# Do Gravitational Waves Exist in Newton's Universe? 

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#### Abstract

This paper shows that gravitational waves in the universe emanate from Newton's laws alone without considering General Relativity, but take into account Mach's principle.


Key words: Newton's laws and the universe, action-at-a-distance, Mach's principle, Gravitational waves, Cosmology

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## 1. Introduction

A previous paper demonstrated that one can already derive the results of the Special Theory of Relativity (SR) on the basis of Newton's Laws ${ }^{11}$. To do this, one must only assume the existence of the universe and apply Mach's principle. One can conclude that the action-at-a-distance-principle of Newton's law of gravity is not in contradiction to a maximum speed at which massive bodies can move. This maximum velocity is determined by the velocitydependence of the mass of a solid body, and is given as in SR by (equation (3.7) in ${ }^{11}$ )

$$
\begin{equation*}
\mathrm{m}=\frac{\mathrm{m}_{0}}{\sqrt{1-\frac{\mathrm{v}^{2}}{\mathrm{~b}_{0}^{2}}}} . \tag{1.1}
\end{equation*}
$$

The quantity of $b_{0}$ can be derived from fundamental parameters of the universe. It is not a natural constant, but depends (among other things) on the position of the moving body within the universe. For a mass positioned at the center of the universe, we found the following relation for this dependence ((4.3) in paper ${ }^{11}$ ):

$$
\begin{equation*}
b_{o}=\sqrt{2 \pi G \rho R_{o}^{2}} \tag{1.2}
\end{equation*}
$$

where $G$ is the gravitational constant, $\rho$ is the density of the universe, and $R_{0}$ is its radius. If the mass is positioned at a distance $r$ from the center of the universe, $b_{0}$ (or now better $b$ ) shows a dependence on this distance.

The value of $b_{0}$ has the dimension of a velocity, and in ${ }^{1)}$ many things indicate that this velocity is identical with the speed of light. However, when deriving the equations in ${ }^{11}$, light and the speed of light do not appear. Electric and magnetic mechanisms are not involved in the phenomena, they play absolutely no role. Therefore, the obvious question is which kind of velocity $b_{o}$ represents and how a possible correlation with the speed of light can be determined. If for $b_{o}$ a physically meaningful and measurable relevance as velocity can be found, then, according to (1.2), this must be in relation to the gravitational constant $G$ and the universal quantity $\rho$ as well as depend upon the extent of the universe $R_{0}$. The latter requirement appears strange at first glance, because it implies that a physical value at a
specific location, i.e. a local value, should depend on the overall extent of the universe. On the other hand, this is not exactly surprising because the theory described in paper ${ }^{1)}$ is based on Newton's action-at-a-distance theory of gravity. The gravitational force acts instantaneously and everywhere, even across the largest distances. The evaluation on the basis of this premise then shows that one must differentiate between the distance dependency of a force and the distance dependency of its impact. This is the required conclusion when one (according to Mach) considers the entirety of all masses in the universe.

If $b_{0}$ on the one hand is to be considered identical to the speed of light and, on the other hand, dependent on gravitation, then we must obviously look for a system that allows oscillations, and that is determined by gravitation as well as by electromagnetic fields. In the previous paper, we presented an initial overview of this. We will take a further step towards this goal here.

## 2. Gravitational Waves in the Scope of Newton's Laws

First let us step back from the previous, very simplified model representation in the paper ${ }^{1)}$, in which we have viewed the universe as static. In other words, a universe that we viewed as existing of spatially fixed masses ("fixed stars"). Let us now look at it more as an entity in which individual volume elements can move against each other, this also corresponds to reality, and call up the properties of the next fundamental simplest oscillating object consisting of point masses, namely that of a linear chain. In the generally known form, this chain consists of individual point masses of size $m$ that are located along a straight line at a distance of a to each other. In the simplest model, they are coupled linearly by springs with the spring constant $D$.

