On a triangle with two parallel sides

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Abstract. We consider the side lengths of a triangle with two parallel sides by division by zero.

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1. Introduction

Let us consider a triangle $ABC$ in the plane such that $a = |BC|$, $b = |CA|$ and $c = |AB|$. Let $\theta_a$ (resp. $\theta_b$) be the angle between $\overrightarrow{BA}$ and $\overrightarrow{AC}$ (resp. $\overrightarrow{BC}$) (see Figure 1). In this note we fix the points $A$, $B$ and the angle $\theta_b$, and consider the side lengths of parallel sides of $ABC$ in the case $\theta_a = \theta_b$ (see Figure 2). We use the definition of division by zero [1, 2]

$$z \div 0 = 0 \text{ for any real number } z.$$  

We use a rectangular coordinate system such that $A$ and $B$ have coordinates $(p, 0)$ and $(q, 0)$, respectively, where we assume $p = c + q$ and the point $C$ lies on the region $y \geq 0$.

2. Side length

The point of intersection of the lines expressed by the equations $y = \tan \theta_a(x - p)$ and $y = \tan \theta_b(x - q)$ coincides with the point $C$, and has coordinates

$$\left( \frac{p \tan \theta_a - q \tan \theta_b}{\tan \theta_a - \tan \theta_b}, \frac{c \sin \theta_a \sin \theta_b}{\sin(\theta_a - \theta_b)} \right).$$

Therefore we get

$$a = \frac{c \sin \theta_b}{\sin(\theta_a - \theta_b)}, \quad b = \frac{c \sin \theta_a}{\sin(\theta_a - \theta_b)}.$$  

If $\theta_a = \theta_b$, then $\sin(\theta_a - \theta_b) = 0$, and we get $a = b = 0$ by (1). Therefore the side length of the parallel sides of a triangle equals 0.

Notice that the $y$-coordinate in (2) also shows that the height corresponding to the base $AB$ equals 0 if $\theta_a = \theta_b$. Also (2) shows that the point $C$ coincides with the origin $(0, 0)$ if $\theta_a = \theta_b$. 

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REFERENCES