# The inherent derivations derived theoretically from one-dimensional Maxwell's equations on the basis of exact differential equation 

The first of sequels on 29 March 2020
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I. An absolute rest, one-dimensional free space with homogeneity, isotropy, linearity and differentiable continuity and orthogonal independent relation between the other spaces and a common time
II. A light beam of duality with electromagnetic mass, momentum density and the wave in the space III. Indeterminacy of independent relationship between mechanical corpuscular system and electromagnetic flux system on the basis of momentum and energy in wave-particle duality


#### Abstract

This paper proposes inherent derivations theoretically from one-dimensional Maxwell's equations on the basis of exact partial differential equations in free orthogonal space. Apart from the Lorentz transformations with no absolute perfection to become infinity at a point that any particle with mass to be unattainable to the speed of light and the quantum mechanics with unequivocal explanation for waveparticle duality. With those background, the first theme derived through a classical process, is the socalled wave-particle duality. New duality in this paper means that electromagnetic energy density $\rho(\mathrm{E})$, momentum $\rho(p)$ and mass density $\rho(M)$ in mechanical corpuscular system can be, respectively, described as one-dimensional wave equation form, traveled at the speed of light equal to reciprocal square root of electromagnetic constant, $\varepsilon(0) \mu(0)$ product of invariant permittivity $\varepsilon(0)$ and permeability $\mu(0)$ in free orthogonal space with homogeneity, isotropy, linearity and differentiable continuity with respect to independent variable space and time on the basis of exact differential equations. The second derived is that the mass density $\rho(\mathrm{m})$ [unit, $\mathrm{kg} / \mathrm{Cub}(\mathrm{m})]$, the momentum density $\rho(\mathrm{p})[\mathrm{kgm} / \mathrm{s} / \mathrm{Cub}(\mathrm{m})]$, the energy density $\rho(\mathrm{E})[\mathrm{J} / \mathrm{Cub}(\mathrm{m})]$, electric flux density $\mathrm{D}(\mathrm{x})[\mathrm{As} / \mathrm{Sq}(\mathrm{m})]$ and magnetic density $\mathrm{B}(\mathrm{y})[\mathrm{Vs} / \mathrm{Sq}(\mathrm{m})]$ is, respectively, able to be expressed as the so-called wave equation function as $\partial(\partial(\mathrm{g}(\mathrm{z}, \mathrm{t}) / \partial \mathrm{z}) / \partial \mathrm{z}=\varepsilon(0) \mu(0)$ $\partial(\partial(\mathrm{g}(\mathrm{z}, \mathrm{t}) / \partial \mathrm{t}) / \partial \mathrm{t}$, through each density replaced $\mathrm{g}(\mathrm{z}, \mathrm{t})$ function into them. So, all of them can travel concurrently at the speed of light in the space: $\rho(\mathrm{m}), \rho(\mathrm{p}), \rho(\mathrm{E})$ in wave form on root mean square unit are able to be described, respectively, $\mathrm{A}(\mathrm{m}) \operatorname{Exp}(4 \pi \mathrm{j} \theta) / 2, \mathrm{~A}(\mathrm{p}) \operatorname{Exp}(4 \pi \mathrm{j} \theta) / 2$, and $\mathrm{A}(\mathrm{E}) \operatorname{Exp}(4 \pi \mathrm{j} \theta) / 2$, $A(D) \operatorname{Exp}(2 \pi j \theta) / \operatorname{Sqrt}(2)$ and $A(B) \operatorname{Exp}(2 \pi j \theta) / S q r t(2)$, where $A(m), A(p), A(E), A(D)$ and $A(B)$ are, respectively, amplitude in wave form. The third derived is that the mass density $\rho(\mathrm{m})$ equals to the product of the energy density $\rho(\mathrm{E})$ and $\varepsilon(0) \mu(0): \rho(\mathrm{m})=\varepsilon(0) \mu(0) \rho(\mathrm{E})$, using new concept specified as mass beam $b(m)$ [unit: kg ], momentum beam and energy beam element assumed from property of the well-known light beam, the mass beam element equals the energy beam element $b(E)[J]$ times $\varepsilon(0) \mu(0)$ : $b(m)=\varepsilon(0) \mu(0) b(E)$, being equivalent to Einstein equation of particle rest mass equivalent to the energy divided by the speed of light. The Forth derived is that an indeterminacy term product of the momentum and indeterminable space interval in mechanical corpuscular system is equivalent to determinable electromagnetic flux in electromagnetic system, so the term is equivalent to the uncertainty principle. From the above-mentioned thought, the Fifth will be able to be postulated that there is an absolute rest free spacetime with orthogonal relationship between a common time and the other spaces with homogeneity, isotropy, linearity and differentiable continuity with respect to independent variable the space and the time on the basis of exact differential equations for the space permits light to travel at invariant speed of light.