If we only consider longitudinal displacements and a force effect only between the adjacent neighbors, then the displacement $\mathrm{s}_{\mathrm{n}}$ affects the adjacent masses by the forces

$$
\begin{align*}
& F_{n, n+1}=D\left(s_{n+1}-s_{n}\right)  \tag{2.1}\\
& F_{n-1, n}=D\left(s_{n-1}-s_{n}\right) . \tag{2.2}
\end{align*}
$$

Thus the equation of motion is

$$
\begin{equation*}
m \ddot{s}_{n}=D\left(s_{n+1}+s_{n-1}-2 s_{n}\right) . \tag{2.3}
\end{equation*}
$$

We find wave equations using the following ansatz

$$
\begin{equation*}
\mathrm{s}_{\mathrm{n}}=\mathrm{s}_{0} \mathrm{e}^{-\mathrm{i}(\mathrm{kan}-\omega \mathrm{t})} . \tag{2.4}
\end{equation*}
$$

The dispersion relation is given by

$$
\begin{equation*}
\omega(\mathrm{k})=2 \sqrt{\frac{\mathrm{D}}{\mathrm{~m}}}\left|\sin \left(\frac{\mathrm{ka}}{2}\right)\right| . \tag{2.5}
\end{equation*}
$$

Let us now consider a chain of linearly aligned masses, separated (at the beginning of the description) by the distance a that do not affect each other with spring forces but which mutually attract each other according to Newton's law of gravity. If we then again take into consideration only the immediate neighbors and only longitudinal displacements, the gravitational force on the $n$-th mass is given by:

$$
\begin{equation*}
K_{n, n \pm 1}= \pm G \frac{m^{2}}{\left(a \pm\left(s_{n \pm 1}-s_{n}\right)\right)^{2}} \tag{2.6}
\end{equation*}
$$

Assuming that the (longitudinal) displacements are much smaller than the distances a, then we can write approximately

$$
\begin{equation*}
K_{n, n \pm 1}= \pm G \frac{m^{2}}{a^{2}}\left(1 \mp 2\left(\frac{s_{\mathrm{n} \pm 1}-s_{n}}{a}\right)\right) . \tag{2.7}
\end{equation*}
$$

With the abbreviation $D_{1}=2 G \frac{m^{2}}{a^{3}}$, the equation of motion is

$$
\begin{equation*}
m \ddot{s}_{n}=-D_{1}\left(s_{n+1}+s_{n-1}-2 s_{n}\right) . \tag{2.8}
\end{equation*}
$$

The change in the "equilibriums distances" a is left out of consideration here. We see these distances as practically static as compared to the changes in the displacements $\mathrm{s}_{\mathrm{n}}$. Furthermore, we have assumed that the velocities $\dot{s}_{n}$ are considerably slower than the light velocity. This means we can set $m \approx m_{0}$. All processes for which the speed of light is a considerable factor are therefore not considered, just as the non-linear processes that we have excluded with the linearization (2.7).

Compared to (2.3), in (2.8) now a negative sign is before the coupling constant, and the ansatz (2.4) in this case leads to purely imaginary values for $\omega(\mathrm{k})$ :

$$
\begin{equation*}
\omega(k)=i 2 \sqrt{\frac{\mathrm{D}}{\mathrm{~m}}} I \sin \left(\frac{\mathrm{ka}}{2}\right) \mathrm{l} . \tag{2.9}
\end{equation*}
$$

Longitudinal wave propagation is obviously not possible on a chain of this type.
Next let us look at transversal waves in a linear chain whose mass points affect each other with next-neighbor gravitational forces. The displacements of the nth mass vertical to the chain line (for example, in the $z$ direction) are designated again with $s_{n}$. For the values of the attractive forces between adjacent masses, the following applies:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{n}, \mathrm{n}} \pm_{1}=\mathrm{G} \frac{\mathrm{~m}^{2}}{a^{2}+\left(\mathrm{s}_{\mathrm{n} \pm 1}-s_{\mathrm{n}}\right)^{2}} . \tag{2.10}
\end{equation*}
$$

The z components of these forces are given by

$$
\begin{equation*}
\mathrm{K}_{\mathrm{n}, \mathrm{n}} \pm_{1, \mathrm{z}}=\mathrm{G} \frac{\mathrm{~m}^{2}}{a^{2}+\left(s_{\mathrm{n} \pm 1}-s_{\mathrm{n}}\right)^{2}} \frac{\mathrm{~s}_{\mathrm{n} \pm 1}-\mathrm{s}_{\mathrm{n}}}{\sqrt{a^{2}+\left(\mathrm{s}_{\mathrm{n} \pm 1}-s_{\mathrm{n}}\right)^{2}}} . \tag{2.11}
\end{equation*}
$$

If we once again ignore terms of the order $s^{2}$ and higher and continue to assume $m=m_{0}$, we have the following equation of motion:

$$
\begin{equation*}
m \ddot{s}_{n}=D_{2}\left(s_{n+1}+s_{n-1}-2 s_{n}\right) \tag{2.12}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathrm{D}_{2}=1 \mathrm{G} \frac{\mathrm{~m}^{2}}{\mathrm{a}^{3}} \tag{2.13}
\end{equation*}
$$

The form of this equation is identical to that of equation (2.3) for a chain with elements that are connected together with linear spring forces. When we now stack these chains next to and over each other, we get a lattice that should permit transversal waves as described in the scope of lattice theory in solid state physics.

