, a highly energetic photon could lose up to $255 \mathrm{keV} .{ }^{81}$ That sounds enormous, but Compton scattering is usually done with highly energetic X - or gamma-rays.
mentioned, the linear momentum of the electron before the interaction is zero, because its (linear) velocity is zero: $\mathrm{p}_{\mathrm{e}}=0$.
${ }^{79}$ We do not want to sound disrespectful when referring to $c^{2}$ as Einstein's constant. It has a deep meaning, in fact. Einstein does not have any fundamental constant or unit named after him. Nor does Dirac. We think $c^{2}$ would be an appropriate 'Einstein constant'. Also, in light of Dirac's remarks on the nature of the strong force, we would suggest naming the unit of the strong charge after him. More to the point, note these steps - finally ! incorporated the directional aspect we needed for the analysis. Note that we also use the rather obvious E = pc relation for photons in the transformation of formulas here.
${ }^{80}$ The energy is inversely proportional to the wavelength: $\mathrm{E}=\mathrm{h} \cdot f=\mathrm{hc} / \lambda$.
${ }^{81}$ The electron's rest energy is about 511 keV .

Could we imagine that a photon loses all of its energy to the electron? No. We refer to Prof. Dr. Patrick LeClair's course notes on Compton scattering ${ }^{82}$ for a very interesting and more detailed explanation of what actually happens to energies and frequencies, and what depends on what exactly. He shows that the electron's kinetic energy will always be a fraction of the incident photon's energy, and that fraction may approach but will never actually reach unity. In his words: "This means that there will always be some energy left over for a scattered photon. Put another way, it means that a stationary, free electron cannot absorb a photon! Scattering must occur. Absorption can only occur if the electron is bound to a nucleus."

## What might actually be happening

A photon interacts with an electron, so we actually think of the photon as being briefly absorbed, before the electron emits another photon of lower energy. The energy difference between the incoming and outgoing photon then gets added to the kinetic energy of the electron according to the law we just derived:

$$
\lambda^{\prime}-\lambda=\Delta f=\frac{h}{\mathrm{~m} c}(1-\cos \theta)
$$

Now, we think of the interference as a process during which - temporarily - an unstable wavicle is created. This unstable wavicle does not respect the integrity of Planck's quantum of action ( $\mathrm{E}=\mathrm{h} \cdot \mathrm{f}$ ). The equilibrium situation is then re-established as the electron emits a new photon and moves away. Both the electron and the photon respect the integrity of Planck's quantum of action again and they are, therefore, stable.

The geometry of the whole thing is simple and difficult at the same time. There is, for example, also a formula for the angle of the outgoing photon, which uses the angle for the incoming photon, but that's all stuff which the reader can look up. The question is: how does this happen, exactly? And what determines the plane which is formed by the outgoing photon and the recoiling electron?

None of the standard textbooks will try to answer that question, because they don't think of electrons and photons as having some internal structure which may explain all of the formulas they get out of their arguments, which are based on the conservation of (1) energy and (2) linear and angular momentum. In contrast, we believe our geometrical models may not only show why but also how all of this happens. Let us walk over the basics of that.

## The Compton wavelength as a circumference

According to our ring current model of an electron, the wavefunction $\psi(x, t)=\psi(x, y, z, t)$ describes the actual position of the pointlike Zitterbewegung charge in its oscillatory motion around some center. ${ }^{83}$

[^19]${ }^{83} \mathrm{We}$ are often tempted to use a semicolon to separate the time variable from the space coordinates. Hence, we would prefer to write $\psi(x, t)=\psi(x, y, z ; t)$ instead of $\psi(x, t)=\psi(x, y, z ; t)$. The semicolon then functions as a serial comma. Of course, we are well aware of Minkowski's view on the relativity of space and time: "Space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality." However, we feel space and time are related but also very separate categories of our mind: perhaps some very advanced minds may claim they can effectively understand the Universe in terms of four-vector algebra, but we surely do not. We understand things in terms of motion, and any equation of motion implies the idea of a motion in space and in time - not in some 'kind of union of the two'. What I am saying is that the oft-used

We may therefore paraphrase Prof. Dr. Patrick LeClair and identify the Compton wavelength with a "distance scale within which we can localize the electron in a particle-like sense." ${ }^{84}$ Why are we so sure of that?

We should first discuss the $2 \pi$ factor. Indeed, we have $h$, not $\hbar$, in the equation. Hence, should we think of the Compton wavelength or the Compton radius of an electron? $2 \pi$ is a sizable factor - a factor equal to about 6.28 - so that is large enough to matter when discussing size. ${ }^{85}$ Of course, we know it is the factor which relates the circumference of a circle with its radius but a wavelength is something linear, isn't it? If we should think of the wavelength of an electron, then what should we imagine it to be anyway? We all know that Louis de Broglie associated a wavelength with the classical momentum $\mathrm{p}=$ mv of a particle but, again, how should we imagine this wavelength? The de Broglie relation says $\lambda=h / \mathrm{p}$ $=h / m v$ goes to infinity ( $\infty$ ) for $v$ going to 0 and $m$ going to $m_{0}$ (the rest mass of the electron). Hence, the de Broglie and the Compton wavelength of an electron are very different concepts:

$$
\lambda_{c}=h / \mathrm{mc} \neq \lambda=h / \mathrm{p}=h / \mathrm{mv}
$$

The illustration below (for which credit goes to an Italian group of $z b w$ theorists ${ }^{86}$ ) helps to make an interesting point. Think of the black circle (in the illustration on the left-hand side below) as circumscribing the Zitterbewegung of the pointlike charge. Think of it as the electron at rest: its radius is the radius of the oscillation of the oscillation of the $z b w$ charge $a=\hbar / \mathrm{mc}$. Note that we actually do ask you to make abstraction of the two-dimensional plane of the oscillation of the Zitterbewegung (zbw) charge, which need not be perpendicular to the direction of motion of the electron as a whole: it can be in the same plane or, most likely, it may be zittering around itself. The point is this: we can introduce yet another definition of a wavelength here - the distance between two crests or two troughs of the wave,

[^20]as shown in the illustration on the right-hand side. ${ }^{87}$


Figure 7: The wavelength(s) of an electron
We should now present a rather particular geometric property of the Zitterbewegung (zbw) motion: the $a=\hbar / \mathrm{mc}$ radius - the Compton radius of our electron - must decrease as the (classical) velocity of our electron increases. That is what is visualized in the illustration on the left-hand side (above). What happens here is quite easy to understand. If the tangential velocity remains equal to $c$, and the pointlike charge has to cover some horizontal distance as well, then the circumference of its rotational motion must decrease so it can cover the extra distance. How can we analyze this more precisely? This rather remarkable thing should be consistent with the use of the relativistic mass concept in our formula for the $z b w$ radius $a$, which we write as:

$$
a=\frac{\hbar}{\mathrm{m} c}=\frac{\lambda_{C}}{2 \pi}
$$

The $\lambda_{c}$ is the Compton wavelength, so that's the circumference of the circular motion. ${ }^{88}$ How can it decrease? If the electron moves, it will have some kinetic energy, which we must add to the rest energy. Hence, the mass $m$ in the denominator ( mc ) increases and, because $\hbar$ and $c$ are physical constants, $a$ must decrease. ${ }^{89}$ How does that work with the frequency? The frequency is proportional to the energy ( $\mathrm{E}=\hbar \cdot \omega=h \cdot f=h / \mathrm{T}$ ) so the frequency - in whatever way you want to measure it - must also increase. The cycle time T , therefore, must decrease. What happens, really, is that we are stretching our Archimedes' screw, so to speak. It is quite easy to see that we get the following formula for our new $\boldsymbol{\lambda}$ wavelength:

[^21]$$
\lambda=v \cdot \mathrm{~T}=\frac{v}{f}=v \cdot \frac{h}{\mathrm{E}}=v \cdot \frac{h}{\mathrm{~m} c^{2}}=\frac{v}{c} \cdot \frac{h}{\mathrm{~m} c}=\beta \cdot \lambda_{C}
$$

Can the (classical or linear) velocity go to $c$ ? In theory, yes, but, in practice, no. The $m$ in the formula is not the mass of the zbw charge but the mass of the electron as a whole. That is non-zero for $v=0$, unlike the rest mass of the zbw charge, which only acquires mass because of its motion. We calculated this relativistic mass of the $z b w$ charge as equal to $1 / 2$ of the electron (rest) mass. The point is this: we are not moving a zero-mass thing here. The energy that is, therefore, required to bring $v$ up to $c$ will be infinitely large: think of the enormous energies that are required to speed electrons up to nearlightspeed in accelerators.