Key words: the wave- mass beam duality, the electromagnetic mass and momentum beam, absolute spacetime, an equivalent equation between mechanical corpuscular system and electromagnetic system, the uncertainty principle

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## 1. Introduction

For the Lorentz transformations ${ }^{1}$ has been properly validated increasing processes that rest mass of a particle tends to infinity as the velocity approaches the speed of light $c$ according to the Lorentz factor ${ }^{2}$, so most people believe that the wave-particle ${ }^{3}$ with mass is incompatible with the Lorentz transformations, despite that the transformations are incomplete at a point which velocity of the particle with mass is unable to attain to the speed of light. With that background, most people have assumed that the wave-particle duality is a classical unsolvable issue and no choice but to tackle in quantum mechanics ${ }^{4}$ that every particle or quantum entity may be described as either a particle or a wave.
So, according to a physics text ${ }^{5}$ approaching through quantum mechanisms, light has a mechanical corpuscular system nature traveling on an axis at the speed of light.
Wherein, apart from the above-mentioned quantum mechanics and the Lorentz transformations, this paper battles, focusing the incomplete point, only through a classical approach, against the duality through theoretical derivations from one-dimensional Maxwell's equations in free orthogonal space with properties of homogeneity, isotropy, continuity, linearity and differentiable continuity with respect of independent variable space $z$ in linear space and a common variable time $t$ under orthogonal relationship with each other.
With the background, the wave-particle duality will be able to proceed to a discussion only at the point of the speed of light in the space through the inherent derivations from one-dimensional Maxwell's equations on the basis of exact differential equations below.

Precaution statement from this point forward.
"Apart from well-known notation convention, to simplify and never to use superscript notation and subscript notation under no specifications to need to describe, so this paper is described as below:
(a) Scalar notation for all terms under no need to specify.
(b) Both permittivity $\varepsilon$ and permeability $\mu$ have no suffix zero: $\varepsilon(0)=\varepsilon, \mu(0)=\mu$.
(c) $\operatorname{Cub}(x)$ means the third power of $x, S q(x)$ means $x$ squared and $\operatorname{Sqrt}(x)$ means the square root of $x$.

To simplify, in case not to leading to misunderstand, we can use them as scalar notation for $\mathrm{D}, \mathrm{B}$ and $\mathrm{dz} / \mathrm{dt}$ are, respectively, function with independent variables $\mathrm{z}, \mathrm{t}$, and D , $B$ and $\mathrm{dz} / \mathrm{dt}$ are, respectively, function just on x axis, just on $y$ axis and just on $z$ axis.
(d) Electric flux density $\mathrm{D}=\mathrm{D}(\mathrm{x}): \partial(\partial \mathrm{D} / \partial \mathrm{z}) / \partial \mathrm{t}$ means the second partial derivative of D with respect to independent variable $\mathrm{x}, \mathrm{z}$ in space and t in time, second partial D by second partial z .
(e) Magnetic flux density $\mathrm{B}=\mathrm{B}(\mathrm{y}): \partial(\partial \mathrm{B} / \partial \mathrm{z}) / \partial \mathrm{z}$ means the second partial derivative of B with respect to independent variable $y, z$ in space and $t$ in time, second partial B by second partial z.
(f) Integral constant is able to be zero through choosing judiciously the point, so constant through the result of integrating will not be described for all terms integrating under no need to specify.
(g) All wave form stands for root mean square (afterward, means r.m.s) for mechanical energy is equivalent to electromagnetic energy"

## 2.Derivations of concerning equations

### 2.1 Space and time characteristics in free space, electric flux and magnetic flux density in the space

In appendix I, given postulates below,
(A) Both space and time have, respectively, properties of homogeneity, isotropy, linearity, differentiable continuity and the space and time is, respectively, geometric orthogonal relationship each other.
(B) Linear the space $z$ and the time $t$, respectively, is independent from each other, and independent variable on each axis.
(C) In consequence, in the geometric space with homogeneity, isotropy, linearity and differentiable continuity, there exists an electromagnetic constant product of invariant permittivity $\varepsilon$ and invariant permeability $\mu$ :

$$
\mathrm{d} \varepsilon=0, \mathrm{~d} \mu=0(0)
$$

(D) Under the above-mentioned postulates, all of the terms in this paper can be described with property of and differentiability on exact differential equations with independent variable, space $z$, time $t$ in free orthogonal space with no field and no media.
In appendix II, one-dimensional Maxwell's equations ${ }^{6}$ are described below.

$$
\begin{array}{r}
\partial \mathrm{D} / \partial \mathrm{z}=-\varepsilon(\partial \mathrm{B} / \partial \mathrm{t}) \\
\partial \mathrm{B} / \partial \mathrm{z}=-\mu(\partial \mathrm{D} / \partial \mathrm{t}) \tag{1-2}
\end{array}
$$

On the other hand, according to mathematical text ${ }^{7}, \mathrm{D}(\mathrm{x})$ and $\mathrm{B}(\mathrm{y})$ are described as exact differential equations, they can express below:

$$
\mathrm{dD}(\mathrm{x})=(\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{z}) \mathrm{dz}+(\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t}) \mathrm{dt}=0 \quad(1-3)
$$

$$
\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t}=-(\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{z})(\mathrm{dz} / \mathrm{dt})(1-4)
$$

Using equation of $\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{z}=-\varepsilon(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t})$ in onedimensional Maxwell's equations, substituting the equation into equation (1-4),

$$
\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t}=\varepsilon(\mathrm{dz} / \mathrm{dt})(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t})(1-5)
$$

