# The Relationship of Gravity and Magnetism 

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#### Abstract

Using the conservation of energy principle, gravity and magnetism are related by the energy required for particle spin. Properties of the electron, including its magnetic moment, its gravitational coupling constant and the gravitational constant $(\mathrm{G})$ are all derived in this paper from this principle.


## 1. Planck Size

The electron is known to have both particle and wave properties, in a phenomenon known as wave-particle duality [1]. The electron is also known to have a charge (a property of the electric force), a magnetic moment (a property of the magnetic force), and a mass (a property of the gravitational force).

The electron's curious wave-particle duality, its properties and forces can be described by waves interacting with an incredibly small particle - a sphere with a radius of Planck length $\left(l_{\mathrm{p}}\right)$. The values and units for all constants in this paper are found in the Appendix and are based on CODATA values [2].

The next figure describes electric waves that converge on a Planck-sized sphere, which are then reflected outwards. These in-waves may not converge equally at all points on the sphere, causing motion or spin, resulting in two types of out-waves. A human-sized analogy of this process may be equivalent to sound waves bouncing off a balloon, while causing a rotation of the balloon at the same time.


Fig. 1.1 - A sphere with radius of Planck length spinning and reflecting waves
In Fig. 1.1, longitudinal in-waves converging on the sphere are described with amplitude of $x_{0}$ in (a). In (b), longitudinal out-waves are reflected with reduced amplitude ( $\mathrm{x}_{-1}$ ). Energy is proportional to wave amplitude, and due to the conservation of energy, the amplitude in (b) is reduced because some energy is used for the spin of the particle
in (c). This spin causes a transverse wave of amplitude $\mathrm{x}_{1}$ at the poles. Longitudinal waves will be shown later to be the electric force, and transverse waves from a spinning particle will be shown to be the magnetic force.

The values for wave amplitude are based on the Planck charge ( $\mathrm{q}_{\mathrm{p}}$ ) and the fine structure constant ( $\alpha_{\mathrm{e}}$ ). The longitudinal in-wave is the Planck charge (a). The decreased longitudinal out-wave is the Planck charge times the square root of the fine structure constant (b), and the new spinning, transverse out-wave is the Planck charge divided by the square root of the fine structure constant (c). These values will be used throughout this paper.

$$
\begin{gather*}
x_{0}=q_{P}  \tag{1.1}\\
x_{-1}=q_{P} \sqrt{\alpha_{e}}  \tag{1.2}\\
x_{1}=\frac{q_{P}}{\sqrt{\alpha_{e}}} \tag{1.3}
\end{gather*}
$$

The longitudinal out-wave amplitude from Eq. 1.2 is better known as the elementary charge ( $\mathrm{e}_{\mathrm{e}}$ ). In this paper, all units of charge (Coulombs) are replaced with units of distance (meters) to express charge as wave amplitude.

$$
\begin{equation*}
x_{-1}=q_{P} \sqrt{\alpha_{e}}=e_{e} \tag{1.4}
\end{equation*}
$$

## 2. Electron

The electron's properties can be described graphically and mathematically based on the wave descriptions from the previous section. Most of the equations in this paper prefer the use of the magnetic constant ( $\mu_{0}$ ) instead of the Coulomb constant ( $\mathrm{k}_{\mathrm{e}}$ ) because wave speed (c) can be separated. The relationship of these constants is as follows:

$$
\begin{equation*}
k_{e}=\frac{\mu_{0} c^{2}}{4 \pi} \tag{2.1}
\end{equation*}
$$

The electron's mass and energy can be described in electrical terms, which was first presented in The Relationship of Mass and Charge [3]. With a core of Planck-length sized particles at the center of the electron, in-waves and out-waves create spherical, longitudinal standing waves to a distance known as the electron's classical radius ( $\mathrm{r}_{\mathrm{e}}$ ). Standing waves have no net propagation of energy, and is therefore by definition stored energy/mass [4]. A physical particle core of Planck length, creating standing waves as the particle's energy, solves the wave-particle duality phenomenon. It is both a particle and a wave.