The point is this: an electron does not become photon-like when moving at near-lightspeed velocities. However, we do see that the circumference of the circle that circumscribes the two-dimensional zbw oscillation of the zbw charges does seem to transform into some linear wavelength when $v$ goes to $c$ !

Of course, we immediately admit this still does not explain what's going on, exactly. It only shows we should not necessarily think of the Compton wavelength as a purely linear feature.

Another remark that we should make here is that, while we emphasize that we should not think of a photon as a charge travelling at lightspeed - it is not - and I mean not at all: photons do not carry charge ${ }^{90}$ - the analysis above does relate the geometry of our zbw electron to the geometry of our photon model. Let us quickly introduce that now as part of a larger reflection of what may or may not be going on in photon-electron interactions.

## Probing electrons with photons

The relation between what we think of as the radius of the Zitterbewegung oscillation of the electric charge ( $a=\hbar / \mathrm{m}_{\mathrm{e}} \mathrm{c}$ ), the Compton wavelength of an electron ( $\lambda_{c}=h / \mathrm{m}_{\mathrm{e}} \mathrm{c}$ ), and the wavelength of a photon $(\lambda=c / f)$ is not very obvious. However, we should not be discouraged because we immediately note on thing, at least: the wavelength of a photon is the same as its Compton wavelength and its de Broglie wavelength. Furthermore, because $v=c$ and, therefore, $\beta=1$, it is also equal to that third wavelength we introduced above: $\lambda=\beta \lambda_{c}$. In short, the wavelength of a photon is its wavelength, so we write:

$$
\lambda=\frac{c}{f}=\frac{c h}{\mathrm{E}}=\frac{c h}{\mathrm{~m} c^{2}}=\frac{h}{\mathrm{~m} c}=\frac{h}{\mathrm{p}}
$$

But so what is that wavelength, really? Indeed, we should probably start by recognizing this: when probing the size of an electron with photons, we had better have some idea of what a photon actually is. Indeed, almost any textbook will tell you that, because of the wave nature of light, there is a limitation on how close two spots can be and still be seen as two separate spots: that distance is of the order of the wavelength of the light that we are using. ${ }^{91}$ There is a reason for that, of course, and it's got as much to do with the wave as with the particle nature of light. Light comes in lumps too: photons. These

[^22]photons pack energy but they also pack one (natural) unit of physical action (h) or - what amounts to the same - one unit of angular momentum ( $\hbar$ ).

Let us recall the basics of what we have actually presented a few times in previous papers already. ${ }^{92}$ Angular momentum comes in units of $\hbar$. When analyzing the electron orbitals for the simplest of atoms (the one-proton hydrogen atom), this rule amounts to saying the electron orbitals are separated by a amount of physical action that is equal to $h=2 \pi \cdot \hbar$. Hence, when an electron jumps from one level to the next - say from the second to the first - then the atom will lose one unit of $h$. The photon that is emitted or absorbed will have to pack that somehow. It will also have to pack the related energy, which is given by the Rydberg formula:

$$
\mathrm{E}_{n_{2}}-\mathrm{E}_{n_{1}}=-\frac{1}{n_{2}^{2}} \mathrm{E}_{R}+\frac{1}{n_{1}^{2}} \mathrm{E}_{R}=\left(\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right) \cdot \mathrm{E}_{R}=\left(\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right) \cdot \frac{\alpha^{2} \mathrm{~m} c^{2}}{2}
$$

To focus our thinking, we considered the transition from the second to the first level, for which the $1 / 1^{2}$ $-1 / 2^{2}$ factor is equal 0.75 . Hence, the energy of the photon that is being emitted will be equal to $(0.75) \cdot E_{R} \approx 10.2 \mathrm{eV}$. Now, if the total action is equal to $h$, then the cycle time $T$ can be calculated as:

$$
\mathrm{E} \cdot \mathrm{~T}=h \Leftrightarrow \mathrm{~T}=\frac{h}{\mathrm{E}} \approx \frac{4.135 \times 10^{-15} \mathrm{eV} \cdot \mathrm{~s}}{10.2 \mathrm{eV}} \approx 0.4 \times 10^{-15} \mathrm{~s}
$$

This corresponds to a wavelength of $\left(3 \times 10^{8} \mathrm{~m} / \mathrm{s}\right) \cdot\left(0.4 \times 10^{-15} \mathrm{~s}\right)=122 \mathrm{~nm}$, which is the wavelength of the light ( $\lambda=c / f=c \cdot T=h \cdot c / \mathrm{E}$ ) that we would associate with this photon energy. ${ }^{93}$ This rather simple calculation is sufficient to illustrate our photon model: we think of a photon as being pointlike but, at the same time, the Planck-Einstein relation tells us it packs one wavelength-or one cycle. The integrity of that cycle is associated with its energy ( E ) and its cycle time ( T ) or - alternatively - with its momentum $(\mathrm{p})$ and its wavelength $(\lambda)$, as evidenced in the $\mathrm{E} \cdot \mathrm{T}=\mathrm{p} \cdot \lambda=h$ relation:

$$
\left.\begin{array}{c}
\mathrm{p}=\mathrm{m} c=\frac{\mathrm{E}}{c^{2}} c=\frac{\mathrm{E}}{c} \Leftrightarrow \mathrm{p} c=\mathrm{E} \\
c=f \lambda=\frac{\mathrm{E}}{h} \lambda=\Leftrightarrow \frac{h c}{\lambda}=\mathrm{E}
\end{array}\right\} \Rightarrow \mathrm{p} \cdot c=\frac{h \cdot c}{\lambda} \Leftrightarrow \mathrm{p} \cdot \lambda=h=\mathrm{E} \cdot \mathrm{~T}
$$

So, yes, the equations above sort of vaguely tell us that if we think of measuring some distance or some time - as we do when we probe an electron with photons - that we will have to content ourselves with measuring it in wavelength units ( $\lambda$ ) or, equivalently, in cycle time units ( $T=1 / f=\lambda / c$ ).

However, when probing electrons as part of Compton scattering processes, we are beating the usual game here: we are actually measuring an effective radius of interference not in terms of the wavelength of the light that we are using but in terms of the difference in the wavelength of the photon that goes in and comes out of the scattering process. That's what the Compton scattering formula tells us:

[^23]$$
\lambda^{\prime}-\lambda=\Delta f=\frac{h}{\mathrm{~m} c}(1-\cos \theta)
$$

We already mentioned that the $1-\cos \theta$ factor on the right-hand side of this equation goes from 0 to 2 as $\theta$ goes from 0 to $\pi$. Hence, the maximum possible change in the wavelength is equal to $2 \lambda_{c}$, which we get from a head-on collision with the photon being scattered backwards at 180 degrees. ${ }^{94}$ However, that doesn't answer the question in regard to the $2 \pi$ factor: the $h / \mathrm{mc}$ factor is still the Compton wavelength, so that is $2 \pi$ times the radius: $\lambda_{c}=2 \pi \cdot a=2 \pi \hbar / \mathrm{mc}$. In fact, mentioning the $2 \lambda_{c}$ value for the maximum difference in wavelength introduces an additional factor 2 .