In the further consideration, we will always limit ourselves to displacements $s_{n} \ll a$. Then there is no passage of a mass from a chain $n$ through the adjacent chain $n+1$ and we can neglect this. In a forthcoming paper, we will try to avoid this limitation. Instead of the simple, linear equation of motion (2.12), we will then have to apply the Frenkel-Kontorova equation, or, if we move to continuous fields, the Sine-Gordon equation ${ }^{314)}$. A detailed description of the relationships and further literature citations can be found in ${ }^{5)}$.

As we know from lattice dynamics, we can describe an oscillating body with mass density $\rho$ (here the universe) by using a continuum model. For reasons of clarity and for the best possible physical understanding of the relationships, we will first continue to consider the point mass lattice model as we did before. We select very large volume elements and then assume that the mass contained within is always placed in the center of the volume element. The volume elements should be so large that the mass density approximately corresponds with the constant density that is found in the universe, experimentally. To prevent misunderstandings: The point masses defined in this manner are not identical to the actual mass accumulations that can be found in the universe, such as stars or galaxies. Rather these stand for the summation of very large numbers of such "real" masses, which are fictively located in the centers of gravity of the selected, very large volume elements.

On the basis of this model conception, we shall now examine the oscillation and wave properties of the universe. In this case, as mentioned previously, we ignore the uniform translational motions in relation to possible wave motions and restrict our focus to the examination of the linearized equation (2.12). Of course, expansion or contraction motions of the universe take place concurrently. Due to the linearization that was undertaken, we assume that these can be described separately and that they do not have a relevant influence on the wave processes to be examined. Therefore, we set a constant distance a between the mass points on the lattice, we are thus observing a "snapshot".

Next we will examine the special case of possible plane waves. Let us think of the universe as divided into discs, for example, so that the disc surfaces are on the $y$-z axis and thus the surface normal points in the $x$ direction. Such a wave that is described by the oscillations of these discs as vertical to the $x$ axis can arguably be seen in very good approximation as a "plane wave", although $R_{0}$ is, of course, not infinitely large. A disc that goes through the origin of the universe, an "equator disc", is characterized by the thickness a and the surface $\pi R_{0}{ }^{2}$. If we first consider only discs in the vicinity of the equator, then their surfaces also approach $\pi R_{0}{ }^{2}$. We label a disc that is at the distance na from the origin with the index $n$, and a shift of the disc in the $z$ direction with $s_{n}$. If we start from the assumption that all volume elements on an $n$ dh disc move in the $z$ direction with almost the same deflection $s_{n}$, thus "collectively" oscillate in the $z$ direction, then we can describe the dynamics of the n adjacent discs in the universe with an (almost) plane wave that propagates in the $x$ direction. This has field components $s_{n}$ of equal size in the $z$ direction overall in the $y$-z plane, however not to infinity, but "only" within the radius $\mathrm{R}_{0}$ of the disc. Due to the linearity of the equations of
motion, we can then of course describe any arbitrary other form of a wave propagation in the $x$ direction by a superposition of such "plane waves" to any wave package. How to stimulate such collective disc oscillations will be examined in a forthcoming publication ${ }^{2)}$. Here we take such a stimulation for granted. The force effect of two discs $n$ and $n+1$ on each other is shown in figure 1.