Fig. 2.1 - Electron particle formed as longitudinal standing waves to its classical radius ( $\mathrm{r}_{\mathrm{e}}$ )
The electron's rest mass $\left(\mathrm{m}_{\mathrm{c}}\right)$ and rest energy $\left(\mathrm{E}_{\mathrm{e}}\right)$ can be solved using the elementary charge (wave amplitude from Eq. 1.4) and the electron's classical radius, which is the transition point from standing waves to traveling waves. This is the reason for using the magnetic constant, as the only difference between Eqs. 2.2 and 2.3 is wave speed (c). Note: refer to the Appendix for corrected units when constants use meters instead of Coulombs for charge.

$$
\begin{equation*}
m_{e}=\frac{\mu_{0}}{4 \pi}\left(\frac{e_{e}^{2}}{r_{e}}\right)=9.11 \times 10^{-31}(\mathrm{~kg}) \tag{2.2}
\end{equation*}
$$

Electron mass

$$
\begin{equation*}
E_{e}=\frac{\mu_{0} c^{2}}{4 \pi}\left(\frac{e_{e}^{2}}{r_{e}}\right)=8.19 \times 10^{-14}\left(\frac{\mathrm{~kg}\left(\mathrm{~m}^{2}\right)}{\mathrm{s}^{2}}\right) \tag{2.3}
\end{equation*}
$$

Electron energy
Longitudinal out-wave energy does not stop at the electron's radius. It continues to flow as longitudinal traveling waves. As it is no longer stored (standing wave) energy, traveling waves are measured as a force because of their ability to change the motion of another particle. This is the electric force.


Fig. 2.2 - Electric force of longitudinal traveling waves beyond the electron's radius
The equation for the electric force is energy at distance, $\mathrm{E}=\mathrm{F} / \mathrm{r}$. It is the same energy equation as Eq .2 .3 , but now with distance squared to become a force. This is Coulomb's equation for two electrons, in magnetic constant format (instead of the Coulomb constant - see Eq. 2.1). Note: when expressed as multiple groups of particles, the elementary charge is multiplied by the number of particles due to constructive wave interference.

$$
\begin{equation*}
F_{e}=\frac{\mu_{0} c^{2}}{4 \pi}\left(\frac{e_{e}^{2}}{r^{2}}\right) \tag{2.4}
\end{equation*}
$$

## Electric force

While the electric force occurs as a result of longitudinal traveling waves, a second force emerges from the electron particle due to the spin of its core. The magnetic force occurs as a result of transverse waves from spin, at two poles illustrated in the next figure. This force will be described mathematically in a section on the atom, deriving the electron's magnetic moment - the Bohr magneton.


Fig. 2.3 - Magnetic force of transverse waves due to electron spin
Due to the conservation of energy, there is a slight difference of longitudinal in-wave energy compared to longitudinal out-wave energy. This is described graphically in the next figure where the net result is longitudinal (electric) energy flowing into the electron particle, which flows out at the poles as transverse (magnetic) energy.


Fig. 2.4 - Gravitational force as a net difference of longitudinal in-waves and out-waves
Described mathematically, the ratio of longitudinal amplitude loss ( $\alpha_{e}$ ) is applied to the core of the electron at Planck length ( $\mathrm{l}_{\mathrm{p}}$ ), and extends to the electron's standing wave radius ( $\mathrm{r}_{\mathrm{e}}$, illustrated in Fig. 2.4. This ratio is known as the gravitational coupling constant for the electron ( $\alpha_{\mathrm{Ge}}$ ) - an incredibly small $2.4 \times 10^{-43}$.

$$
\begin{equation*}
\alpha_{G e}=\frac{l_{P}^{2}}{r_{e}^{2}}\left(\alpha_{e}\right)=2.4 \times 10^{-43} \tag{2.5}
\end{equation*}
$$

Electron gravitational coupling constant
For a single particle, this energy loss in longitudinal (electric) energy is very small ( $\mathrm{E}_{\mathrm{Ge}}$ ). The gravitational coupling constant from Eq. 2.5 is applied to the electron's energy, where $E=m_{e} c^{2}$. Gravity is better expressed as a force, where force is energy at a distance. When applied at the electron's radius, the force of gravity on a single electron ( $\mathrm{F}_{\mathrm{Ge}}$ ) at its radius is described in Eq. 2.7. This will be used in the next section to derive the gravitational constant (G).

$$
\begin{align*}
& E_{G e}=m_{e} c^{2}\left(\alpha_{G e}\right)  \tag{2.6}\\
& F_{G e}=\frac{m_{e} c^{2}\left(\alpha_{G e}\right)}{r_{e}} \tag{2.7}
\end{align*}
$$

## 3. Two Electrons

Gravity is a shading effect of longitudinal traveling waves (electric force). In a single particle, described in Fig. 2.4, the net result is equal pressure from all sides of the sphere. However, when considering two or more particles within proximity of each other, there will be a shading effect as some energy that is absorbed by a particle is not reflected to another particle. This creates unequal energy/pressure on the exterior sides of each particle, forcing two particles together. Yet, this pressure is very slight, and due to the dominance of longitudinal traveling waves that are reflected, the electric force dominates over gravity for two electrons.