Who ordered that? Let us advance a possible explanation: we are not saying it is the explanation, but it may be an explanation. It goes like follows.

## The electron-photon excitation: a temporary spin-2 particle?

We think the energy of the incident photon - as an electromagnetic oscillation, that is - is temporarily absorbed by the electron and, hence, the electron is, therefore, in an excited state, which is a state of non-equilibrium. As it returns to equilibrium, the electron emits some of the excess energy as a new photon, and the remainder gives the electron as a whole some additional momentum.

How should we model this? One intriguing possibility is that the electron radius becomes larger because it must now incorporate two units of $h$ or, when talking angular momentum, two units of $\hbar$. So we should, perhaps, re-do our calculation of the Compton radius of our electron as follows:

$$
\left.\begin{array}{c}
\left.\begin{array}{c}
\mathrm{E}=\mathrm{m} c^{2} \\
\mathrm{E}=2 h f=2 \hbar \omega
\end{array}\right\} \Rightarrow \mathrm{m} c^{2}=2 \hbar \omega \\
c=a \omega \Leftrightarrow a=\frac{c}{\omega} \Leftrightarrow \omega=\frac{c}{a}
\end{array}\right\} \Rightarrow \mathrm{m} a^{2} \omega^{2}=2 \hbar \omega \Rightarrow \mathrm{~m} \frac{c^{2}}{\omega^{2}} \omega^{2}=\frac{2 \hbar c}{a} \Leftrightarrow a=\frac{2 \hbar}{\mathrm{~m} c}
$$

This may actually work - as long as we remember the energy and mass factors here are the combined energies and masses of the electron and the photon. Of course, this triggers the next question: what's the typical energy or mass of the incoming photon? To demonstrate the Compton shift, Compton used X-ray photons with an energy of about 17 keV , so that's about $3.3 \%$ of the energy of the electron, which is equal to 511 keV . For practical purposes, we may say the photon doesn't change the energy of the electron very much but, of course, it is significant enough to cause a significant change in the state of motion of the electron. ${ }^{95}$

Hence, the excited state of an electron may involve a larger radius - twice as large, approximately. Do we think it explains the above-mentioned factor 2 ? We will let the reader think for himself here as we haven't made our mind up on it yet.

[^24]
## Planck's quantum of action as a vector

What is uncertainty in quantum mechanics? How does it relate to the philosophical concepts of freedom and determinism? We already briefly discussed this, but let us walk over the basics once more. Richard Feynman notes that, "from a practical point of view", we have "indeterminability in classical mechanics as well." ${ }^{\prime 96}$ He gives the example of water splashing over a dam:
"If water falls over a dam, it splashes. If we stand nearby, every now and then a drop will land on our nose. This appears to be completely random, yet such a behavior would be predicted by purely classical laws. The exact position of all the drops depends upon the precise wigglings of the water before it goes over the dam. How? The tiniest irregularities are magnified in falling, so that we get complete randomness. Obviously, we cannot really predict the position of the drops unless we know the motion of the water absolutely exactly."

While it is, effectively, not practically possible to know the motion of the water "absolutely exactly", the deeper question is: is it theoretically possible? In classical mechanics, we assume it is. In quantum mechanics, we assume it is not. Quoting Feynman once again, we may say that "the Uncertainty Principle protects quantum mechanics." Why should this be so?

We think it's plain nonsense: there is no reason whatsoever to assume the uncertainty in classical mechanics in any different from the uncertainty in quantum mechanics. The uncertainty in quantum mechanics is not anything more than the Ungenauigkeit-imprecision, which is the term Heisenberg first used to describe the apparent randomness in our mathematical description of Nature ${ }^{97}$ —we encounter in classical mechanics as well. It is just an imprecision, indeed-as opposed to the weird metaphysical quality which Heisenberg would later claim it to be ${ }^{98}$ and which, without any precise
${ }^{96}$ See: Feynman's Lectures, III-2-6.
${ }^{97}$ We gratefully acknowledge the Wikipedia article on Heisenberg for this information. It states that it was effectively in Copenhagen - in 1927, to be precise - that Heisenberg developed his uncertainty principle but that in the letter he wrote to Wolfgang Pauli to describe it - he used the term Ungenauigkeit rather than Ungewissheit or Unbestimmtheit (indefiniteness). The reader should note that German was probably more important than English or French as a language of science back then.
${ }^{98}$ We think the term 'indefiniteness' (Ungewissheit or Unbestimmtheit in German) might better capture this metaphysical quality. When saying this, we hastily add we do not believe such metaphysical quality applies to quantum mechanics: probing electrons with photons is a complicated process but we do not believe it is fundamentally different from, say, a blind man probing some object by taking it in his hands and trying to feel it or - if you prefer a more accurate metaphor - carefully scanning some object by firing spherical elastic balls at it and carefully keeping track of how they bounce back.

Needless to say, we do acknowledge the force is not a simple contact force here: the structure of the electromagnetic force is complicated, as evidenced from the fact that Volume II of Feynman's Lectures is the most voluminous of all three, even if all it does is go through the consequences of Maxwell's set of equations. In fact, Feynman shows that, when using four-vector notation, Maxwell's equations reduce to two only:

$$
\begin{aligned}
\square^{2} A_{\mu} & =\frac{1}{\epsilon_{0}} j_{\mu} \\
\nabla_{\mu} j_{\mu} & =0
\end{aligned}
$$

The $A_{\mu}$ and $j_{\mu}$ are four-vectors $-A_{\mu}$ combines the classical electric potential and the (magnetic) vector potential $\boldsymbol{A}$, so $A_{\mu}=(\phi, \boldsymbol{A})$ - and so I should probably use boldface rather than straight face (yes, I just invented that term right
definition, physicists now refer to as 'uncertainty'. The concept of 'uncertainty' effectively mixes two different connotations:
(3) an imprecision in our measurements that is due to the wave-particle nature of electrons and other elementary particles - a duality which the ring current model, combining the idea of charge and its motion, captures from the outset (as opposed to trying to cope with the duality in some wave equation or other dual mathematical descriptions); and
(4) a rapid magnification of what Feynman refers to as 'the tiniest irregularities' but which are, in practice, purely statistical.

This is reflected in what Feynman refers to as the 'great equation of quantum mechanics'-the wave equation for an electron in free space ${ }^{99} 100$ :

$$
\frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}}+\frac{\partial^{2} \phi}{\partial z^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} \phi}{\partial t^{2}}=\frac{\mathrm{m}^{2} c^{2}}{h^{2}} \phi=\frac{1}{\lambda_{C}^{2}} \phi
$$

We replaced the $\mathrm{m}^{2} c^{2} / h^{2}$ by the Compton wavelength $\lambda_{c}=h / \mathrm{mc}$ in the formula above because we effectively do think as the distance scale within which we can localize the electron in a particle-like

[^25]The implications of this 're-write' of Maxwell's equations are indeed quite deep. From the first equation, we get this:

$$
\frac{j_{\mu}}{A_{\mu}}=\square^{2} \epsilon_{0}
$$

The relation is, obviously, extremely beautiful: the ratio of two physical (but four-dimensional) realities equals the product of a mathematical operator and a (one-dimensional) physical (scalar) constant. I honestly don't have a clue about what it might mean, but it must be a very importantly relation. Why? Because the electric constant $\varepsilon_{0}$ is related to lightspeed and the fine-structure constant. We will let you google those relationships. Again, if you feel you make sense of them, please do not hesitate to email us: I think about them pretty much all of the time. Feynman had to get his Nobel Prize and, therefore, probably deceives us here and there. However, in Vol. II, he is a genius at work and, for me, I think the depth of his equations is quite obvious here:

Remember that Eqs. (25.21) could be deduced from Maxwell's equations only if we imposed the gauge condition

$$
\begin{equation*}
\frac{\partial \phi}{\partial t}+\boldsymbol{\nabla} \cdot \boldsymbol{A}=0 \tag{25.23}
\end{equation*}
$$

which just says $\nabla_{\mu} A_{\mu}=0$; the gauge condition says that the divergence of the four-vector $A_{\mu}$ is zero. This condition is called the Lorenz condition. It is very convenient because it is an invariant condition and therefore Maxwell's equations stay in the form of Eq. (25.22) for all frames.
${ }^{99}$ This is just one of the many expressions of it. We took it from Feynman's Lectures, section I-48-6. Feynman did not substitute $m^{2} c^{2} / h^{2}$ for $1 / \lambda c^{2}$ in his presentation. We must, therefore, assume he did not think of any possible association or relation with the Compton wavelength and/or radius of an electron.
${ }^{100}$ Needless to say, free space means we don't think of any electromagnetic field or any other force field here. Schrödinger adapted this equation to include a central force field: a hydrogen nucleus (a proton) attracting the electrons around it. It is good to note that Dirac refers to such wave equations as the equations of motion of an electron, because that's what they are.
sense. The dissipation which this wave equation models is, therefore, not the dissipation of the electron itself but, effectively, our rapidly increasing uncertainty about where it might be at a later point in time.

This, of course, triggers the question: why would it increase so rapidly? The answer is simple. It's just scale: Planck's quantum of action can (also) be written as the product of momentum over some distance $(h=p \cdot \lambda)^{101}$ and, in light of the very tiny distance scale $(\lambda)$, the momentum ( $p$ ) is, effectively, relatively large - which is why any imprecision in its magnitude or direction quickly translates into huge uncertainties concerning the actual position of the elementary particle that we are trying to describe.

This is clear enough ${ }^{102}$, but the smart reader will, of course, immediately ask for more clarity: do we think the uncertainty is in the direction or in the magnitude of the momentum? This is an essential question, indeed! If the uncertainty is purely statistical, then what's the hidden variable? We think the hidden variable may be in the motion of the plane of rotation of the Zitterbewegung charge. Hence, we think the statistical uncertainty has to do with the direction rather than the magnitude of the momentum. We may, perhaps, focus the idea of the reader with the illustration below. It shows the precessional motion of the plane of rotation of a spinning top. It reminds us of the vector quality of the angular momentum. We should, therefore, probably write Planck's quantum as a vector: $\boldsymbol{h}$ (or, to use the right unit here, $\boldsymbol{\hbar}$ ) instead of $h$ (or $\hbar$ )—_just like we would use $L$ (the boldface to denote a vector) rather than $L$ for angular momentum. Look at the wobbling angular momentum of the spinning top below, and then think of the plane of rotation of our pointlike charge.


Figure 8: A wobbling plane of rotation ${ }^{103}$
Of course, we should remind ourselves we only have regularity in this motion because of the force field, which is gravitational in the example of the spinning top (it's the $F_{g}$ vector). In quantum mechanics, the force field would be some magnetic field: think of the Stern-Gerlach experiment here. ${ }^{104}$ The question

[^26]then becomes: what's the motion in the absence of a magnetic field?
We do not have the answer to that question but we do not believe that motion would be entirely random: one would expect that interactions with other charged and non-charged particles (think of the interaction with photons which we just described) would not only result in some change of the linear state of motion of an electron but in changes in the direction of its angular momentum as well. These are, of course, assumptions we may not be able to test because the measurement itself - the probing with photons, or the measurement of the angular momentum by applying some magnetic field - would cause such changes themselves! This is, in fact, what Heisenberg's Ungenauigkeit is all about! ${ }^{105}$

Hence, in the absence of a magnetic field (read: for a particle in free space), we would not be able to ascertain the direction of Planck's quantum - written as a vector ( $\boldsymbol{h}$ or, in its reduced form, $\boldsymbol{\hbar}$ ) but, while we would think of it as being fairly random, we would not doubt its direction is the result of the motion of the plane of rotation of the Zitterbewegung charge which, itself, is the result of previous interactions and events. Such randomness would, therefore, be random but it would not be mysterious!

So how would we write the Uncertainty Principle then? Simple, we just write $\boldsymbol{h}$ and $\mathbf{p}$ as vectors. We get the $\boldsymbol{p} \cdot \boldsymbol{\lambda}=\boldsymbol{h}$ expression from the Planck-Einstein relation and we just apply it to the ring current geometry (with $\lambda$ the de Broglie wavelength ${ }^{106}$ ). We must then introduce the idea of a non-precise direction of $\boldsymbol{h}$ so we can write something like this:

$$
\Delta \boldsymbol{h}=\Delta(\mathbf{p} \cdot \lambda)
$$

Plain vector algebra - and remembering $\Delta \boldsymbol{h}$ denotes an uncertainty in the direction of the $\boldsymbol{h}$ and $\mathbf{p}$ vectors, not of its magnitude - then gives us the expression you are used to-Heisenberg's (in)famous
charged particles. Stern-Gerlach experiments are always done with electrically neutral particles, such as potassium atoms (see, for example, this rather typical lab experiment for students) or, in the original experiment, silver atoms. There is a very practical reason for this, of course: it's because any electric charge in the magnetic field in the Stern-Gerlach apparatus would be subject to a Lorentz force which would be much larger than the force resulting from the magnetic moment. At the same time, I find it a rather weird state of affairs: most, if not all, of mainstream quantum theory hinges on the assumption that an electron has two spin states only (up versus down) but, at the same time, no experiment has actually verified this. It is not that no one ever suggested it. H. Batelaan, T. J. Gay, and J. J. Schwendiman, for example, wrote a rather intriguing letter to the Physical Review journal in 1997, explaining in very much detail how the Stern-Gerlach experiment could be modified to also split an electron beam based on the magnetic moment being up or down. Their proposal is based, in fact, on a proposal of the French physicist Léon Brillouin. Brillouin was an active participant in the 1927 Solvay Conference and his proposal, which he published in 1928 (L. Brillouin, Proc. Natl. Acad. Sci. U.S.A. 14, 755, 1928), can be traced straight back to the discussions there. In light of the venerable history of this proposal, it is very strange no one has followed up on it. One would think this should be a higher priority than the new US $\$ 600 \mathrm{~m}$ accelerator project which Dr. Consa talks about in his scathing review of the current state of affairs in the field of experimental research.
${ }^{105}$ We may usefully remind the reader Heisenberg's very first arguments were entirely based on this kind of physical reasoning. Think of Heisenberg's thought experiment involving a microscope using gamma rays instead of optical light: it is very obvious Heisenberg does not think of uncertainty as something metaphysical here! Instead, he plainly relates the uncertainty to the interaction between the observer - the act of probing or measuring - and what is being observed or measured.
${ }^{106}$ For a geometric interpretation of the de Broglie wavelength, we refer to our discussion of the Compton wavelength and what happens to it when the Zitterbewegung charge inside of the electron is also subjected to some linear motion.

Uncertainty Principle:

$$
h=\Delta p \cdot \Delta \lambda
$$

That's all what it is. By way of conclusion, we may, perhaps, add one more remark: why are we so sure the uncertainty must be in the direction of the angular momentum of the electron, rather than in its magnitude?