Fig. 1: Gravitational forces between mass elements of adjacent discs

We see the mass elements $\mathrm{dm}_{\mathrm{n}}=\rho \operatorname{ardr} \mathrm{d} \varphi$ (in the disc n ) and $\mathrm{dm}_{\mathrm{n}+1}$ (in the disc $\mathrm{n}+1$ ) as rigid bodies, but not the discs themselves. For the magnitude of the force between the center of the mass element $\mathrm{dm}_{\mathrm{n}}$ in the disc n and the center of the mass element $\mathrm{dm}_{\mathrm{n}+1}$ in the disc $n+1$, the following relationship applies

$$
\begin{equation*}
\mathrm{dK}_{\mathrm{n}, \mathrm{n}+1}=\mathrm{G} \frac{(\rho \mathrm{a})^{2} \mathrm{rdrd} \mathrm{~d} \mathrm{r}^{\prime} \mathrm{dr}^{\prime} \mathrm{d} \varphi^{\prime}}{\mathrm{a}^{2}+\mathrm{r}^{\prime 2}} . \tag{2.14}
\end{equation*}
$$

The force components in the x direction are given by $\mathrm{dK}_{n, n+1, \mathrm{x}}=\mathrm{dK}_{\mathrm{n}, \mathrm{n}+1} \cos \vartheta$, whereby

$$
\begin{equation*}
\cos \vartheta=\frac{\mathrm{a}}{\sqrt{\mathrm{a}^{2}+\mathrm{r}^{\prime 2}}} . \tag{2.15}
\end{equation*}
$$

Thus

$$
\mathrm{K}_{\mathrm{n}, \mathrm{n}+1, \mathrm{x}}=\mathrm{G}(\rho \mathrm{a})^{2} \operatorname{ardrd} \varphi \int_{0}^{\mathrm{R}_{0}} \frac{\mathrm{r}^{\prime} \mathrm{dr}^{\prime}}{\left(\mathrm{a}^{2}+\mathrm{r}^{\prime}\right)^{3 / 2}} \int_{0}^{2 \pi} \mathrm{~d} \varphi^{\prime}
$$

or, after integration and considering that $\mathrm{a} \ll \mathrm{R}_{0}$ :

$$
\begin{equation*}
\mathrm{K}_{\mathrm{n}, \mathrm{n}+1, \mathrm{x}}=\mathrm{G}(\rho \mathrm{a})^{2} \operatorname{ardrd} \varphi \frac{1}{\mathrm{a}} 2 \pi . \tag{2.16}
\end{equation*}
$$

One can see that only those mass elements of $\mathrm{dm}_{n+1}$ provide contributions for which $\mathrm{r}^{\prime}$ is not much larger than the distance a between the middle surfaces of the two discs.

If we now also integrate across a sub-area of the left disk, namely from $r=0$ to $r=a$, and across the entire angle $\varphi^{\prime}$ from 0 to $2 \pi$, then we see:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{n}, \mathrm{n+1}, \mathrm{x}}^{\prime}=2 \pi \mathrm{G}(\rho \mathrm{a})^{2} \pi \mathrm{a}^{2} . \tag{2.17}
\end{equation*}
$$

We can write this relation in the following form:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{n}, \mathrm{n}+1, \mathrm{x}}^{\prime}=2 \mathrm{G} \frac{\rho \mathrm{ama}^{2} \rho a \pi \mathrm{a}^{2}}{\mathrm{a}^{2}} . \tag{2.18a}
\end{equation*}
$$

This is identical to the description of the force between two point masses of the size $\mathrm{dm}=$ $\rho а \pi \mathrm{a}^{2}$ at a distance of a on the x-axis, however with an "effective" gravitation constant of G ' = 2 G. This reflects the attraction force between two sub-areas $\mathrm{dF}=\pi \mathrm{a}^{2}$ that face each other in the x -axis on the two vertical discs n and $\mathrm{n}+1$.

We determine the entire force between the two disks by stacking the sub-masses next to and on top of each other, whereby now none of the force components in the $y$ and $z$ direction are to be considered anymore (these are already contained in the "effective" gravitational constant $\mathrm{G}^{\prime}$ ). Therefore, in (2.18a) we replace the sub-sections dF with the complete area of the discs $F=\pi R_{0}{ }^{2}$. This means we replace the sub-masses $d m=\rho a \pi a^{2}$ with $M_{s}=\rho a \pi R_{0}{ }^{2}$ :

$$
\begin{equation*}
K_{n, n+1, x}^{\prime \prime}=2 G \frac{M s M s}{a^{2}} . \tag{2.18b}
\end{equation*}
$$