Fig. 3.1 - Gravity as a shading effect between two or more particles
The dominance of the electric force $\left(\mathrm{F}_{\mathrm{e}}\right)$ compared to the gravitational force $\left(\mathrm{F}_{\mathrm{g}}\right)$ is expressed in the gravitational coupling constant for the electron ( $\alpha_{\mathrm{Ge}}$ ), as $\mathrm{F}_{\mathrm{g}} / \mathrm{F}_{\mathrm{e}}$. By no coincidence, it is the same value as the ratio of the Planck length and electron radius found earlier in Eq. 2.5. This ratio can be interpreted to mean that it would take roughly $10^{43}$ number of electrons experiencing the gravitational force to be equivalent to the force of two electrons experiencing the electric force at the same distance.

$$
\begin{equation*}
\alpha_{G e}=\frac{F_{g}}{F_{e}}=\frac{G m_{e}^{2}}{k_{e} e_{e}^{2}}=2.4 \times 10^{-43} \tag{3.1}
\end{equation*}
$$

In Eq 3.1, the separation distance of particles ( r ) is not shown because the distance cancels with both forces. They both decline at the square of distance $\left(r^{2}\right)$. The complete form of the gravitational force equation for two masses is the following. In this example, two electrons of mass $\mathrm{m}_{\mathrm{e}}$.

$$
\begin{equation*}
F_{g}=\frac{G m_{e} m_{e}}{r^{2}} \tag{3.2}
\end{equation*}
$$

## Gravitational force

Because the force of gravity $\left(\mathrm{F}_{\mathrm{g}}\right)$ uses two masses $(\mathrm{m})$ in the equation, the gravitational constant G must account for the extra mass in the equation. In SI units, a force is expressed as $\mathrm{kg}^{*} \mathrm{~m} / \mathrm{s}^{2}$, which has one mass ( kg ). Using the gravitational force equation of a single electron, found earlier in Eq. 2.7, the electron's mass ( $\mathrm{m}_{\mathrm{e}}$ ) is flipped to the denominator and the electron's radius ( $\mathrm{r}_{\mathrm{e}}$ ) moves to the numerator. This allows the force to be expressed now as the multiple of two masses by canceling one mass in the gravitational constant. The following becomes the derivation of the gravitational constant $G$, correct in both value and units.

$$
\begin{equation*}
G=\frac{r_{e} c^{2}\left(\alpha_{G e}\right)}{m_{e}}=6.67 \times 10^{-11}\left(\frac{m^{3}}{k g\left(s^{2}\right)}\right) \tag{3.3}
\end{equation*}
$$

Gravitational constant (G)

## 4. Hydrogen Atom

An electron paired with a proton is the simplest atom - hydrogen. The electron orbits the proton at a most probable distance known as the Bohr radius ( $\mathrm{a}_{0}$ ). A proton and electron will neutralize the electric force. In an atom, the magnetic force is also neutralized as electron pairings in orbitals require electrons of opposite spin.

The force of magnetism is found in static magnets with the alignment of many electrons. It is also found in electromagnetism when electrons flow through a conductor. But it is a single electron, and its magnetic moment, that is addressed here. The electron's magnetic moment is known as the Bohr magneton $\left(\mu_{\mathrm{B}}\right)$. It has units of joules per Tesla (J/T). When these units are dissected further, the true nature of the electron's magnetic moment can be explained. When the units of charge (Coulombs) are replaced with units of distance for wave amplitude (meters), the SI units for the Bohr magneton become $\mathrm{m}^{3} / \mathrm{s}$. These units are a volumetric flow rate. The correct derivation of units:

$$
\begin{equation*}
\frac{J}{T}=\frac{\frac{k g\left(m^{2}\right)}{s^{2}}}{\frac{k g}{(C) s}}=\frac{\frac{k g\left(m^{2}\right)}{s^{2}}}{\frac{k g}{(m) s}}=\frac{m^{3}}{s} \tag{4.1}
\end{equation*}
$$

A visual of transverse (magnetic) energy flowing from the poles of the electron is shown again in Fig. 4.1. It flows with a wave speed of the speed of light (c).