As the result, we can get equation below.
$\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t}-\varepsilon(\mathrm{dz} / \mathrm{dt})(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t})=\partial(\mathrm{D}(\mathrm{x})-\varepsilon(\mathrm{dz} / \mathrm{dt})$ $\mathrm{B}(\mathrm{y})) / \partial \mathrm{t}=0(1-6)$
By judiciously choosing the point, under $\mathrm{D}(\mathrm{x})=0$ and $B(y)=0$ at a point of $x=0, y=0$ and $t=0$, we can obtain new equation below.

$$
\begin{array}{r}
\mathrm{D}=\varepsilon(\mathrm{dz} / \mathrm{dt}) \mathrm{B}(1-7) \\
\mathrm{D}(\mathrm{x})=\varepsilon(\mathrm{dz} / \mathrm{dt}) \mathrm{B}(\mathrm{y})(1-7-2)
\end{array}
$$

where $\mathrm{D}=\mathrm{D}(\mathrm{x}), \mathrm{B}=\mathrm{B}(\mathrm{y})$ to simplify under conditions that each term is only one direction.
In an analogous way above,

$$
\begin{aligned}
\mathrm{dB}(\mathrm{y})= & (\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{z}) \mathrm{dz}+(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t}) \mathrm{dt}=0(1-8) \\
& \partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t}=-(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{z})(\mathrm{dz} / \mathrm{dt})(1-9)
\end{aligned}
$$

By judiciously choosing the point: $\mathrm{B}(\mathrm{y})=0$ at a point of $\mathrm{y}=0$ and $\mathrm{t}=0$,
Therefore,

$$
\begin{array}{r}
\mathrm{B}=\mu(\mathrm{dz} / \mathrm{dt}) \mathrm{D}(1-10) \\
\mathrm{B}(\mathrm{y})=\mu(\mathrm{dz} / \mathrm{dt}) \mathrm{D}(\mathrm{x})(1-10-2)
\end{array}
$$

For equation (1-7) and (1-10), changing the scalar notations into vector notations, we can get equations.

$$
\begin{aligned}
\mathrm{D}(\mathrm{x}) \mathbf{i} & =\varepsilon \mathrm{B}(\mathrm{y}) \mathbf{j} \times(\mathrm{dz} / \mathrm{dt}) \mathbf{k}(1-11) \\
\mathrm{B}(\mathrm{y}) \mathbf{j} & =(\mathrm{dz} / \mathrm{dt}) \mathbf{k} \times \mu \mathrm{D}(\mathrm{x}) \mathbf{i}(1-12)
\end{aligned}
$$

where bold $\mathbf{i}, \mathbf{j}, \mathbf{k}$ is, respectively, unit vector in x direction on x axis, on y axis and on z axis, and $\times$ means vector product.
Next, using equations of (1-7) and (1-10) in scalar notations, multiplying both right hands and left hands in equations, so that, $\mathrm{BD}=\varepsilon \mu \mathrm{Sq}(\mathrm{dz} / \mathrm{dt}) \mathrm{BD}$, therefore, dividing the equation above by BD , we can get equation below

$$
\varepsilon \mu \mathrm{Sq}(\mathrm{dz} / \mathrm{dt})=1(1-13)
$$

Through invariant permittivity $\varepsilon$ and permeability $\mu$, the product of $\varepsilon$ and $\mu$ is invariant, derivative of the equation above, so we get an equation below.

$$
\mathrm{d}(\mathrm{Sq}(\mathrm{dz} / \mathrm{dt}))=0(1-14)
$$

so that,

$$
\mathrm{dz} / \mathrm{dt}=0 \quad(1-15)
$$

All of electromagnetic terms can travel at the invariant speed of light in space with invariant $\varepsilon$ and $\mu$.
Next, multiplying both sides in equation (1-7) by $\mu \mathrm{D}$ and both sides in equation (1-10) by $\varepsilon \mathrm{B}$, using equation (1-13), we can get equation below.

$$
\varepsilon \mathrm{Sq}(\mathrm{~B})=\mu \mathrm{Sq}(\mathrm{D})
$$

### 2.2 Main light properties, electromagnetic momentum and mass density

According to text with respect to light ${ }^{8}$, we know light properties as follows:
(2-a) going straight on a directional axis to travel in orthogonal free space from an evidence of beam ray.
(2-b) possessing momentum from an evidence of photoelectric effect.
(2-c) traveling at the speed of light in the abovementioned space and time.
(2-d) there is a form describable with orthogonal vector of electric and magnetic density in the space.
Electromagnetic momentum density with the above-mentioned properties in vector notation is defined below.