Now, when we consider a shift of the discs $n$ and $n+1$ against each other in the $z$ direction, we find (similar to (2.11)):

$$
\begin{equation*}
\mathrm{K}_{\mathrm{n}, \mathrm{n}}^{\prime \prime} \pm_{1,2}=2 \mathrm{G} \frac{(\mathrm{Ms})^{2}}{a^{2}+\left(s_{\mathrm{n} \pm 1}-s_{\mathrm{n}}\right)^{2}} \frac{s_{\mathrm{n} \pm 1^{-}}-s_{\mathrm{n}}}{\sqrt{a^{2}+\left(s_{\mathrm{n} \pm 1}-s_{\mathrm{n}}\right)^{2}}} \tag{2.19}
\end{equation*}
$$

Just as in (2.11) to (2.13) we see the following, if $s_{n \pm 1}-s_{n} \ll a$ :

$$
\begin{equation*}
M_{s} \ddot{s}_{n}=D_{2}^{\prime}\left(s_{n+1}+s_{n-1}-2 s_{n}\right) \tag{2.20a}
\end{equation*}
$$

or
with

$$
\begin{align*}
& \ddot{s}_{n}=D^{‘}\left(s_{n+1}+s_{n-1}-2 s_{n}\right)  \tag{2.20b}\\
& D^{‘}=2 G \frac{\rho a \pi R_{0}{ }^{2}}{a^{3}} . \tag{2.21}
\end{align*}
$$

With the wave ansatz (2.4), this results in:

$$
\begin{equation*}
\omega=\frac{2}{\mathrm{a}} \sqrt{2 \pi \rho \mathrm{GR}_{0}^{2}}{ }^{2} \sin \frac{\mathrm{ka}}{2} . \tag{2.22}
\end{equation*}
$$

And the group velocity is then:

$$
\begin{equation*}
\frac{\mathrm{d} \omega}{\mathrm{dk}}=\sqrt{2 \pi \rho \mathrm{GR}_{0}{ }^{2}} \cos \frac{\mathrm{ka}}{2} . \tag{2.23}
\end{equation*}
$$

Maximum values (also negative) of $\frac{\mathrm{d} \omega}{\mathrm{dk}}$ result when $\mathrm{ka} \ll 1$, or in general when

$$
\begin{equation*}
\mathrm{ka}=\mathrm{n} 2 \pi . \tag{2.24a}
\end{equation*}
$$

In this case

$$
\begin{equation*}
\frac{\mathrm{d} \omega}{\mathrm{dk}}=\mathrm{b}_{0} . \tag{2.25}
\end{equation*}
$$

In comparison to the group velocity of the linear chain described by (2.12), the value for the parallel oscillating disc group velocity is higher by a factor of $\sqrt{2}$.

The relations (2.22) to (2.25) are exactly identical as in a crystal lattice with (linear) spring forces. However, for every crystal, the value a is set, whereby we have left this undetermined here so far. The possible consequences of this will be discussed later in this paper.

Even solely on the basis of Newton's laws, it appears that "gravitational waves" are possible in the universe. This is actually not surprising when we simply consider a space volume that is large enough. We then have a number of gravitationally interacting spatial objects that are represented in our simple model by a discrete, oscillating lattice of mass points. There is a maximum group velocity of the possible waves limited by the extent of the universe $\mathrm{R}_{0}$, given by (2.23), and in the proximity of the origin it corresponds with the amount $b_{0}$ for which we have found indications in the paper ${ }^{1)}$ that this could be identical with the speed of light c . As a very essential result of this paper ${ }^{1}$, we also discovered that a body that moves through the "universal gas" - unlike with a normal gas - cannot reach a velocity larger than $b_{0}$. Here there is no "supersonic speed". Each body is a "masson", and its speed is limited by the relation (1.1).

In addition, we have now found another result of particular interest, namely that only transversal wave propagation is possible (cf. to the result (2.9) for a longitudinal wave approach). Contrary to a solid body that is modeled by spring forces, in the universe with its gravitational forces there is only one "sound velocity"!

Independent of the question as to when these universal sound waves can be excited and whether and how there is a correlation between them and electromagnetic waves ("light"!) (this is examined in ${ }^{2}$ ), we will now examine the mathematical description of the "universal lattice" in more depth. In doing so, we will go beyond the previous approximation where we have only considered the effect of immediately adjacent discs on each other. Due to the long-distance nature of gravitation, this approximation is possibly too rough.