Fig. 4.1 - Electron's magnetic moment - the flow rate of transverse waves due to spin
There are two poles for the electron. Flowing from one pole (1/2), the volumetric flow rate is based on the wave speed (c), electron's radius ( $\mathrm{r}_{\mathrm{e}}$ ) and the increased amplitude from Fig. 1.1 and Eq. 1.3, which is the Planck charge ( $\mathrm{q}_{\mathrm{p}}$ ) divided by the square root of the fine structure constant $\left(\alpha_{c}\right)$. This flow rate is the Bohr magneton - the electron's magnetic moment in value and corrected units.

$$
\begin{equation*}
\mu_{B}=\frac{1}{2} \frac{q_{P} r_{e}{ }_{e}}{\sqrt{\alpha_{e}}}=9.274 \cdot 10^{-24}\left(\frac{m^{3}}{s}\right) \tag{4.2}
\end{equation*}
$$

## Bohr magneton

Due to the relationship of the Planck length and electron's radius, the Bohr magneton can also be derived using the electron's gravitational coupling constant ( $\alpha_{\mathrm{Ge}}$ ), illustrating the true relationship between magnetism and gravity.

$$
\begin{equation*}
\mu_{B}=\frac{1}{2} \frac{q_{P} l_{P} c}{\sqrt{\alpha_{G e}}}=9.274 \cdot 10^{-24}\left(\frac{m^{3}}{s}\right) \tag{4.3}
\end{equation*}
$$

As mentioned previously, most atoms are stable both electrically and magnetically. In magnetic terms, it will be shown mathematically that the electron's rest energy $\left(\mathrm{E}_{\mathrm{e}}\right)$ is equal to the magnetic energy between the electron and proton in a hydrogen atom $\left(\mathrm{E}_{\mathrm{m}}\right)$ when the electron is at a distance of the Bohr radius ( $\mathrm{a}_{0}$ ). This is the most probable orbital distance for the electron when it orbits a proton.


Fig. 4.2 - Magnetic energy $\left(\mathrm{E}_{\mathrm{m}}\right)$ in a hydrogen atom - an electron $(-)$ and proton $(+)$ at the Bohr radius $\left(\mathrm{a}_{0}\right)$.

The electron's energy was derived earlier in Eq. 2.3 as $8.19 \times 10^{-14} \mathrm{~kg}^{*} \mathrm{~m}^{2} / \mathrm{s}^{2}$. Now, it will be shown that the magnetic energy between the electron and proton in hydrogen is the same value. $\mathrm{E}_{\mathrm{m}}=\mathrm{E}_{\mathrm{c}}$. The volume $(\mathrm{V})$ between the electron and proton can be described like a cylinder, with surface area $\pi^{*} \mathrm{r}_{e}{ }^{2}$ and length a $\mathrm{a}_{0}$. This is Eq. 4.4. When the Bohr magneton flow rate is squared for both directions into and out of the electron $\left(\mu_{\mathrm{B}}{ }^{2}\right)$, in the aforementioned cylindrical volume $(\mathrm{V})$ with linear density $\left(\mu_{0}\right)$, it becomes an energy equation. This is the magnetic energy $\left(\mathrm{E}_{\mathrm{m}}\right)$ flowing between the electron and proton, expressed in Eq. 4.6, exactly matching the electron's rest energy in value and units.

$$
\begin{gather*}
V=a_{0}\left(\pi r_{e}^{2}\right)  \tag{4.4}\\
E_{m}=\mu_{0} \frac{1}{V}\left(\mu_{B}^{2}\right)  \tag{4.5}\\
E_{m}=\frac{\mu_{0}}{a_{0}\left(\pi r_{e}^{2}\right)}\left(\mu_{B}^{2}\right)=8.19 \times 10^{-14}\left(\frac{\mathrm{~kg}\left(\mathrm{~m}^{2}\right)}{\mathrm{s}^{2}}\right) \tag{4.6}
\end{gather*}
$$

## 5. Large Bodies

As found in the gravitational coupling constant for the electron (Eq. 3.1), it takes a gravitational force of roughly $10^{43}$ electrons to equal the electric force of just two electrons. It takes about $10^{37}$ protons for the same electric force. A significant number of particles are required before gravity is detected. A large body, such as a planet like Earth, has many neutralized particles that create the atomic elements found on the planet. The Earth has roughly $10^{50}$ atoms [5]. The shading effect from two particles, illustrated earlier in Fig 3.1, is now detected in large bodies such as the Earth and Moon. This shading effect of the electric force, being absorbed by particles for spin, now becomes the force of gravity. Gravity is a result of unequal pressure, forcing two large bodies together.


