Refutation of Ruitenburg theorem

Abstract: We evaluate the an equation of the geometric component of the semantic proof of Ruitenburg's theorem as not tautologous, to refute the conjecture. These results form a non tautologous fragment of the universal logic $\mathcal{V}_4$.

We assume the method and apparatus of Meth8/$\mathcal{V}_4$ with Tautology as the designated proof value, $F$ as contradiction, $N$ as truthity (non-contingency), and $C$ as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Let

\[
\begin{align*}
\sim & \text{ Not, } \neg ; \\
+ & \text{ Or, } \lor, \cup ; \\
- & \text{ Not Or; } \\
\& & \text{ And, } \land, \cap, \cdot, \otimes ; \\
\setminus & \text{ Not And; } \\
> & \text{ Imply, greater than, } \rightarrow, \Rightarrow, \supset, \rightarrow ; \\
< & \text{ Not Imply, less than, } \epsilon, \subset, \forall, \exists, \leftarrow, \leq ; \\
= & \text{ Equivalent, } \equiv, \equiv, \leftrightarrow, \Delta, \approx ; \\
\% & \text{ possibility, for one or some, } \exists, \exists, \diamondsuit, M ; \\
\# & \text{ necessity, for every or all, } \forall, \square, L ; \\
(z=z) & \text{ T as tautology, } T, \text{ ordinal 3; } \\
(%z>\#z) & \text{ F as contradiction, } \emptyset, \text{ Null, } \bot, \text{ zero;} \\
(%z<\#z) & \text{ N as non-contingency, } \Delta, \text{ ordinal 1; } \\
(~(y<x) & \text{ (x \leq y), (x \subseteq y, (x \subseteq y); (A=B) (A\sim B).} \\
\text{Note for clarity, we usually distribute quantifiers onto each designated variable.}
\end{align*}
\]


The goal of this talk is to supply a semantic proof of Ruitenburg Theorem.

The geometric component

As geometric environment, we consider the category $\mathcal{P}_0$ of finite rooted posets (with $p$-morphisms) and the category of sheaves over them with the canonical (Grothendieck) topology.

A poset $\langle P, \leq \rangle$ is rooted iff it has a greatest element $\rho P$.

$f : Q \rightarrow P$ is a $p$-morphism iff it is order-preserving and moreover satisfies the following condition for all $q \in Q, p \in P$

\[
p \leq f(q) \Rightarrow \exists q' \in Q (q' \leq q \& f(q') = p).
\]

Remark 49.1.2: Eq. 49.1.2 as rendered is not tautologous, to refute the conjectured equation of the geometric component of the semantic proof of Ruitenburg's theorem.