Let's stay with the model of a discrete three-dimensional lattice of mass points and continue to examine the special case that discs only oscillate in directions parallel to their disc surface. However let's now not only consider the interaction between directly adjacent discs, but also the interaction between the disc $n$ and all discs $n+1$ and $n-l$, respectively. We envision the nth disc again as the "equator disc" near the origin of the universe. Instead of (2.19), for a disc that is at a distance of la from the nth disc, we get the following:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{n} \pm 1, \mathrm{n}, \mathrm{Z}}=\mathrm{G} \frac{\left(\mathrm{M}_{1}\right)^{2}}{(\mathrm{la})^{3}}\left(\mathrm{~s}_{\mathrm{n} \pm 1}-\mathrm{s}_{\mathrm{n}}\right)+\mathrm{O}\left(\frac{\left(\mathrm{~s}_{\mathrm{n} \pm 1}-\mathrm{s}_{\mathrm{n}}\right)^{3}}{(\mathrm{la})^{5}}\right) . \tag{2.26}
\end{equation*}
$$

If we neglect the non-linear terms, we get the equation of motion
with

$$
\begin{equation*}
\ddot{\mathrm{s}}_{\mathrm{n}}=\gamma \sum_{\mathrm{l}=1}^{\mathrm{l}_{\max }} \frac{\mathrm{M}_{1}}{1^{3}}\left(\mathrm{~s}_{\mathrm{n}+1}+\mathrm{s}_{\mathrm{n}-1}-2 \mathrm{~s}_{\mathrm{n}}\right) \tag{2.27}
\end{equation*}
$$

$$
\begin{equation*}
\gamma=\frac{2 \mathrm{G}}{\mathrm{a}^{3}} \tag{2.28}
\end{equation*}
$$

The mass of the disc at the distance la from the equator disc is given by

$$
\begin{equation*}
M_{1}=\pi \rho a\left(R_{0}^{2}-l^{2} a^{2}\right) \tag{2.29}
\end{equation*}
$$

We now find wave solutions using the following slightly changed ansatz

$$
\begin{equation*}
s_{n+1}=s_{0} e^{-\mathrm{i}(\mathrm{ka}(\mathrm{n}+\mathrm{l})-\omega \mathrm{t})}, \tag{2.30}
\end{equation*}
$$

and we arrive at

$$
\begin{equation*}
\omega^{2}=4 \gamma \pi \rho a \mathrm{R}_{0}^{2} \sum_{\mathrm{l}=1}^{\mathrm{l}_{\max }} \frac{\left(1-\mathrm{I}^{2} \frac{\mathrm{a}^{2}}{\left.\left.\mathrm{R}_{0}\right)^{2}\right)}\right.}{\mathrm{I}^{3}} \sin ^{2} \frac{\mathrm{kla}}{2} . \tag{2.31}
\end{equation*}
$$

The maximum value for $l=l_{\text {max }}$ is determined by the value of $a$ :

$$
\begin{equation*}
l_{\max }=\frac{R_{0}}{a} . \tag{2.32}
\end{equation*}
$$

Therefore, the following always applies

$$
\begin{equation*}
1-l^{2} \frac{\mathrm{a}^{2}}{\mathrm{R}_{0}{ }^{2}} \leq 1 \tag{2.33}
\end{equation*}
$$

Therefore, the sum in (2.31) is not the sum of a harmonic series. Rather, the series elements are reduced by the numerator to about zero when $I$ approaches $I_{\text {max }}$. For this reason, but also because $\sin ^{2} \frac{\text { kla }}{2} \leq 1$ for all kla, converges the series.

In our model, the distance a between two discs is a parameter with a numerical value that has not yet been determined. Therefore the value of (2.31) is also not determined numerically. We must therefore keep an eye out for a physical principle that determines the value.