The honest answer is: we cannot be sure. We are basically just siding with Einstein's instinct here: the velocity of light is the velocity of light, Planck's quantum of action is Planck's quantum action, and the classical velocity of the electron is the classical velocity of the electron. Hence, if we assume the Compton radius of the electron and the motion of the charge inside is defined by $h$ and $c$, and if we assume the linear motion of the same charge is defined by the classical (linear) velocity of the electron $(v)$, then there is no room for uncertainty! The uncertainty must, therefore, be in the wobbling of $\boldsymbol{h}$. Nothing more. Nothing less. ${ }^{107}$

## Conclusions

The professional physicist will point out that (1) the lines above were definitely unscientific and (2) despite our claim we'd give you a real explanation of Compton scattering, we failed to produce all of the details of the process. Indeed, how does it work, exactly?

We effectively did not work out all of the details, but we do have some very strong clues here on how a definitive explanation - including all relevant variables, including the mentioned plane formed by the outgoing photon and the recoiling electron - may look like. The plane of the Zitterbewegung oscillation, for example, must - without any doubt - play a crucial role in determining the angles of the incoming as well as the outgoing photon.

Also, it is quite obvious that the circular motion of the $z b w$ charge must explain the $\pi$ or $2 \pi$ factor. It is not a size factor: the size of the electron is of the order of the Compton radius (a) but, because of the circular motion, it is actually the circumference of the motion $(\lambda=2 \pi a)$ that must enter the Compton scattering formula. The image of a hand sling throwing a stone comes to mind, of course-but that's probably too simplistic: throwing some mass out by converting circular to linear motion is easy enough to imagine, but here we're not talking mass.

At the same time, our models are all 'mass without mass' models and so they show that energy is, ultimately, motion: an oscillating force on a charge in case of the electron - or, in case of the photon, an oscillating electromagnetic field. The motion that's associated with an electron is circular, while the motion of the oscillating electromagnetic field is linear. As such, we may - figuratively speaking - say that 'circular' motion is being converted into 'linear' motion, and vice versa.

In terms of the 'mechanics' of what might actually be going on when a photon is absorbed or emitted by an electron, that's all what we can give you for the time being! You'll have to admit we can't quite show you what's going on inside of the box, but we did open it at least, didn't we?

[^27]Just in case you think we didn't, we invite you - once again - to google the papers explaining the $S$ matrix of Compton scattering processes. ${ }^{108}$ We are sure you will prefer to re-read our paper and - who knows? - start doing some pretty calculations yourself!

QED: Quod erat demonstrandum.

[^28]
[^0]:    ${ }^{1}$ We are, obviously, referring to the invention of the concept of strangeness by Murray Gell-Man and Kazuhiko Nishijima in the 1950s. Feynman's treatment of it in his 1963 Lectures on physics shows that the concept of strangeness - and the related conservation law - is a rather desperate assumption to explain the decay of Kmesons (kaons). Unfortunately, the concept of strangeness started a strange life of its own and would serve as the basis for the quark hypothesis which - for a reason I find even stranger than the concept of strangeness itself was officially elevated to the status of a scientific dogma by the Nobel Prize Committee for Physics.

    As for the invention of a new force or a new potential, we are, obviously, referring to the Yukawa potential. This hypothesis - which goes back to 1935 - might actually have been productive if it would have led to a genuine exploration of a stronger short-range force on an electric charge - or, if necessary, the invention of a new charge. Indeed, if the electromagnetic force acts on an electric charge, it would be more consistent to postulate some new charge - or some new wave equation, perhaps - matching the new force. Unfortunately, theorists took a whole different route. They invented a new aether theory instead: it is based on the medieval idea of messenger or virtual particles mediating forces.
    ${ }^{2}$ Paul A.M. Dirac, Principles of Quantum Mechanics, $4^{\text {th }}$ revised edition (1958), section 73: Theory of the positron.

[^1]:    ${ }^{3}$ The idea is usually this: the potential energy of a charge at some large distance (infinity, for calculational purposes) is defined as zero. When bringing that charge closer to some other charge, work is done with or against the force. Its energy is, accordingly, negative or positive. The potential energy of an electron in an electron orbital, for example, is, obviously, negative.
    ${ }^{4}$ Paul A.M. Dirac, Theory of electrons and positrons, Nobel Lecture, 12 December 1933. Needless to say, we associate the elementary wavefunction with this motion. To be precise, we think the $a \cdot e^{-i \cdot \theta}$ or $a \cdot e^{+i \cdot \theta}$ function gives us the spatial position of the oscillating elementary charge at every point in time. In other words: the elementary wavefunction represents what Erwin Schrödinger referred to as the Zitterbewegung of the electron. The Zitterbewegung of an electron is a physical application of Wheeler's 'mass without mass' idea: all of the energy (and, therefore, all of the equivalent mass) of the electron is in the oscillatory motion of the Zitterbewegung (zbw) or elementary charge. This explains the dual wave-particle character of an electron and, as such, it also explains diffraction and the interference of an electron with itself when being forced through a single or double slit. As mentioned, we will come back to this later in the text and the reader should, therefore, just make a mental note of this and proceed his or her reading.
    ${ }^{5}$ In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1x). We quote this example for the same reason as why we use Feynman's Lectures as a standard reference: it's authoritative and, more importantly, also available online so the reader can check and explore for himself.
    ${ }^{6}$ Even neutrons have a magnetic moment and, therefore, spin! One should also note that photons - despite their

[^2]:    spin-one property - cannot not come in a zero-spin state. When studying Feynman's Lectures, I found that to be one of the weirdest things: Feynman first spends several chapters on explaining spin-one particles to, then, in some obscure footnote suddenly write this: "The photon is a spin-one particle which has, however, no "zero" state." The issue is related to another glaring inconsistency: in the first three chapters of his Lectures on physics, he talks about adding wavefunctions and the basic rules of quantum mechanics, and it all happens with a plus sign. However, in his chapter on the theoretical distinction between bosons and fermions, he suddenly says we should be adding the amplitudes of fermions combine with a minus sign. In any case, we cracked our own jokes on the boson-fermion theory and so we should leave it at that.

    We should add one note on the zero-spin state of helium, however: the electrons and protons inside the helium atom will line up in a way that minimizes energy. The helium atom, as a composite particle, may, therefore, be said to come in a zero-spin state. One may think of this situation as the exception which confirms the general rule which applies to all charged particles-including composite particles, neutral or not. [The latter idea - that of a neutral charged particle - obviously refers to a composite particle in which there are as many positive as negative electric charges.]
    ${ }^{7}$ We are obviously not talking about the zero-spin state of, say, a spin-one atom here. We talk about elementary particles only here.

[^3]:    ${ }^{8}$ The quantum-mechanical argument is technical, and I did not reproduce it in this book. I encourage the reader to glance through it, though. See: Euler's Wavefunction: The Double Life of -1 . Note that the $e^{+i \pi} \neq e^{-i \pi}$ expression may look like horror to a mathematician! However, if he or she has a bit of a sense for geometry and the difference between identity and equivalence relations, there should be no surprise. If you're an amateur physicist, you should be excited, because it actually is the secret key to unlocking the so-called mystery of quantum mechanics. Remember Aquinas' warning: quia parvus error in principio magnus est in fine. A small error in the beginning can lead to great errors in the conclusions, and we sure think of this as an error in the beginning!
    ${ }^{9}$ Simple experiments - such as the Stern-Gerlach experiment - involve a magnetic or an electric field only. However, many experiments and apparatuses - such as the Penning trap, for example - will involve a combination of both.
    ${ }^{10}$ The illustration was taken from Feynman's Lecture on the ammonia maser. We gratefully acknowledge Michael Gottlieb and Rudolf Pfeiffer, who have worked for decades to get those Lectures online and, therefore, claim copyright on Feynman now.