$$
\rho(\mathrm{p}) \mathbf{k}=\mathrm{Di} \times \mathrm{Bj}(2-1)
$$

where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ is, respectively, unit vector in x direction on x axis, y axis, and z axis.
$\rho(\mathrm{p})$ : electromagnetic momentum density
D: electric flux density only on an axis with variable x
B : magnetic flux density only on an axis with variable y
According to scalar notation, we can get the electromagnetic momentum density below,
$\rho(\mathrm{p})=\mathrm{DB}[\mathrm{kgm} / \mathrm{s} / \mathrm{Cub}(\mathrm{m})](2-2)$ Given conditions that the momentum has property of differentiable continuity with respect to variable z and time independent each other and describable as exact differential equation in orthogonal free space above-mentioned in 2.1, differentiating equation (2-2), using equations of (1-7-2) and (1-10-2)
$\mathrm{d} \rho(\mathrm{p})=\mathrm{BdD}+\mathrm{DdB}$

$$
=(\mathrm{dz} / \mathrm{dt})(1 / 2) \mathrm{d}(\mu \mathrm{Sq}(\mathrm{D})+\varepsilon \mathrm{Sq}(\mathrm{~B}))(2-3)
$$

Therefore, electromagnetic mass density is defined as below.
$\rho(\mathrm{m})=(1 / 2)(\mu \mathrm{Sq}(\mathrm{D})+\varepsilon \mathrm{Sq}(\mathrm{B}))$
$[\mathrm{kg} / \mathrm{Cub}(\mathrm{m})](2-4)$
Using equation $\varepsilon \mathrm{Sq}(\mathrm{B})=\mu \mathrm{Sq}(\mathrm{D})$ (1-16), we can get equations below.

$$
\begin{aligned}
& \rho(\mathrm{m})=\mu \mathrm{Sq}(\mathrm{D})[\mathrm{kg} / \mathrm{Cub}(\mathrm{~m})](2-4-2) \\
& \rho(\mathrm{m})=\varepsilon \mathrm{Sq}(\mathrm{~B})[\mathrm{kg} / \mathrm{Cub}(\mathrm{~m})](2-4-3)
\end{aligned}
$$

Replacing equation (2-3) with equation (2-4), we can get new relationship between the momentum and mass density is expressed below.

$$
\mathrm{d} \rho(\mathrm{p})=(\mathrm{dz} / \mathrm{dt}) \mathrm{d} \rho(\mathrm{~m})(2-3)
$$

By judiciously choosing the point, that is, $\rho(\mathrm{p})$ $=0$ and $\rho(m)=0$ at two points of $x=0, y=0$, and $\mathrm{t}=0$, removing the derivative sign d from each hand in equation (2-3), we can get equation
below.
$\rho(\mathrm{p})=(\mathrm{dz} / \mathrm{dt}) \rho(\mathrm{m})[\mathrm{kgm} / \mathrm{s} / \mathrm{Cub}(\mathrm{m})](2-4)$
Using each the equation (1-7) and (1-10), we can simplify the equation (2-4) into each equation below.

$$
\rho(\mathrm{p})=\mu(\mathrm{dz} / \mathrm{dt}) S q(\mathrm{D})[\mathrm{kgm} / \mathrm{s} / \mathrm{Cub}(\mathrm{~m})](2-4-2)
$$

$$
\rho(\mathrm{p})=\varepsilon(\mathrm{dz} / \mathrm{dt}) \mathrm{Sq}(\mathrm{~B})[\mathrm{kgm} / \mathrm{s} / \mathrm{Cub}(\mathrm{~m})](2-4-3)
$$

Besides, multiplying both sides in equation (2-4) by (dz/dt), we can get new energy density equation (2-5) defined as electromagnetic energy density $\rho(\mathrm{E})$ below.
$\rho(\mathrm{E})=(\mathrm{dz} / \mathrm{dt}) \rho(\mathrm{p})=\mathrm{Sq}(\mathrm{dz} / \mathrm{dt}) \rho(\mathrm{m})$
$[\mathrm{J} / \mathrm{Cub}(\mathrm{m})](2-5)$
Using equation (1-7) and (1-10), we can simplify the equation (2-5) into equation below.

$$
\begin{equation*}
\rho(\mathrm{E})=(1 / 2)(\mathrm{Sq}(\mathrm{D}) / \varepsilon+\mathrm{Sq}(\mathrm{~B}) / \mu) \tag{m}
\end{equation*}
$$

$$
\rho(\mathrm{E})=\mathrm{Sq}(\mathrm{D}) / \varepsilon[\mathrm{J} / \mathrm{Cub}(\mathrm{~m})](2-6-2)
$$

$$
\rho(\mathrm{E})=\mathrm{Sq}(\mathrm{~B}) / \mu[\mathrm{J} / \mathrm{Cub}(\mathrm{~m})](2-6-3)
$$

So, dividing both sides in equation (2-6) by $\mathrm{Sq}(\mathrm{dz} / \mathrm{dt})$ equal to reciprocal product of permittivity and permeability and using equation (1-13), we can obtain electromagnetic energy density defined as below.

$$
\rho(\mathrm{m})=\varepsilon \mu \rho(\mathrm{E})(2-7)
$$

Given that electromagnetic momentum $\rho(\mathrm{p})$, energy $\rho(E)$ and mass density $\rho(m)$ is describable to exact differential equations, using equation (1$1)$ and $(1-2), d \rho(m)=0, d \rho(E)=0$ and $d \rho(p)=0$, so we can get equations below.