Fig. 5.1 - Gravity as a shading effect of the electric force in a large body
The absorption of energy for particle spin appears in magnetism. Each particle spins, and when aligned with other particles, magnetic poles may form in large bodies like the Earth. Fig. 5.2. is an illustration of the Earth's north and south magnetic poles.


Fig. 5.2 - Magnetic poles of a large body
The Earth's magnetic field is intricately linked to its gravitational force as a result of every spinning particle on and within the Earth.

## Conclusion

When redefining the unit of charge as a unit of distance, due to charge being measured as wave amplitude, multiple properties of the electron particle can be linked together:

- The electron particle can be described as longitudinal standing waves, with it measured as energy or as mass (energy without wave speed).
- The electric force can be described as longitudinal, traveling waves, measured as a force (energy at a distance).
- The magnetic force can be described as transverse waves, with the electron's magnetic moment measured as a volumetric flow rate.
- The gravitational force is due to unequal longitudinal wave amplitude on a particle - a shading effect between two or more particles.

Magnetism and gravity are linked due to the conservation of energy. Energy into a particle is perfectly conserved, yet it has two forms as it leaves a particle. Some energy is used for spin, creating magnetic energy. The vast majority of the energy returns as electric energy. It is the slight difference of inbound electric energy compared to outbound electric energy that is the unequal pressure on particles that becomes the force known as gravity. The energy used for magnetism is the lost energy of gravity.

## Appendix

## Constants

The following constants are used in this paper (CODATA values).

| Symbol | Definition | Value (units) |
| :--- | :--- | :--- |
| c | Wave velocity (speed of light) | $299,792,458(\mathrm{~m} / \mathrm{s})$ |
| $\mathrm{E}_{\mathrm{e}}$ | Electron energy | $8.1871 \times 10^{-14}\left(\mathrm{~kg}^{*} \mathrm{~m}^{2} / \mathrm{s}^{2}\right)$ |
| $\mathrm{m}_{\mathrm{e}}$ | Electron mass | $9.1094 \times 10^{-31}(\mathrm{~kg})$ |
| $\mathrm{k}_{\mathrm{e}}$ | Coulomb constant | $8.9876 \times 10^{9}\left(\mathrm{~kg} \mathrm{~m}^{*} / \mathrm{s}^{2}\right)^{\mathrm{a}}$ |
| $\mu_{0}$ | Magnetic constant | $1.2566 \times 10^{-6}(\mathrm{~kg} / \mathrm{m})^{\mathrm{a}}$ |
| $\mu_{\mathrm{B}}$ | Bohr magneton | $9.2740 \times 10^{-24}\left(\mathrm{~m}^{3} / \mathrm{s}\right)^{\mathrm{a}}$ |
| $\mathrm{qP}_{\mathrm{p}}$ | Planck charge | $1.8756 \times 10^{-18}(\mathrm{~m})^{\mathrm{a}}$ |
| $\mathrm{e}_{\mathrm{e}}$ | Elementary charge | $1.6022 \times 10^{-19}(\mathrm{~m})^{\mathrm{a}}$ |
| $\alpha_{\mathrm{e}}$ | Fine structure constant | 0.00729735 |
| $\alpha_{\mathrm{Ge}}$ | Gravitational coupling (electron) | $2.4005 \times 10^{-43}$ |
| G | Gravitational constant | $6.6741 \times 10^{-11}\left(\mathrm{~m}^{3} / \mathrm{kg}^{*} \mathrm{~s}^{2}\right)$ |
| $\mathrm{l}_{\mathrm{P}}$ | Planck length | $1.6162 \times 10^{-35}(\mathrm{~m})$ |
| $\mathrm{r}_{\mathrm{e}}$ | Electron classical radius | $2.8179 \times 10^{-15}(\mathrm{~m})$ |
| $\mathrm{a}_{0}$ | Hydrogen 1s radius (Bohr radius) | $5.2918 \times 10^{-11}(\mathrm{~m})$ |

a - Corrected units when units of Coulombs (C) is replaced with distance (meters).
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