To simplify the formulae, we first define the following:

$$
\begin{equation*}
\mu=\mathrm{ka} . \tag{2.34}
\end{equation*}
$$

Then we can rewrite (2.31):

$$
\begin{equation*}
\omega=\sqrt{2 \mathrm{G}^{2} \mathrm{R}_{0}^{2}} \sqrt{4 \sum_{\mathrm{l}=1}^{\mathrm{l}_{\max }} \frac{\left(1-\frac{\mathrm{l}^{2}}{1_{\max }{ }^{2}}\right)}{\mathrm{I}^{3} \mu^{2}} \sin ^{2} \frac{\mathrm{l} \mu}{2}} \mathrm{k} . \tag{2.35}
\end{equation*}
$$

If we only consider regions that are not too far away from the "equator disc", then the following applies:

$$
\begin{equation*}
\omega=\sqrt{2 \pi \mathrm{GRR}_{0}^{2}} \sqrt{4 \sum \frac{1}{1^{3} \mu^{2}} \sin ^{2} \frac{\mathrm{l}}{2}} \mathrm{k}=\sqrt{2 \pi \mathrm{G} \mathrm{\rho R}_{0}^{2}} \mathrm{f}_{\mathrm{G}} \mathrm{k} . \tag{2.36}
\end{equation*}
$$

For the group velocity, this results in:

$$
\begin{equation*}
\frac{\mathrm{d} \omega}{\mathrm{dk}}=\mathrm{b}_{0} \frac{\sum_{\frac{1}{1^{2}} \sin \mu}^{\sqrt{4 \sum_{1^{3}} \sin ^{2} \frac{1 \mu}{2}}} . . . . . ~}{\text {. }} \tag{2.37}
\end{equation*}
$$

The quotient in (2.37) only depends on $\mu$. Since we have not yet set the value of a in our lattice model, the value of $\mu$ is also not determined, even if we set the value of k . However, as a consequence of the existence of a maximum speed of $b_{0}$ for a moving mass (see ${ }^{1)}$ (3.10) or (3.11)), there must be the same maximum speed for a wave packet (that can carry mass or energy). Otherwise, a wave packet would be able to move faster than with $\mathrm{b}_{0}$. Consequently, we can determine $\mu$ by asking for which value of $\mu$ the size of the quotient in
(2.37) equals 1. Again, this condition is fulfilled for $\mu \ll 1$ or in general if the relation (2.24a) is valid:

$$
\begin{equation*}
\mu \approx \mathrm{n} 2 \pi \tag{2.24b}
\end{equation*}
$$

In consideration of the propagation velocity of the gravitational waves, it apparently does not play a role whether the gravitative interaction only affects immediately adjacent discs, but also discs that are far apart.

The values described by (2.24b) are however not the only solutions for $\mu$. With a numerical calculation of the series in (2.37) (using ${ }^{6}$ ), we can discover (by trial and error), that $\mu=0.7$ is also a possible value. We can possibly answer whether this also makes physical sense with additional consideration. There is actually a second possibility to determine the value of $\mu$. If we assume the validity of Fermat's Principle, then we can ask for which value $\mu$ the group velocity reaches a maximum value. Therefore, the following would be required:

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{~d} \mu}\left(\frac{\mathrm{~d} \omega}{\mathrm{dk}}\right)=0 . \tag{2.38}
\end{equation*}
$$

If we insert (2.37) here, then after a few transformations the following results:

$$
\begin{equation*}
4 \sum \frac{1}{1^{3}} \sin ^{2} \frac{1 \mu}{2} \sum \frac{1}{1} \cos l \mu=\left(\sum \frac{1}{1^{2}} \sin l \mu\right)^{2} . \tag{2.39}
\end{equation*}
$$

We see immediately that (2.39) is fulfilled, if $\mu \ll 1$ or again if

$$
\begin{equation*}
\mu=\mathrm{n} 2 \pi . \tag{2.24c}
\end{equation*}
$$

In this case, the numerical examination with the help of ${ }^{6}$ also returns the value (2.24c). The numerical value of $\mu=0.7$ determined above is not returned here, and therefore this can be presumably physically excluded. However, we are not going to examine this further.

In a crystal lattice, a is a fixed value. Therefore, in the description of the physics of crystal lattices, it is sufficient to limit this to the first Brillouin zone. However, in our description of the universe with a lattice model, a is not determined initially. The requirements ( $2.24 a-c$ ) thus permit various values for a. However, as a result of the existence of a maximum speed for the velocity of masses, these values are restricted. This is a novel and surprising aspect that can potentially be traced back to the (too) simple construction of our lattice model. However, if this was not the case, and this limitation of $\mu$ were actually determined by the properties of the universe, then this limitation of the product of a space differential and the wavenumber would show a certain similarity to Heisenberg's uncertainty relation. Of course it is an interesting question as to whether there is a connection here; however this is a question that we will not follow up on but will leave for further research.