[^4]:    ${ }^{11}$ The field that separates the two states is, therefore, electric. Other typical quantum-mechanical systems will typically involve magnetic dipoles which are, therefore, separated by magnetic fields.
    ${ }^{12}$ Indeed, most of us should be very familiar with the proton spin flipping mechanism because of its use in magnetic resonance imagining. I must assume that - if you have ever had an MRI - you also googled and watched one or more YouTube videos explaining the physics underpinning this amazing technology. If not, I warmly recommend you do so. It will give you a better intuitive grasp of the basics of quantum mechanics than any of the standard introductory courses. For starters, it gives you a great engineering feel for what might or might not be going on. MRI machines do not only manipulate the direction of the proton spin but - in order to localize the region that is being scanned, it also manipulates its phase. It is all very sophisticated but beautiful to watch. If anything, it shows how real spin - the motion of the elementary charge within a proton or an electron - actually is. You can google some videos for yourself but, in case you'd appreciate some advice here, we would single out the one from Doctor Klioze: https://www.youtube.com/watch?v=djAxjtN 7VE. It is almost half an hour long, but well worth spending the time! Enjoy!
    ${ }^{13}$ We refer the reader to section 7 of Chapter 10 of Feynman's Lectures on quantum mechanics for a comparison with Feynman's original quote. Back in 1963, Feynman modeled proton spin using the Pauli matrices, which is still very useful, of course!

[^5]:    ${ }^{14}$ We refer the reader to our papers for what we usually refer to as a classical explanation of various quantummechanical phenomena, including Mach-Zehnder interference and other previously mysterious experimental results. As for tunneling through a wall, think of this: when you are going to crash a projectile into a wall, it is not only its momentum that is going to matter. The direction of your projectile is going to matter at least as much, doesn't it? Taking a hit directly or sideways make a lot of difference, doesn't it? ©)
    ${ }^{15}$ We will do so in one of the next section(s). As for now, the reader may just want to note our argument is mathematical and philosophical: the very idea of motion includes the idea of moving in time, but in one direction only. A particle may go back to a place where it was before, or it may stay where it was, but if we'd allow it to go back in time, we would not be able to capture its motion in a well-behaved function. Well-behaved equations of motion imply the same particle may re-visit the same place in space, but not the same place in time. We do not need any complicated arguments in terms of entropy or - worse - imagined symmetries that cannot exist in reality.
    ${ }^{16}$ The CODATA value is its measured value as published by the Committee on Data (CODATA) of the International Science Council. This Committee regularly makes informed decisions on what experiments count (or don't count) and, more importantly, to what degree of precision their results are supposed to be valid.
    ${ }^{17}$ You are, of course, aware of the so-called anomaly in the magnetic moment. It is a difference between the theoretical and measured value. The difference is of the order of Schwinger's factor ( $\alpha / 2 \pi$ ). Mainstream physicists explain it using quantum field theory. We explain it by assuming the Zitterbewegung charge has a tiny but non-zero size itself.
    ${ }^{18}$ We are aware this may sound shocking to those who have been brainwashed in the old culture. If so, make the switch. It should not be difficult: a magnetic moment - any magnetic moment, really - is generated by a current. The magnetic moment of elementary particles is no exception.

[^6]:    ${ }^{19}$ To determine what is up or down, one has to apply the ubiquitous right-hand rule.
    ${ }^{20}$ The use of the subscripts in the magnetic moment may be confusing, but shouldn't be: we use -e for an electron and +e for a positron. We do so to preserve the logic of denoting the (positive) elementary charge as $\mathrm{q}_{\mathrm{e}}$ (without a + or a - in the subscript here).
    ${ }^{21}$ In case the reader wonders why we associate the ++++ signature with the positron rather than with the electron, the answer is: convention. Indeed, if I am not mistaken (which may or may not be the case), it is the ++++ metric signature which is the one which defines the usual righthand rule when dealing with the direction of electric currents and magnetic forces.
    ${ }^{22}$ If we should summarize our quest for truth in physics, we might say this was our drive: we wanted an explanation for these so-called intrinsic properties of elementary particles: their mass or energy, their size, and their magnetic moment. We think we have succeeded in finding that explanation. Having said that, we must admit the physicality of pair creation and annihilation still puzzles us deeply. We therefore think we greatly reduced the Mystery, but we sure did not eliminate it completely!

[^7]:    ${ }^{28}$ To be precise, Heisenberg got a postponed prize from 1932. Erwin Schrödinger and Paul A.M. Dirac jointly got the 1933 prize. Prof. Pleijel acknowledges all three in the introduction of his speech: "This year's Nobel Prizes for Physics are dedicated to the new atomic physics. The prizes, which the Academy of Sciences has at its disposal, have namely been awarded to those men, Heisenberg, Schrödinger, and Dirac, who have created and developed the basic ideas of modern atomic physics."
    ${ }^{29}$ We will slightly nuance this statement later but we will not fundamentally alter it. We think of matter-particles as an electric charge in motion. The centripetal force that keeps the particle together is electromagnetic in nature, because it acts on a charge. Matter-particles, therefore, combine wave-particle duality. Of course, it makes a difference when this electromagnetic oscillation, and the electric charge, move through a slit or in free space. We will come back to this later. The point to note is: matter-particles do not dissipate. Feynman actually notes that at the very beginning of his Lectures on quantum mechanics, when describing the double-slit experiment for electrons: "Electrons always arrive in identical lumps."
    ${ }^{30}$ The interpretation of the Planck-Einstein relation is another matter, but one doesn't need matter-waves for it.

[^8]:    The Planck-Einstein relation basically tells us particles have a unique frequency: a clock speed which is equal to the cycle time $\mathrm{T}=1 / \mathrm{f}=\mathrm{h} / \mathrm{E}=\mathrm{h} / \mathrm{mc}^{2}$. Combining this idea with the reality of the magnetic moment of elementary particles, one arrives at the conclusion the frequency must be equal to the frequency of the motion of the electric charge, which both Erwin Schrödinger as well as Paul A.M. Dirac paid much attention to. We will come back to this.
    ${ }^{31}$ The reader should note that we have very decent classical explanations for both. See our paper on the electron and the anomalous magnetic moment as well as our paper on the Lamb shift.
    ${ }^{32}$ Oliver Consa, Something is Rotten in the State of QED, February 2020.
    ${ }^{33}$ John P. Ralston, How To Understand Quantum Mechanics, 2017. We must add that, while we find Dr. Ralston's criticism of his own profession fairly enlightening, we do not think his own interpretation of quantum mechanics yet another model, yes - is very convincing. Not at all, actually. :-/
    ${ }^{34}$ We could have quoted other founding fathers of quantum mechanics who ended up skeptical about the theory they had created (or Einstein, of course). Dirac, for example, was rather vocal about it. However, we opted to quote W.E. Lamb because he is, perhaps, one of the lesser known-together with John Stewart Bell. The case of J.S. Bell is very particular, because he was nominated for a Nobel Prize for the (in)famous No-Go Theorem, which tells us there are no hidden variables that can explain the interference in some kind of classical way? Bell's Theorem is what it is: a mathematical theorem. Nothing more. Nothing less. It, therefore, respects the GIGO principle: garbage in, garbage out. John Stewart Bell himself - a third-generation quantum physicist, we may say did not like his own 'proof' and thought that some "radical conceptual renewal" might disprove his conclusions (John Stewart Bell, Speakable and unspeakable in quantum mechanics, pp. 169-172, Cambridge University Press, 1987). J.S. Bell kept exploring alternative theories - including Bohm's pilot wave theory, which is a hidden variables theory - until his death at a relatively young age (the year when he got nominated for the Nobel Prize, in fact).
    ${ }^{35}$ We should be precise: Willis Eugene Lamb Jr. got half of the 1955 Nobel Prize in Physics for his discovery. He had to share it with Polykarp Kusch. To be precise, Lamb got his half of the prize "for his discoveries concerning the fine structure of the hydrogen spectrum" (which is the Lamb shift), while Polykarp Kusch got it "for his precision determination of the magnetic moment of the electron" (the so-called anomaly in the magnetic moment). The reader can look it all up on the website of the Nobel Prize for Physics.

