Relativistic Newtonian Gravity Makes Dark Energy Superfluous?

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Abstract

This paper shows that a simple and relativistic extension of Newtonian gravity leads to predictions that fits supernova observations of magnitude versus redshift very well without having to rely on the hypothesis of dark energy. In order to test the concept, we look at 580 supernova data points from the Union2 database.

Some relativistic extensions of Newtonian gravity have been investigated in the past, but we have reason to believe the efforts were rejected prematurely, before their full potential was investigated. Our model suggests that mass, as related to gravity, is also affected by standard relativistic velocity effects, something that is not the case in standard gravity theory, and this adjustment gives supernova predictions that fit the observations. Our findings are reflected in several recent research papers that follow the same approach; that work will also will be discussed in this paper.

Key Words: Supernovas, redshift, dark energy, relativistic Newton modification.

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1 Relativistic Newton Extension

Somewhat ad-hoc relativistic extensions of Newtonian gravity have been suggested in the past. In the 1980s, Bagge (1981) and Phillips (1986), for example, both suggested the following relativistic extension of Newtonian gravity

$$F = \frac{M \frac{m}{\sqrt{1 - \frac{w^2}{c^2}}}}{R^2}$$
(1)

This idea was in line with Einstein's (1905) special relativity theory, where a moving mass can be seen as a relativistic mass of the form $\frac{m}{\sqrt{1-\frac{w^2}{c^2}}}$. The idea must be that the small mass is moving relative to the larger gravitational mass and therefore appears to be relativistic from the perspective of the larger gravitational mass.

Phillips initially claimed that his relativistic Newton extension led to a prediction of the perihelion precession of Mercury that was equal to the predictions from Einstein's (1916) general relativity, which has been confirmed by experiments. Peters (1987) claimed that Phillips had made a mistake in his calculations and in reality, the Phillips extension only predicted half of the needed perihelion precession of Mercury. Shortly after that revelation, Phillips (1987) acknowledged the mistake, but claimed that the approach was still interesting and should be investigated further. The Phillips and Bagge model was, for similar reasons, also criticized by Ghosal and Chakraborty (1991) and Chow (1992).

However, we will claim that an additional mistake appears in the relativistic logic here. Looking at the astronomical system, it is mainly the Sun that acts on Mercury and naturally both the Sun and the Mercury are moving relative to the Earth. In this case, Haug (2020a; 2020c) has therefore suggested that the relativistic model should be extended as follows

$$F = \frac{\frac{M}{\sqrt{1 - \frac{v_M^2}{c^2}}} \frac{m}{\sqrt{1 - \frac{v_m^2}{c^2}}}}{R^2 \left(1 - \frac{v_M^2}{c^2}\right)}$$
(2)

That is, we are simply using standard logic from special relativity. We are standing in a third reference frame, Earth, and are observing two masses moving relative to us, namely the Sun and Mercury. Clearly, both masses must be relativistic, not only one of them. Further, Rmust represent the center to center distance between Mercury and the Sun and this length, as seen by an observer outside this system (the Earth), will undergo length contraction. This simply follows relativistic logic. Previous researchers may have missed the point that, in the case of the Sun's effect on Mercury, we are outside observers. Haug (2020c) has recently shown that this more logical relativistic extension of Newtonian gravity seems to give the correct precession of Mercury. As with the Philips prediction, the work should be checked by several independent researchers before any final conclusion is made.

A central issue in such investigations is that relativistic extensions to Newtonian gravity must follow logic in a realistic manner. If we are standing on Earth, for example, and are completing gravitational predictions about the Moon then the large mass is the Earth, and we are at rest with respect to the Earth. The Moon is a relativistic mass relative to Earth, and the radius, center to center, from Earth to the Moon is observed from the rest frame, that is from Earth. This means that the relativistic effects will only be present for the small mass, and in this case we should have the Bagge and Philips formula:

$$F = \frac{M \frac{m}{\sqrt{1 - \frac{w^2}{c^2}}}}{R^2} \tag{3}$$

Another case to consider would address a different phenomenon - observing the redshift of a supernova as viewed from Earth, for example. In this case, the large mass M is the supernova, and the lab frame is the small mass and the relativistic Newton formula must actually be

$$F = \frac{\frac{M}{\sqrt{1 - \frac{w^2}{c^2}}}m}{R^2}$$
(4)