$$
\begin{array}{r}
\partial \rho(\mathrm{p}) / \partial \mathrm{t}=-(\mathrm{dz} / \mathrm{dt}) \partial \rho(\mathrm{p}) / \partial \mathrm{z}(2-8) \\
\partial \rho(\mathrm{m}) / \partial \mathrm{t}=-(\mathrm{dz} / \mathrm{dt}) \partial \rho(\mathrm{m}) / \partial \mathrm{z}(2-9) \\
\partial \rho(\mathrm{E}) / \partial \mathrm{t}=-(\mathrm{dz} / \mathrm{dt}) \partial \rho(\mathrm{E}) / \partial \mathrm{z}(2-10)
\end{array}
$$

### 2.3 Electromagnetic wave functions

Since $\partial(\partial \mathrm{g}(\mathrm{z}, \mathrm{t}) / \partial \mathrm{z}) / \partial \mathrm{t}$ and $\partial(\partial \mathrm{g}(\mathrm{z}, \mathrm{t}) / \partial \mathrm{t}) / \partial \mathrm{z}$ are continuous by hypothesis in orthogonal spacetime, the cross derivative $\partial(\partial \mathrm{g}(\mathrm{z}, \mathrm{t}) / \partial \mathrm{t}) / \partial \mathrm{z}$ and $\partial(\partial \mathrm{g}(\mathrm{z}, \mathrm{t}) / \partial \mathrm{t}) / \partial \mathrm{z}$ are also continuous and therefore equal ${ }^{9}$, using equation (2-4), (2-5) and (2-7), equation (1-1) and (1-2) in Maxwell's equations, under the above-mentioned postulates, we can get an equation described electromagnetic characteristic function $\mathrm{g}(\mathrm{z}, \mathrm{t})$ as follows.
$\partial(\partial g(z, t) / \partial z) / \partial t=\mu \varepsilon \partial(\partial g(z, t) / \partial t) / \partial z(3-1)$ where we can replace function $g(z, t)$ with terms:
(a) Electric flux density D, and the density squared, $\mathrm{Sq}(\mathrm{D})$
(b) Magnetic flux density B, and the density squared $\mathrm{Sq}(\mathrm{B})$
(c) Electromagnetic energy density
(d) Electromagnetic momentum density
(e) Electromagnetic mass density

This equation $\mathrm{g}(\mathrm{z}, \mathrm{t})$ is called as one-dimensional
wave equation ${ }^{10}$, however, the equation (3-1) includes momentum and mass density incompatible with so-called wave function, so in this paper, the equation is called electromagnetic characteristic function, wave-particle function or the duality function.
Besides, given conditions that all of terms is zero at a start point of $\mathrm{z}=0$ and $\mathrm{t}=0$, using the equation (3-1), electric flux density D and the density squared are, respectively, expressed wave form with root mean square base below.
$\mathrm{D}=\mathrm{A}(\mathrm{D}) \operatorname{Exp}(2 \pi j \theta) / \mathrm{Sq}(2)$
$[\mathrm{As} / \mathrm{Sq}(\mathrm{m})](3-2-1)$
$\mathrm{Sq}(\mathrm{D})=\mathrm{A}(\mathrm{DD}) \operatorname{Exp}(4 \pi \mathrm{j} \theta) / 2$
$[\mathrm{Sq}(\mathrm{As} / \mathrm{Sq}(\mathrm{m}))](3-2-2)$
Next, magnetic flux density $B$ and the density squared
$\mathrm{B}=\mathrm{A}(\mathrm{B}) \operatorname{Exp}(2 \pi \mathrm{j} \theta) / \mathrm{Sq}(2)[\mathrm{Vs} / \mathrm{Sq}(\mathrm{m})](3-3-1)$ $\mathrm{Sq}(\mathrm{B})=\mathrm{A}(\mathrm{BB}) \operatorname{Exp}(4 \pi \mathrm{j} \theta) / 2$

$$
[\mathrm{Sq}(\mathrm{Vs} / \mathrm{Sq}(\mathrm{~m}))](3-3-2)
$$

where
phase function $\theta$ with respect to frequency $f$ and wave number $k$ at an observer's eye on rest position,

$$
\theta=\mathrm{ft}-\mathrm{kz}(3-9)
$$

Substitute the equation of $\operatorname{Sq}(\mathrm{D})$ into the equation (2-4-2), we can get an equation of electromagnetic mass density $\rho(\mathrm{m})$ below.

$$
\rho(\mathrm{m})=\mu \operatorname{Sq}(\mathrm{D})=\mathrm{A}(\mathrm{~m}) \operatorname{Exp}(4 \pi \mathrm{j} \theta)(3-10)
$$

where

$$
\mathrm{A}(\mathrm{~m})=\mu \mathrm{A}(\mathrm{DD})(3-11)
$$

Square root of both hand in equation (3-10), we can get equation in mass form below.

$$
D=\operatorname{Sqrt}(\rho(\mathrm{m}) / \mu)
$$

$$
=\operatorname{Sqrt}(\mathrm{A}(\mathrm{~m}) / \mu) \operatorname{Exp}(2 \pi \mathrm{j} \theta)(3-12-1)
$$