[^9]:    ${ }^{36}$ One should, indeed, note that both Lamb as well as Kusch had measured the Lamb shift and the anomaly respectively, but both had left the explaining of it to (other) physicists - some more famous names you probably are more acquainted with, such as Julian Schwinger and Richard Feynman, who - together with Tomonaga - got a Nobel Prize in physics "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"-read: their explanations of this and other previously unexplained measurements.
    ${ }^{37}$ See: W.E. Lamb Jr., Anti-photon, Appl. Phys. B 60, 77-84 (1995).
    ${ }^{38}$ The $S$ in $S$-matrix refers to scattering, not to the German Spur (the trace of a matrix) or some other term. I am inserting this rather pedantic note because part of the mystery of quantum mechanics is stuff like this: abbreviations or concepts that never get a proper definition or an accurate portrayal of the archaeology or genealogy of the idea. According to the Wikipedia article on it, a proper $S$-matrix was first introduced by John Archibald Wheeler in a 1937 article on the composite wavefunction for multi-particle nuclei.

[^10]:    ${ }^{40}$ Lamb's remark on Niels Bohr leaving the actual calculations to postdocs may or may not reflect a remark of Feynman - I can't recall if it's in his Lectures or in his Strange Theory of Light and Matter - on the rather extraordinary amount of calculus one needs to calculate anything based on quantum field theory or Feynman's own path integral formulation of quantum mechanics and the fact that, therefore, such calculus is usually left to doctoral students who, in Feynman's words, have to "keep going at it for their PhD." We need to add a lot of the heavy lifting on the discovery of the Lamb shift was actually done by one of Lamb's own postdoc students, R. Retherford, who did not get much recognition for it-except for a brief mention in Lamb's Nobel Prize speech.

    As for the calculations themselves, we may briefly mention that Stefano Laporta claims - in a 2017 article (other researchers have come up with similar articles in the meanwhile) - to have calculated 891 four-loop contributions to the anomalous magnetic moment, resulting in a precision of 'up to 1100 digits of precision' of the electron's actual g-ratio (slightly different from a theoretical value of 2 , which is - in my not-so-humble view - simply just is what it is: a theoretical value based on mathematical idealizations). One gets an uncanny feeling here: if one has to calculate a zillion integrals all over space using 72 third-order diagrams to calculate the $12^{\text {th }}$ digit in the anomalous magnetic moment, or 891 fourth-order diagrams to get the next level of precision, then there might something wrong with the theory-read: our mathematical idealizations. We refer to our own paper(s) for a more classical explanation of the anomalous magnetic moment.

[^11]:    ${ }^{42}$ We will say more about this later.
    ${ }^{43}$ The $h=\mathrm{E} \cdot \mathrm{T}$ and $h=\mathrm{p} \cdot \lambda$ are complementary but equivalent descriptions of the electron in the ring current model.
    ${ }^{44}$ In case the reader would be interested in our own convictions here, we will just say this: we think it's a ridiculous argument. If God is hiding Him- or Herself, He (or She) hides in much better places. Think of what we refer to as love here or - more mundane, perhaps - this strange order that comes out of disorder.

[^12]:    ${ }^{45}$ Of course, these vectors are a bit different than the usual Cartesian coordinate vectors that you are used to but the idea of base vectors (base states) and combining or adding them remains the same. Physicists will solemnly talk of a Hilbert space but that is just jargon meant to impress you.
    ${ }^{46}$ I did this randomly just now and these two recent articles came out on top: https://arxiv.org/abs/1512.06681 and https://arxiv.org/abs/1810.11455. You may get others. This input-output calculus approach allows for endless refinements and, hence, endless research-which is always nice from an academic point of view, of course.
    ${ }^{47}$ The reader may be interested to know the author of this paper is an economist who has, effectively, dabbled in both macro- as well as micro-economic modeling and the training in statistics that comes with it. We are, therefore, very familiar with Marc Twain's 1907 quote: "There are three kinds of lies: lies, damned lies, and statistics."
    ${ }^{48}$ The quote is, apparently, from a May 1963 paper or article in Scientific American. We didn't analyze the context in which Dirac is supposed to have said this.
    ${ }^{49}$ See: Feynman's Lectures, III-1-1.

[^13]:    be excited, because it actually is the secret key to unlocking the so-called mystery of quantum mechanics.
    Remember Aquinas' warning: quia parvus error in principio magnus est in fine. A small error in the beginning can lead to great errors in the conclusions, and we sure think of this as an error in the beginning!
    ${ }^{64}$ We should also apologize for the (partial) overlap between this and our introductory discussions. We will try to rationalize and shorten the next version of this paper.

[^14]:    ${ }^{65}$ We wish the reader who would want to try using not-so-well-behaved functions to arrive at some kind of description of reality the best of luck.
    ${ }^{66}$ We actually do not like the concept of spacetime very much: time and space are related (through special and

[^15]:    general relativity theory, to be precise) but they are not the same. Nor are they similar. We do, therefore, not think that some 'kind of union of the two' will replace the separate concepts of space and time any time soon, despite Minkowski's stated expectations in this regard back in 1908. Grand statements and generalizations are not often useful in physics.
    ${ }^{67}$ While a photon is generally considered to be a spin-one particle, it does not have a zero state. In contrast, spinone atoms are composite particles and, hence, its component particles may effectively line up in a way that produces a spin-zero state.
    ${ }^{68}$ We find the term somewhat confusing but a polarized beam is a beam of particles with the same spin.
    ${ }^{69}$ See: Feynman's Lectures, III-5-5 (Interfering Amplitudes) and III-5-6 (The Machinery of Quantum Mechanics).

[^16]:    ${ }^{70}$ The absolute square is, obviously, a shorthand for the absolute value of the square.
    ${ }^{71}$ You may think of this like follows. The energy in an oscillation - any physical oscillation - is always proportional to the square of the (maximum) amplitude of the oscillation, so we can write this: $\mathrm{E} \propto a^{2}$. To be precise, the energy of an electron in our ring current model is equal to $E=m \cdot a^{2} \cdot \omega^{2}$. We get this from (1) Einstein's mass-energy equivalence relation ( $E=m \cdot c^{2}$ ) and (2) our interpretation of $c$ (lightspeed) as a tangential velocity ( $c=a \cdot \omega$ ). We think the mass of the electron is just the equivalent mass of the energy of the Zitterbewegung charge in its oscillation. We, therefore, think it makes sense to think that the probability of actually finding the charge here or there will be proportional to the energy density here and there.
    ${ }^{72}$ The reader should carefully note what is commutative and what not here!
    ${ }^{73}$ A careful analysis of the processes involving K-mesons in Feynman's rather enigmatic Lecture on it reveals that the quark hypothesis results from a rather unproductive approach to analyzing disintegration processes: inventing new quantities that are supposedly being conserved, such as strangeness, is... Well... As strange as it sounds. We, therefore, think the concept of quarks confuses rather than illuminates the search for a truthful theory of matter. See our introduction on Smoking Gun Physics in our general paper on the power of classical physics. This text is based on an earlier critical appraisal of the theory of virtual particles.

[^17]:    ${ }^{74}$ This rather extreme example of before and after states is, perhaps, less extreme than it appears to be: the proton-proton collisions in CERN's LHC colliders are quite violent too, albeit at a much smaller scale. However, as mentioned, one should not necessarily expect that the disintegration processes involved in the creation of such disequilibrium situations would or should be reversible.
    ${ }^{75}$ Feynman provides such argument based on placing an open filter (no masks) in-between two other filters. See the above-mentioned reference to Feynman's Lectures (III-5-5).