In this case, the large gravity mass is the supernova that is moving relative to Earth. Further, the distance R runs from the center of the supernova to the Earth, and this distance is not undergoing length contraction, because the distance between the Earth and the supernova itself is not moving, even though it is expanding due to the supernova moving relative to us. It seems we must have three versions of the relativistic Newton formula depending on the situation. These three situations are illustrated in Figure 1. In the upper panel, the Earth is the laboratory frame and at the same time it is the mass M, and the moon is the mass m. Here the small mass is relativistic relative to the lab frame. In the middle panel, we are studying the Sun's gravitational effect on Mercury from the Earth. That is, both the Sun and Mercurv must be moving relative to the observer-frame (the Earth). Here both of the masses in the Newton formula must be relativistic. In the lower panel, we are observing the gravity effect on light sent out from a supernova from our base on Earth. That is, the large mass M is the supernova, and the supernova is moving relative to the Earth. These all follow from the most logical steps in relativity theory as applied to Newtonian gravity. To our surprise, even after an extensive "library" search, only the formula in the upper panel has been discussed in the physics literature, and it was, in our view, incorrectly applied to the situation in the second panel. This led to an incorrect prediction and may have caused researchers to abandon further investigation of relativistic Newton extensions prematurely.

Using the correct relativistic extension based on special relativistic logic combined with Newtonian gravity even fits supernova redshift data very well using baryonic matter only. We will look at this in the next sections and show a striking result - that no dark energy is needed.

2 Do We Need Dark Energy?

Here we will look at our Newton relativistic model to see if it predicts supernova data correctly. Our model needs

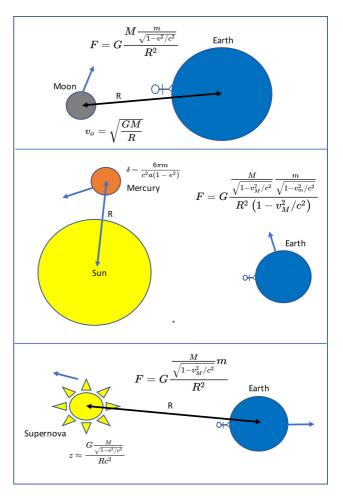


Figure 1: The figure shows two different situations and how they affect which masses (also in gravity) should be seen as relativistic and which mass is a rest-mass. This depiction follows standard logic from SR.

to take both Newton relativistic effects and relativistic Doppler effects into account. The Einstein relativistic Doppler shift is given by

$$z = \frac{1 - \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} - 1 \tag{5}$$

Solved with respect to v, this gives the well-known formula

$$v_{pec}(z) = c \frac{(1+z)^2 - 1}{(1+z)^2 + 1} \tag{6}$$

where the luminous distance (as a first approximation) is given by

1

$$D \approx z \frac{c}{H_o} \frac{(1+z)^2 - 1}{(1+z)^2 + 1}$$
(7)

where H_o is the Hubble constant. However, in the time light takes to travel the distance D, that is $t_D = \frac{D}{c}$, then

the emitting object will move away $t_D v_{pec}$. This was not taken into account by Davis and Lineweaver (2004), for example, as first pointed out by MacLeod (2004) and also discussed by Brissenden (2020). The corrected emitter time is therefore given by

$$t_e = t_D + \frac{t_D v_{pec}}{c} = t_D (1 + v/c)$$
(8)

This means the relationship between the proper time at the emitter t_e and the proper time at the observer t_o is given by

$$t_o = \frac{t_e}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t_D(1 + v/c)}{\sqrt{1 - \frac{v^2}{c^2}}} = t_D \sqrt{\frac{1 - v/c}{1 + v/c}} = t_D(1+z)$$
(9)

This means the luminous distance is actually given by

$$D = t_D c = \frac{c}{H_o} \frac{(1+z)^2 - 1}{(1+z)^2 + 1} (1+z)$$
(10)

The effective magnitude is found by using the following formula is

$$m_B(z) = 5\log H_o D + M_B \tag{11}$$

where M_B is the absolute magnitude in the B-band at the maximum light curve and log is the log with base 10. Be aware that the Hubble constant cancels out with the Hubble constant in the distance formula, so the formula is, in reality, not dependent on the Hubble constant. Davis and Lineweaver suggest that M_B should have a value of around 3.45, as found from data; we will use the same value here. We can say this is what special relativity alone will predict. Davis and Lineweaver compare the SR model with supernova data and conclude, "SR fails this observational test dramatically being 23σ from the general relativistic ACDM model?". The SR model Davis and Lineweaver uses is the one before taking into account the supernova moves during the time it takes for the light to travel from the supernova to the observer, so it is better than they claimed. Still, we agree that SR cannot explain the supernova data alone. In addition, we need to take relativistic effects in the Newton theory into consideration. The supernova is the large mass Min the Newton formula, and we are observing it from the Earth. If the supernova is moving, the mass will be a relativistic mass in relation to us and is not mathematically identical to the rest-mass, as it is in the standard Newton formula. That is, the following formula is relevant

$$F = \frac{\frac{M}{\sqrt{1 - \frac{w^2}{c^2}}}m}{R^2}$$
(12)