Using equation (2-4-3) and applying the same process, we can get equation in mass form below. $B=\operatorname{Sqrt}(\rho(\mathrm{m}) / \varepsilon)$

$$
=\operatorname{Sqrt}(\mathrm{A}(\mathrm{~m}) / \varepsilon) \operatorname{Exp}(2 \pi \mathrm{j} \theta)(3-12-2)
$$

Besides, substituting the equation of $\rho(\mathrm{m})$ into the equation (3-1) under invariant frequency $f$ and wave number k , we can get an equation below.

$$
\mathrm{Sq}(\mathrm{f})=\mu \varepsilon \mathrm{Sq}(\mathrm{k})(3-13)
$$

Analogy to the same process with respect to the mass density, we can describe the momentum and energy density in wave form below.

$$
\begin{aligned}
\rho(\mathrm{p}) & =A(\mathrm{p}) \operatorname{Exp}(4 \pi \mathrm{j} \theta)(3-14) \\
\rho(\mathrm{E}) & =A(\mathrm{E}) \operatorname{Exp}(4 \pi \mathrm{j} \theta)(3-15)
\end{aligned}
$$

### 2.4 The indeterminacy principle in mechanical corpuscular system and determinacy principle in electromagnetic system

Reviewing momentum density equation (2-2) with respect to electromagnetic momentum flux
density, right hand in this equation and left hand is, respectively, able to review as mechanical corpuscular system, as electromagnetic system.
So, multiplying right hand by element area $\mathrm{dA}(\mathrm{x}$, $z)=d x d z$ on an $x-z$ plane and $d A(y, z)=d y d z$ on an $y-z$ plane, in the right hand, so that we can be defined as electromagnetic flux $\phi(\mathrm{DB})$ below.

$$
\phi(\mathrm{DB})=\phi(\mathrm{D}) \phi(\mathrm{B})[\mathrm{Js}](4-1)
$$

where
electric flux: $\phi(\mathrm{D})=\mathrm{D}$ dA(x, z) $=$ Ddxdz (4-2-1) magnetic flux: $\phi(\mathrm{B})=\mathrm{BdA}(\mathrm{y}, \mathrm{z})=\mathrm{Bdydz}(4-2-2)$ Next, multiplying left hand by element of dxdydzdz, given momentum flux $\phi(\mathrm{p})$ defined as one product of momentum flux density $\rho(p)$ in $z$ direction and element area dxdy, so that, in the left hand, we can be defined as electromagnetic linear momentum flux $\phi$ (p) and electromagnetic momentum beam element $b(p)$ below.

$$
\text { left hand }=\rho(p) d x d y d z d z=b(p) \times d z(4-3)
$$

where,

$$
\phi(\mathrm{p})=\rho(\mathrm{p}) \times \mathrm{d} x \mathrm{dy}[\mathrm{kgm} / \mathrm{s} / \mathrm{m}]
$$

viewing as electromagnetic momentum beam element $b(p)$ product of linear momentum flux and the length element dz , we can get equation below.

$$
\mathrm{b}(\mathrm{p})=\phi(\mathrm{p}) \times \mathrm{dz}[\mathrm{~J} / \mathrm{Cub}(\mathrm{~m})](4-5)
$$

In consequence, equation (4-4) in right hand equals equation (4-5) in left hand, so that, we can obtain equation below.

$$
\mathrm{b}(\mathrm{p}) \times \mathrm{dz}=\phi(\mathrm{DB})(4-6)
$$

This equation means that a bundled momentum beam element $b(p)$ in mechanical corpuscular system is equivalent to an electromagnetic flux $\phi(\mathrm{DB})$ in electromagnetic flux system and left hand in this equation has underminable specific length dz , on the other right hand, the electromagnetic flux $\phi(\mathrm{DB})$ is determinable.
Furthermore, using $d z=(d z / d t) d t$, given a finite difference $\delta z$ on an z and a finite difference $\delta t$ on an $t$,

$$
\begin{aligned}
\mathrm{dz} & \approx \delta \mathrm{z}(4-7) \\
\mathrm{dt} & \approx \delta \mathrm{t}(4-8)
\end{aligned}
$$

so that we can review indeterminacy equations with respect to momentum and energy in a mechanical corpuscular system for electromagnetic flux system with no indeterminacy as follows.

$$
\begin{aligned}
& \mathrm{b}(\mathrm{p}) \times \delta \mathrm{z}=\phi(\mathrm{DB})(4-9-1) \\
& \mathrm{b}(\mathrm{p}) \times \mathrm{dz}=\phi(\mathrm{DB})(4-9-2)
\end{aligned}
$$

Analogy to the above processing with respect to electromagnetic mass and energy density, we can get mass and energy flux defined as, respectively, $\phi(\mathrm{m})$ and $\phi(\mathrm{E})$ and electromagnetic mass beam element $b(m)$, electromagnetic energy beam
element $b(E)$.
where

$$
\begin{array}{r}
\phi(\mathrm{m})=\rho(\mathrm{m}) \mathrm{dxdy}[\mathrm{~kg} / \mathrm{m}](4-10) \\
\mathrm{b}(\mathrm{~m})=\phi(\mathrm{m}) \times \mathrm{dz}[\mathrm{~kg}](4-11) \\
\phi(\mathrm{E})=\rho(\mathrm{E}) \mathrm{dxdy}[\mathrm{~J} / \mathrm{m}](4-12) \\
\mathrm{b}(\mathrm{E})=\phi(\mathrm{E}) \times \mathrm{dz}[\mathrm{~J}](4-13)
\end{array}
$$