[^18]:    ${ }^{76}$ This is, once again, a standard textbook equation but - if the reader would require a reminder of how this formula comes out of special relativity theory - we may refer him to the online Lectures of Richard Feynman. Chapters 15 and 16 offer a concise but comprehensive overview of the basics of relativity theory and section 5 of Chapter 6 gives the reader the formula he needs here. It should be noted that we dropped the 0 subscript for the rest mass or energy: $m_{0}=m$. The prime symbol (') takes care of everything here and so you should carefully distinguish between primed and non-primed variables.
    ${ }^{77}$ We realize we are cutting some corners. We trust the reader will be able to google the various steps in-between.
    ${ }^{78}$ We could have used boldface to denote vectors, but the calculations make the arrow notation more convenient here. So as to make sure our reader stays awake, we note that the objective of the step from the first to the second equation is to derive a formula for the (linear) momentum of the electron after the interaction. As

[^19]:    ${ }^{82}$ See the exposé of Prof. Dr. Patrick R. LeClair on Compton scattering.

[^20]:    concept of spacetime is not something we can easily imagine: space and time are fundamentally different categories of the mind. We cannot go backward in time, for example. Not in our reality, at least. Why not? Because the $\boldsymbol{x}=(x, y, z)=\left(f_{x}(t), f_{y}(t), f_{z}(t)=f(t)\right.$ - the fundamental equation of motion - would no longer be a proper function: a physical object cannot be at two different places at the same point in time. It can return to a place where it was (or it can simply stay where it is), but it will then be there at a different point of time. That's why time goes by in one direction only. Why do we highlight this point? Because some physicists - including Feynman seem to suggest we should think of antimatter as particles traveling back in time. We think that is plain nonsensical.
    ${ }^{84}$ See the reference above.
    ${ }^{85}$ The nature of a $2 \pi$ factor is definitely very different from that of a 2 or $1 / 2$ factor, which we often got when analyzing something using non-relativistic equations. A $2 \pi$ factor is associated with something circular, so we need to explain what circular feature, and not in approximate but in exact terms-which is not easy in this particular case.
    ${ }^{86}$ Vassallo, G., Di Tommaso, A. O., and Celani, F, The Zitterbewegung interpretation of quantum mechanics as theoretical framework for ultra-dense deuterium and low energy nuclear reactions, in: Journal of Condensed Matter Nuclear Science, 2017, Vol 24, pp. 32-41. Don't worry about the rather weird distance scale ( $1 \times 10^{-6} \mathrm{eV}^{-1}$ ). Time and distance can be expressed in inverse energy units when using so-called natural units ( $c=\hbar=1$ ). We are not very fond of using natural units because we think they may hide rather than clarify or simplify some of the more fundamental relations. Just note that $1 \times 10^{-9} \mathrm{eV}^{-1}=1 \mathrm{GeV}^{-1} \approx 0.1975 \times 10^{-15} \mathrm{~m}$. As you can see, the zbw radius (for $v=0$ ) is of the order of $2 \times 10^{-6} \mathrm{eV}^{-1}$ in the diagram, so that's about $0.4 \times 10^{-12} \mathrm{~m}$, which is more or less in agreement with the Compton radius as calculated ( $a_{\mathrm{v}=0} \approx 0.386 \times 10^{-12} \mathrm{~m}$ ).

[^21]:    ${ }^{87}$ Because it is a wave in two or three dimensions, we cannot really say there are crests or troughs, but the terminology might help you with the interpretation of the rather particular geometry here, which is that of an Archimedes screw, but that's only because of the rather particular orientation of the plane of the zbw oscillation, which we ask you not to accept for granted: you should, instead, imagine the plane of oscillation itself is probably not stable: the (in)famous uncertainty in quantum mechanics may actually be related to our lack of knowledge in regard to the plane of the zbw oscillation: it may itself be zittering around.
    ${ }^{88}$ Needless to say, the C subscript stands for the C of Compton, not for the speed of light (c).
    ${ }^{89}$ We advise the reader to always think about proportional ( $\mathrm{y}=k \mathrm{x}$ ) and inversely proportional ( $\mathrm{y}=\mathrm{x} / k$ ) relations in our exposé, because they are not always intuitive.

[^22]:    ${ }^{90}$ This is, in fact, the quintessential difference between matter-particles and energy carriers such as photons (for the electromagnetic force) and neutrinos (for the strong(er) force).
    ${ }^{91}$ See, for example, Feynman's discussion of using photons to try to detect electrons as part of the two-slit experiment.

[^23]:    ${ }^{92}$ See, for example, our paper on Relativity, Light, and Photons.
    ${ }^{93}$ Just so you can imagine what we are talking about, this is short-wave ultraviolet light (UV-C). It is the light that is used to purify water, food or even air. It kills or inactivate microorganisms by destroying nucleic acids and disrupting their DNA. It is, therefore, harmful. Fortunately, the ozone layer of our atmosphere blocks most of it.

[^24]:    ${ }^{94}$ The calculation of the angle of the outgoing photon involves a different formula, which the reader can also look up from any standard course. See, for example, Prof. Dr. Patrick R. LeClair's lecture on Compton scattering, which we referenced already. The reader should note that the $1-\cos \theta$ is equal to -1 for $\pm \pi$, and that there is no change in wavelength for $\theta=0$, which is when the photon goes straight through, in which case there is no scattering.
    ${ }^{95}$ Arthur Compton actually did not fire photons into free electrons but into electron shells bound into atoms: it is only because the binding energy between the nucleus and the orbital electron is much lower than the energy of the X -ray photons that one could think of the electrons as being free. In fact, the experiment knocked them out of their orbitals!

[^25]:    here and now). If you are reading this footnote, it means you are damn serious in trying to figure this out-so, yes, please do email me if you have some ideas here. I promise I will reply. The point that I want to make here is this: the first relation above models the dynamics of Maxwell's equations. The second just states the principle of charge conservation. Combine that with the idea of various modes of an oscillation in space and in time, and... Well... That's as close as I got to modelling the Great Mystery in Physics.

[^26]:    ${ }^{101}$ The $h=\mathrm{E} \cdot \mathrm{T}$ and $h=\mathrm{p} \cdot \lambda$ are complementary but equivalent descriptions of the electron in the ring current model.
    ${ }^{102}$ The reader will immediately recognize this is just the same mechanism as the (statistical) mechanism which makes chaos theory comprehensible and, therefore, rational and logically true.
    ${ }^{103}$ The illustration is taken from Wikimedia Commons under the CC-BY-SA 2.5 license so I need to acknowledge the author: Xavier Snelgrove.
    ${ }^{104}$ The reader will know that - in order to separate particles based on their magnetic moment being up or down the magnetic field cannot be homogenous: a field gradient is necessary so as to produce an actual force on the particle. We can, therefore, not prove that an electron - or whatever other elementary particle with spin - will effectively line up in a homogenous magnetic field as well. In fact, we should note something which may surprise the reader: an actual Stern-Gerlach experiment has actually never been done with electrons-or with other

[^27]:    ${ }^{107}$ We are very much aware of our rather arrogant tone here. I am happy to get other opinions here. To be sure, I would treat them as being philosophical rather than scientific-as philosophical as Einstein's or our own intuition here. However, that should not prevent one from having a healthy logical discussion on scientific matters.

[^28]:    ${ }^{108}$ As mentioned in the introduction, we rather randomly googled the following two papers:
    https://arxiv.org/abs/1512.06681 and https://arxiv.org/abs/1810.11455. You may get others. This input-output calculus approach allows for endless refinements and, hence, endless research - which is always nice from an academic point of view, of course.