It must still be examined whether there is a lowest value for a, thus whether a can approximate zero. In our model, a is the distance between the middle layers of two "equator discs" (and this therefore simultaneously describes the thickness of a disc). The mass of a disc is given by $M_{s}=\rho a \pi R_{0}{ }^{2}$. If a were to approach zero, then the total mass of a disc would also approach zero. To be able to assume the average value of the density of the universe for $\rho$, we certainly need a certain minimum volume and therefore a certain minimum mass of a disc. If our theory is to also apply for waves with a wave length of $\gamma$-radiation ( $\lambda \approx 10^{-12} \mathrm{~cm}$ ) then from $\mu \ll 1$ the following would be required:

$$
\begin{equation*}
\mathrm{a} \ll 0,1 \quad 10^{-12} \mathrm{~cm} . \tag{2.40}
\end{equation*}
$$

If, for example $a=10^{-15} \mathrm{~cm}$, then this would result in (with $\rho=5 \times 10^{-30} \mathrm{~g} / \mathrm{cm}^{3}$ and $\mathrm{R}_{0}=2 \times 10^{28}$ cm, see ${ }^{11}$ )

$$
\begin{equation*}
M_{s}=6 \times 10^{14} \mathrm{~g} . \tag{2.41}
\end{equation*}
$$

This is a considerable mass, although the disc according to (2.40) is very thin. Our model assumption that we are viewing this mass as evenly distributed over the disc does not appear to be falsified by this, but of course also not as proven.

When we move from the model of a discrete lattice to a continuum model, meaning instead of considering the displacements $s_{n}$ at the lattice point $n$ we consider the field $s(x)$, the equation of motion (2.26) transitions to a wave equation for $s(x){ }^{5)}$, namely for the propagation of transversal waves with the "sound velocity" $\mathbf{b}_{0}$. The parameter $\mu$ no longer appears, the determination and containment of this is therefore no longer needed. However, we must now examine whether the transformation from the discrete lattice to the continuum can be performed at all in the usual manner (see also ${ }^{5}$ ).

## 3. Summary and Outlook

Our investigation has shown that gravitational waves are possible even solely in a universe that is described exclusively on the basis of Newton's mechanics and Newton's law of gravity. If we take into account "distant masses" (Mach's principle), we can see that a "displacement field" is possible for centers of mass in sufficiently large space volumes. Contrary to Newton's field of gravity itself, this displacement field indicates local degrees of freedom. The general theory of relativity (GR) is not required for this. The theory used as a basis here (see ${ }^{1)}$ ) does not contradict the special theory of relativity (SR) nor the principle of equivalence. We cannot compare the results with those from the GR since this paper only considers a universe of uniformly distributed masses with low speeds. It cannot describe phenomena with unequally distributed masses or velocities near the speed of light. It is possible to expand the theory to these cases as well and was already demonstrated to some degree in the paper ${ }^{11}$. Of course, it is still to be clarified whether the theory is compatible with the GR after an expansion. The clarification of this question is far beyond the scope of this paper and will be left for further research.

However, in further research it would be very interesting to examine how to describe a solid sample mass that can move over larger distances ( $x>$ a) and distort the surrounding lattice. The presence of such a particle can possibly be seen as an "impurity" in this lattice and as a result would be treated with the methods to describe the movement of impurities in solidstate crystals. The fundamental start point for this treatment is the Frenkel-Kontorova or the Sine-Gordon equation. These equations have been used in the detailed examination of dislocation shifts in crystal lattices. The "particle properties" of stable propagation solutions from the Sine-Gordon equation (solitons, breathers) are related with their "wave properties". With a view to the theory developed here, it seems that there is necessarily a universal wave-particle correlation that is present at every position in the universe. There is reason to hope that basic quantum phenomena could be explained by this model of the universe, namely that the universe shows properties of an oscillating lattice.

The formal relationship of many solution properties for an oscillating lattice to the theory of relativity was pointed out early (cf. the extensive and detailed presentation by Günther ${ }^{5}$ ). We "only" need to transfer these results to the conditions in the universe. Additionally, we must take into account the results in ${ }^{1)}$ and in this paper, which show that the universe can (or must?) be considered as a "lattice". Whereby, instead of the spring forces in a crystal, the forces of gravity are to be set between masses. (However, these masses cannot be equated with the galactic individual masses such as stars or galaxies, but appear as foci of large volume elements of the universe).

## 4. References

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