In consequence, using $\mathrm{dz}=(\mathrm{dz} / \mathrm{dt}) \mathrm{dt}$ and $\delta \mathrm{z}=$ $(\mathrm{dz} / \mathrm{dt}) \delta t$, using beam element equation (4-11) and (4-13), we can get a relationship between electromagnetic flux system and mechanical corpuscular system below.

$$
\begin{array}{r}
\phi(\mathrm{DB})=\mathrm{b}(\mathrm{~m})(\mathrm{dz} / \mathrm{dt}) \times \mathrm{dz}(4-14) \\
\phi(\mathrm{DB})=\mathrm{b}(\mathrm{~m}) \mathrm{Sq}(\mathrm{dz} / \mathrm{dt}) \times \mathrm{dt}=\mathrm{b}(\mathrm{p})(\mathrm{dz} / \mathrm{dt}) \times \mathrm{dt}= \\
\mathrm{b}(\mathrm{E}) \times \mathrm{dt}(4-15)
\end{array}
$$

So, putting each equation above into finite difference variable $\delta \mathrm{z}$ and $\delta$ t, we can a finite difference relationship between electromagnetic flux system and mechanical corpuscular system below.

$$
\begin{array}{r}
\phi(\mathrm{DB})=\mathrm{b}(\mathrm{~m})(\mathrm{dz} / \mathrm{dt}) \times \delta \mathrm{z}(4-16) \\
\phi(\mathrm{DB})=\mathrm{b}(\mathrm{~m}) \mathrm{Sq}(\mathrm{dz} / \mathrm{dt}) \times \delta \mathrm{t}=\mathrm{b}(\mathrm{p})(\mathrm{dz} / \mathrm{dt}) \times \delta \mathrm{tt}= \\
\mathrm{b}(\mathrm{E}) \times \delta \mathrm{t}(4-17)
\end{array}
$$

## 3. Conclusions

The above-mentioned equations will lead to conclusions below.

### 3.1 Absolute spacetime

According to equation (3-1), given electromagnetic constant product of invariant permittivity $\varepsilon$ and invariant permeability $\mu$ in a free orthogonal space with homogeneity, isotropy, linearity and differentiable continuity with respect to independent variable of a directional z and time $t$ on the basis of exact differential equations, electromagnetic flux density terms (electric, magnetic, mass, momentum and energy) are able to be described as wave form with the same invariant speed of light squared equal to reciprocal of the constant. In consequence, a postulate will be able to make that there is an absolute rest space for the space permits light to travel at the speed of light equal to the reciprocal square root of the constant in the space with no media and no field so that all of electromagnetic terms with corpuscular property can travel like bullet in the space.
In other word, we will be able to postulate an existence of absolute spacetime with each equal interval of space and time in the space and each orthogonal axis. In other word, the absolute space and time have, respectively, invariant space interval unit and invariant time interval unit, so a frame with the space and time is absolutely at rest. At the first setout, if an absolute spacetime
defined here is assumed, second, proposed the spacetime with a property of the invariant speed of light, third, showed up the wave-mass beam duality derived from Maxwell's equations, light bending will be derived under star emitted huge light intensity with big gravitation and collisions of the radiated light with the light reflected.
If into reverse, space will have to be bend.
In consequence, we will need no bending space.
3.2 The wave - mass beam duality of both properties in electromagnetic flux system and in a mechanical corpuscular system
The electromagnetic mass, that is, photon mass ${ }^{11}$ has denied up to the present time because the Lorentz transformations is non-permissive for an existence of the mass. From equations (3-1), (46 ), (4-11) and (4-13) with respect to electromagnetic momentum, mass and energy traveled on a directional axis, we can find out that electromagnetic beam duality has both forms of electromagnetic wave in the system and a bundled mass beam element in a mechanical corpuscular system.
Besides, from each equation (3-12-1) and (3-122 ), the electric flux density D and magnetic density B show, respectively, wave form with electromagnetic mass. In consequence, each electric and magnetic flux density, momentum, mass and energy can undulate in mass form and all of them can travel concurrently at the speed of light on each axis under no field condition like bullet in free space.
3.3 The uncertainty principle, equivalent equation between the momentum beam with an indeterminacy specific length on a direction for light to travel in a mechanical corpuscular system and determinacy electromagnetic flux in electromagnetic system
The indeterminacy equation (4-10) with respect to the momentum and (4-11) with respect to the energy in a mechanical corpuscular system for electromagnetic flux system with no indeterminacy is, respectively, equivalent to socalled the uncertainty principle ${ }^{12}$ under postulate that the electromagnetic flux $\phi$ product of electric flux $\phi(\mathrm{D})$ and magnetic flux $\phi(\mathrm{B})$ is equal to Planck constant h, we can describe below.
$(\mathrm{dz} / \mathrm{dt}) \phi(\mathrm{m}) \delta \mathrm{z}=\mathrm{h}$