This again leads to a gravitational redshift (as observed from a weak gravitational field) of approximately

$$z_{gr} \approx \frac{G \frac{M}{\sqrt{1 - \frac{v^2}{c^2}}}}{Bc^2} \tag{13}$$

This is equal to the standard GR gravitational redshift multiplied by $\gamma_M = \frac{1}{\sqrt{1-\frac{w^2}{c^2}}}$. It is important that this can be derived independent on GR, as shown by researchers in the past, something we discuss in more detail in Appendix A. We must adjust our luminous distance with this factor to get the correct prediction. This will then give what we can call an apparent distance that, in addition to distance, reflects the missing relativistic mass adjustment for the supernova. Our model will therefore simply be

$$m_B(z) = 5\log(H_o D\gamma_M) + M_B \tag{14}$$

The difference between this and pure SR is the additional γ_M factor. Our result is mathematical "equivalent" to a theory recently suggested by Brissenden (2020). However, in his theory the γ factor comes from a speculative idea that proper velocity is what is relevant and not the velocity, so he claims $v_{pec}\gamma$ is relevant. Proper velocity is defined as taking the distance, as measured from the laboratory frame ("stationary" frame), divided by the time traveled as measured in the moving frame. Brissender has shown that this fits very well with supernova observations. However, there is no deep theory on why we should use proper velocity here rather than the velocity, and, in fact, we will claim the logic falls in the opposite direction; that it is the velocity that should be used and not the proper velocity.

In other approaches, Kipreso (2014) has what we would call a structural equivalent mathematical model, where he argues that his γ factor adjustment relative to SR is due to the need for a suggested time-contraction ratio. In order to get a γ factor here, he needs to use the Mansouri and Sexl (1976) transformation rather than the Lorentz transformation, something we will get back to later. So, Kipreos work is basically mathematical very similar to ours in that the time must be adjusted by a $\gamma_z = 1/\sqrt{1 - \frac{v_{pec}^2}{c^2}}$. However, again, there is no deep theory for why he has to do this suggested time-contraction ratio (by this we do not mean the lack of a solid transformation theory, but rather, why he needs the time-contraction in the first place).

We propose that we may have found the correct explanation for why there is time adjustment needed, namely because we also need to do relativistic adjustments for the mass in the Newtonian gravity formula. In short, we need to take into account that masses moving relative to an observer are observed as relativistic masses and not rest-masses. We will discuss this further in the coming sections.

3 Testing the model against 580 supernova 1A data points

To assess the prediction power of our model, we test it against 580 data points from the Union-2 supernova 1A database, Amanullah et al. (2010). Figure 2 show the observations in black points. The red line corresponds to our Newton relativistic model in addition to SR. The red line will be the predictions from the Brissender model and also from the Kipreos model. However, these approaches have very different interpretations than our model. The blue line is when we are only taking standard SR into account; as stated previously, our model is simply a logical extension of SR. That is, we assume any mass when moving relative to us is a relativistic mass. Further, we do not see the logic for why there should be different masses for gravitational and non-gravitational phenomena.

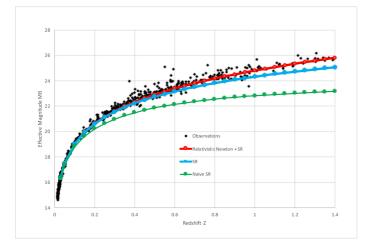


Figure 2: The figure shows 580 supernova data points from the Union-2 database, in black. The red line represents the predictions from our relativistic Newton gravity, the blue line is predictions from SR only, and the green line is what we call the naive SR model where we do not take into account that the supernova is moving during the time it takes for the light to move from the supernova to the observer.

The green line is an even more naive use of SR, where we neglect to take into account that the supernova has moved even further away in the time it takes for light to travel the distance from the supernova to the observer (Earth); this corresponds to using formula 7.

