Solving the Flatness and Horizon Problems via Self-Organized Criticality

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Abstract

Self-organized criticality (SOC) is a universal mechanism for self-sustained critical behavior in large-scale systems evolving outside equilibrium. The trademark signature of SOC is two-fold: a) it occurs in complex ensembles of multiple interacting components and b) it is characterized by power-law distribution of "avalanche" sizes. This brief report suggests that both flatness and horizon problems of cosmology may be explained away through the universal features of SOC. The explanation stems from the so-called *finite scaling ansatz* (FSS) of SOC, which is a generic paradigm for the emergence of complexity in Nature. Our approach is straightforward and evades traditional solutions involving fine-tuning, particle horizons or inflation.

Key words: flatness problem, horizon problem, inflation, fine tuning, self-organized criticality, finite scaling ansatz, minimal fractal manifold.

1. From SOC to the minimal fractality of spacetime

Consider a large-scale system of size L undergoing a second-order phase transition. The transition is driven by the control parameter λ as it approaches the critical value λ_c. Near the critical point and for systems of infinite extent ($L \to \infty$), the correlation length ξ diverges as [10-12]

$$\xi \sim (\lambda - \lambda_c)^{-\nu} \; ; \; L \to \infty, \; \lambda \to \lambda_c \tag{1}$$

In the transition region, a relevant variable of the system is also a diverging quantity which scales as

$$A_\infty(\lambda) \sim |\lambda-\lambda_c|^{-\zeta} \; ; \; L\to\infty,\; \lambda\to\lambda_c \tag{2}$$

where ζ is a critical exponent. In what follows, we introduce the notation

$$\tau_s = -(\zeta/\nu) \tag{3}$$

There are two distinct cases associated with the power-law (2). If the size of the system greatly exceeds the correlation length, $L >> \xi$, by (1) and (2) we write

$$A_L(\lambda) \sim \xi^{-\tau_s} \; ; \; (L >> \xi,\; \lambda\to\lambda_c) \tag{4}$$

In the opposite case, $L << \xi$, the system size takes over the scaling behavior and (2) turns into

$$A_L(\lambda) \sim L^{-\tau_s} \; ; \; (L << \xi,\; \lambda\to\lambda_c) \tag{5}$$

Taken together, (4) and (5) define the *finite-size scaling* (FSS) ansatz [1, 10-11]

$$A_L(\lambda) = \xi^{-\tau_s}\Phi(L/\xi) \; ; \; (L\to\infty,\; \lambda\to\lambda_c) \tag{6}$$

where the scaling function controls the finite-size effects of critical behavior and is defined as

$$\Phi(x) = \begin{cases} const; & |x| >> 1 \\ x^{-\tau_s} \; ; & x\to 0 \end{cases} \tag{7}$$

To transition from the framework of critical phenomena to SOC, one simply identifies the correlation length with the concept of *avalanche-size*, i.e.,

$$s = \xi \; ; \quad s_{cr} = L \tag{8}$$

The probability distribution defining the FSS ansatz in SOC is a natural extrapolation of (6) and takes the form of a probability distribution [11]

$$\boxed{P(s,L) \sim s^{-\tau_s} \Phi\left(s/s_{cr} \right) \text{ for } s >> 1,\ L >> 1} \tag{9a}$$

$$\boxed{s_{cr}(L) \sim L^{D_0} \text{ for } L >> 1} \tag{9b}$$

in which τ_s and D_0 are called the *avalanche-size exponent* and the *avalanche dimension*, respectively. Quite generally, (9) shows that, for a system of finite extent and large size avalanches, the avalanche-size probability behaves as a fractal function times a generic scaling function. To enable all moments of (9) to exist, the scaling function must decay sufficiently fast. One obtains the following representation of the scaling function upon power expanding it around zero,

$$\Phi(x) \sim \begin{cases} \Phi(0) + \Phi'(0)x + \frac{1}{2}\Phi''(0)x^2 + ..., & x << 1 \\ \to 0, & x >> 1 \end{cases} \tag{10}$$

The avalanche-size probability must be normalized to unity and its average be diverging along with $L \to \infty$, which leads to the following constraints

$$\sum_{s=1}^{\infty} P(s;L) = 1 \qquad \text{for } L < \infty, \tag{11}$$

$$\langle s\rangle = \sum_{s=1}^{\infty} sP(s;L) \to \infty \text{ for } L \to \infty \tag{12}$$

Under the assumption that $\Phi(0) \neq 0$, the behavior of (9) for an infinite system size may be approximated as

$$\lim_{L\to\infty} P(s;L) \sim s^{-\tau_s}\Phi(0) \tag{13}$$

Furthermore, to comply with (11) and (12), the avalanche-size exponent must fall in the range

$$1 < \tau_s \leq 2 \tag{14}$$

We have extensively discussed in [6 - 9] the physical significance of the *minimal fractal manifold* (MFM), a spacetime continuum characterized by arbitrarily small and scale-dependent deviations from four dimensions ($\varepsilon = 4 - D << 1$). The MFM reflects an evolving setting that starts far-from-equilibrium and gradually reaches the equilibrium conditions mandated by field theory in the limit of four-dimensional spacetime ($\varepsilon = 0$). There are well-motivated reasons to believe that dimensional fluctuations driven by ε are asymptotically compatible with the internal structure and dynamics of the Standard Model of particle physics [6-9].

Based on these premises, we advance below the hypothesis that the dimensional deviation ε and the avalanche-size s are interchangeable concepts via

$$\varepsilon = 4 - D = s^{-1} << 1 \tag{15}$$

Furthermore, since ε flows with the energy scale, it likely reaches its uppermost observable value close to the formation of the cosmic microwave background (CMB) [13]. Thus, the maximal dimensional deviation is set to

$$\varepsilon_{cr} = \varepsilon_{\max} \approx 10^{-5} \; ; \; \varepsilon << \varepsilon_{cr} \tag{16}$$

which turns (9) into

$$P(\varepsilon, \varepsilon_{cr}) \sim \varepsilon^{\tau_s} \Phi(\varepsilon_{cr}/\varepsilon) \; , \; \varepsilon << 1 \tag{17a}$$

$$\varepsilon_{cr}(a) \sim a^{D_0} \, , \; a >> 1 \tag{17b}$$

where $a = a(t)$ is the scale factor describing the Universe expansion.

Next paragraph deploys these ideas towards solving the flatness and horizon problems, two major topics of contemporary cosmology [2-5].

2. The asymptotic approach to flatness and homogeneity

The considerations outlined so far suggest that the large-scale dynamics of the Universe may be naturally interpreted as a *global SOC process*. In light of this viewpoint, an evolving cosmological parameter – be it the deviation from spacetime flatness or the homogeneity across causally disconnected patches of the Universe – slowly converges towards a quasi-stationary value representing a *non-equilibrium steady state (NESS)*. Moreover, it is conceivable that, while dimensional fluctuations induced by $\varepsilon = 4 - D << 1$ provide the *driving* mechanism of cosmological SOC, Universe expansion acts as a *dissipation* reservoir. One may reasonably infer from (9) and (15) - (17) that the observed value of such a parameter $A_{a(t)}(\varepsilon)$ flows with ε as in

$$A_{a(t)}(\varepsilon) = A_{a_0}(\varepsilon)[1-\varepsilon^{\tau_s}\Phi(\varepsilon/\varepsilon_{cr})] \ , \ \ \varepsilon << 1, \ a(t) >> 1 \tag{18}$$

in which a_0 denotes the scale factor associated with the NESS of Universe expansion. Scaling (18) can be cast in the equivalent form

$$\frac{\Delta A_{a_0}(\varepsilon)}{A_{a_0}(\varepsilon)} = \frac{A_{a_0}(\varepsilon) - A_{a(t)}(\varepsilon)}{A_{a_0}(\varepsilon)} = \varepsilon^{\tau_s}\Phi(\varepsilon/\varepsilon_{cr}) \tag{19}$$

or

$$\frac{\Delta A_{\infty}(s)}{A_{\infty}(s)} = \varepsilon^{\tau_s}\overline{\Phi}(\varepsilon_{cr}/\varepsilon) \tag{20}$$

where

$$\overline{\Phi}(\varepsilon_{cr}/\varepsilon) = \Phi(\varepsilon/\varepsilon_{cr}) = \Phi(\varepsilon_{cr}/\varepsilon)^{-1} \tag{21}$$

Power expanding (21) by analogy with (10) yields

$$\overline{\Phi}(x) \sim \begin{cases} \overline{\Phi}(0) + \overline{\Phi}'(0)x + \frac{1}{2}\overline{\Phi}''(0)x^2 + ..., & x << 1 \\ \to 0, & x >> 1 \end{cases}$$

which leads to

$$\boxed{\Delta A_{a_0}(\varepsilon) = 0 \ ; \ \overline{\Phi}(\varepsilon_{cr}/\varepsilon) \to 0, \ \varepsilon << \varepsilon_{cr} \ , \varepsilon \to 0} \tag{22}$$

It is apparent from (22) that the end-state of the asymptotic approach to flatness and homogeneity matches the classical limit of four-dimensional spacetime ($D = 4$).

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