4 Summary and Discussion

In this paper, we have introduced three basically mathematical very similar models that all predict supernova observations without relying on dark matter. Naturally there may be some debate on whether or not they are very similar mathematically, as the inputs and assumptions are not the same for all of them. To be clear, these are then three considerably different models in their assumptions and interpretation of explanatory causes that give the correct prediction of supernovas. However, these models are very similar, from a structural mathematical point of view. We summarize the three alternatives plus the standard GR, Newton model below

- 1. The relativistic modified Newtonian model presented here simply states that we need to take into account relativistic masses for moving objects. Gravitational masses are no exception. Since the standard model does not do this, we get a correction factor of $\gamma_z = 1/\sqrt{1 - v_{pec}^2/c^2}$. This gives a nice fit to supernova observations. This model was suggested in 2020 and is fully consistent with Haug (2020a) recently presented quantum gravity theory that unifies gravity with quantum mechanics.
- 2. The model presented by Brissender in 2019 in relation to predicting supernovas is mathematically very similar to the model above, but it has very different interpretation. In the Brissender model, the γ adjustment factor is due to what we would claim is a speculative idea: that it is the proper supernova velocity that is relevant and not the standard velocity. We doubt this explanation. That said, the Brissender paper together with the Kipreos model made us think more deeply about the need for a γ correction to get the correct supernova predictions. Taken together with our view that we must also have relativistic masses in the Newton gravity model, this helps to develop a new perspective.
- 3. The Kipreos (2014) model published in 2014, where the end result is mathematically very similar to the two models described above. This model also has a γ factor that it claims is a correction factor for time dilation of the signal sent in the past versus the one received in the present. The explanation for exactly why this should be the case seems to be the weakness in this approach. Still, this paper strongly indicates that something may be missing in the standard theory and pointed us towards the need for further research.
- 4. The standard gravity model. This model needs dark energy in order to be consistent with supernova data, but we strongly suspect that dark energy is simply a fudge factor. Unfortunately, much money has gone into this model and it has considerable prestige. Many of the best-known physicists working in cosmology today are heavily invested in this model, which leads to biases against considering the findings from the three alternative models explained in this paper.

The Kipreos (2014) interpretation relies on the absolute transformation by Mansouri and Sexl (1976) that

again is linked to absolute simultaneity rather than relativity of simultaneity, something that has been supported by some recent research Spavieri et al. (2018; 2019). The Mansouri and Sexl transformation has the same length transformation as the Lorentz transformation, but the time transformation is just $t' = t\gamma$ compared to the Lorentz time transformation of $t' = (t - vx/c^2)\gamma$. The suggested interpretation in this paper seems to be compatible both with SR and with the Mansouri and Sexel transformation. This is because it does not rely on any time-transformation; the same seems to be the case with the Brissender interpretation. Therefore, the essential question is: what is the most likely cause for the γ adjustment that leads to a good fit of supernova predictions? Is it the time adjustment suggested by Kipreos? Is it that we should rely on proper velocity? Or is it simply that the masses in the Newton gravity formula are also relativistic? We believe that the last suggestion, as presented in this paper, makes the most sense. In particular, this is true since this method also seems to lead to the correct prediction of Mercury precession, gives an escape velocity that is compatible with a Planck mass particle, and is, in general, a more consistent and logical theory, see Haug (2020a).

It is worth pointing out that there is no Lorentz transformation of a relativistic mass. Recent research indicates there are strong parallels between mass and clocks. Elementary particles are likely 'ticking" at the Compton time interval, as suggested by Haug (2020a). That mass is linked to the Compton frequency also has some recent observational support, see Lan et. al. (2013) and Dolce and Perali (2015). The mass follows a transformation similar to simple time dilation, and not equal to Lorentz time transformation. This means the mass follows a transformation equal to the time transformation in the Mansouri and Sexl transformation. The link between our theory and the Kipreos (2014) approach is therefore closer than it may seem at first sight. We have pointed out that the clock is the mass, and that the relativistic mass indeed follows a transformation identical to the time transformation in the Mansouri and Sexl transformation. However, the relativistic mass is also consistent with special relativity.

We will claim it is not very logical to assume that masses are observed to be relativistic in SR, but then suddenly should not be observed to be relativistic in relation to gravity. Even in a weak gravitational field why should the gravitational observation not be affected by the velocity of the gravitational object? We think the original error occurred in the 1980s and 1990s, when Bagge and Phillips suggested that only the small mass was relativistic. In other words, they did not explore the idea fully before it was rejected by the science community, perhaps prematurely.

5 Broken Lorentz Invariance?

Most researchers on gravity have assumed that a theory of gravity must be Lorentz invariant. Our suggested Newton modification above does not seem to follow this criteria. However, we believe that it would be a mistake to reject this theory at this stage due to this issue. First of all, the modified Newton gravity presented here is not a fully developed gravity theory, but mostly provides logical reasoning around how Newton's theory could be relativistically adjusted based on simple concept that all masses moving relative to the observer are not exempt from special relativity. It could very well be that this is still only a good approximation that requires further adjustment.

More important is the fact that several quantum gravity theories, as well as superstring theory predict Lorentz invariance breakdown. However, despite extensive searching no sign of the Planck scale breakdown has been observed. One main thought on this is that the Planck energy is an enormously high energy, much higher than one can achieve in the Large Hadron Collider, for example. However, attempts have been made to predict effects from the Planck scales at much lower energies, yet even after extensive testing, such effects have not been observed. Still, there are many paths not fully explored here, as mentioned in the following review paper Hees et al. (2017). It had recently been suggested that gravity itself is a breakdown of Lorentz invariance at the Planck scale and that that all observable gravity phenomena are, remarkably, a detection of this, see Haug (2020a).

It is also interesting to see this in a historical perspective. In 1899, Max Planck first suggested the natural units, today known as the Planck units. He obtained these units by assuming that there were three fundamental universal constants, namely Newton's gravitational constant, the speed of light, and the Planck constant. From these constants, he used dimensional analysis to find a length, a time, a mass, and an energy that he thought had an important meaning for physics. Then in 1916, Einstein suggested that in order to move forward in thinking about gravity, one had to come up with a quantum gravity theory, or in his own words "Because of the intra-atomic movement of electrons, the atom must radiate not only electromagnetic but also gravitational energy, if only in minute amounts. Since, in reality, this cannot be the case in nature, then it appears that the quantum theory must modify not only Maxwell's electrodynamics but also the new theory of gravitation.".

A few years later in 1922, Eddington suggested that gravity likely was linked to the Planck length and he stated "But it is evident that this length must be the key to some essential structure. It may not be an unattainable hope that someday a clearer knowledge of the process of gravitation may be reached.". For many years Planck and Eddington were ignored and even ridiculed for these ideas by other famous physicists, such as Bridgman (1962), see Gorelik (1992) for more on that history. Even to this day there is disagreement on the importance of the Planck length; while the majority think it is the shortest possible length Padmanabhan (1985), Adler (2010), Hossenfelder (2012) and that it likely plays an important role in quantum gravity, other physicists claim that the Planck length is of no use and merely a mathematical artifact, see Unzicker (2020), for example.

If the Planck length can only be calculated with dimensional analysis and there is no sign of it in any measurement, then why not simply assume it is not there, just as Einstein once did with the ether? However, other pathways are also possible. Recently, Haug (2020b) has published strong evidence showing that the Planck length can be found from basically any gravity phenomena without any knowledge of the gravity constant or the Planck constant¹. This analysis supports the idea that all gravity phenomena are linked to the Planck scale and that this is directly measurable. Further, if the Planck length is invariant, then all gravity phenomena are breaks in Lorentz invariance. This should not be taking for granted, but we think this view deserves further study before it can be confirmed or rejected. It is possible that a modified Newton theory similar to the one we have suggested here could be part of the puzzle as well. After all, the Planck scale and even such things as dark matter have not been detected according to the standard theory. Nevertheless, with the recent findings strongly pointing towards a detection of the Planck scale, a detection that lies within gravity itself, we could have a game changing perspective in development now.

6 Conclusion

Both Kipreos (2014) and Brissender (2020) have recently suggested basically very similar mathematical models that seems to predict supernova 1A data very well without relying on dark energy. However, we question the assumptions behind their models. Here we have shown a third alternative that is mathematically very similar to the two other models, but provides a very different explanation for why this adjustment is needed. We claim the mass in gravity and, more precisely, in the Newton formula the mass is not excluded from velocity related relativistic effects. By taking into account relativistic mass correctly, we create a model that predicts supernova 1A data, and that addresses redshift versus magnitude correctly without relying on dark energy.

This also shows the importance of letting speculative ideas be published and circulate, even if they are in conflict with mainstream models (such as the dark energy hypothesis). The Kipreos model and the Brissender model, for example, are much more speculative than our model. Still, they both show how a small adjustment to SR doppler shift leads to correct prediction of supernovas without relying on dark energy. Instead of ignoring this work, we have investigated these lines of thought further and have developed a robust and sound explanatory model for both simple multi-body planetary system and also for supernovas. It is logical and consistent to say that there only is one type of mass at the most fundamental level and that this mass is affected by velocity from the perspective of the observer as well as in relation to gravity. Further study is merited.

Disclosure statement

We have no conflict of interest to declare.

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Data Availability

The data used in this paper are from the Union-2 supernova 1A database

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¹And as discussed in great detail in that paper, this is not simply a matter of hiding G and \hbar by rewriting the units or any similar "tricks".

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