$$
\mathrm{Sq}(\mathrm{dz} / \mathrm{dt}) \phi(\mathrm{m}) \delta \mathrm{t}=\mathrm{h}
$$

where electromagnetic flux $\phi$ is equal to Planck constant: $\phi=\mathrm{h}$

At present, on the basis of the above-mentioned
equations, current preparations papers are made below.
Photoelectric effect (prior notice), Lorentz's and Coulomb's force (prior notice), Relationship between electron mass, spherical radius and elementary charge (prior notice), Electron substructure with fractal charge and relationship between the electron and a nucleus with positive fractal charge at the center of an atom (prior notice), Conservation law with respect to electromagnetic mass, momentum and energy at all points in time like mechanical system with potential energy and kinetic energy (prior notice), Light bending under a star with huge gravitational force and with the surface radiated a massive amount of light and satellite reflected the light (prior notice), Double slits, Reflection, Refraction, Interference, Diffraction, Polarization (prior notice)

## Appendix I

Under postulates below.
(A) Both space and time have, respectively, properties of homogeneity, isotropy, linearity, differentiable continuity and the space and time is, respectively, geometric orthogonal relationship each other.
(B) Linear the space $z$ and the time $t$, respectively, is independent from each other, and independent variable on each axis.
(C) In consequence, in the geometric space with homogeneity, isotropy, linearity and differentiable continuity, there exists an electromagnetic constant product of invariant permittivity $\varepsilon$ and invariant permeability $\mu$ :
$\mathrm{d} \varepsilon=0, \mathrm{~d} \mu=0(\mathbf{I}-0)$
(D) Under the above-mentioned postulates, all of the terms in this paper can be described with property of and differentiability on exact differential equations with independent variable, space $z$, time $t$ in the space with no field and no media.

## Appendix II

Under thus conditions above-mentioned, according to mathematical text ${ }^{13}$, electric flux density and magnetic flux density expressed through exact differential equation is, respectively, able to describe below.

$$
\mathrm{dD}=0, \partial(\partial \mathrm{D} / \partial \mathrm{z}) / \partial \mathrm{t}=\partial(\partial \mathrm{D} / \partial \mathrm{t}) / \partial \mathrm{z}(\mathbf{I I}-1)
$$

$$
\mathrm{dB}=0, \partial(\partial \mathrm{~B} / \partial \mathrm{z}) / \partial \mathrm{t}=\partial(\partial \mathrm{B} / \partial \mathrm{t}) / \partial \mathrm{z}(\mathbf{I I}-2)
$$

According to electromagnetic text ${ }^{14}$ concerning to one-dimensional Maxwell's equations,

$$
\begin{equation*}
\partial \mathrm{D} / \partial \mathrm{z}=-\varepsilon(\partial \mathrm{B} / \partial \mathrm{t}) \tag{II-3}
\end{equation*}
$$

$\partial \mathrm{B} / \partial \mathrm{z}=-\mu(\partial \mathrm{D} / \partial \mathrm{t}) \quad(\mathbf{I I}-4)$
Substituting equations (II-3) and (II-4) into (II-1) and (II-2), we can get one-dimensional wave function below.

$$
\begin{aligned}
& \partial(\partial \mathrm{D} / \partial \mathrm{z}) / \partial \mathrm{z}=\varepsilon \mu \partial(\partial \mathrm{D} / \partial \mathrm{t}) / \partial \mathrm{t}(\mathbf{I I}-5) \\
& \partial(\partial \mathrm{B} / \partial \mathrm{z}) / \partial \mathrm{z}=\varepsilon \mu \partial(\partial \mathrm{B} / \partial \mathrm{t}) / \partial \mathrm{t}(\mathbf{I I}-6)
\end{aligned}
$$

On the other hand, $\mathrm{D}(\mathrm{x})$ and $\mathrm{B}(\mathrm{y})$ are described as exact differential equations, they can express below:

$$
\begin{array}{r}
\mathrm{dD}(\mathrm{x})=(\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{z}) \mathrm{dz}+(\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t}) \mathrm{dt}=0 \\
\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t}=-(\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{z})(\mathrm{dz} / \mathrm{dt})(\mathbf{I I}-7) \\
\text { Using equation of } \partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{z}=-\varepsilon(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t}), \\
\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t}=\varepsilon(\mathrm{dz} / \mathrm{dt})(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t})(\mathrm{II}-8) \\
\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t}-\varepsilon(\mathrm{dz} / \mathrm{dt})(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t}) \\
=\partial(\mathrm{D}(\mathrm{x})-\varepsilon(\mathrm{dz} / \mathrm{dt}) \mathrm{B}(\mathrm{y})) / \partial \mathrm{t}=0(\mathbf{I I}-9)
\end{array}
$$

By judiciously choosing the point: $\mathrm{D}(\mathrm{x})=0$ at a point of $\mathrm{x}=0$,
Equation above,

$$
\mathrm{D}(\mathrm{x})=\varepsilon(\mathrm{dz} / \mathrm{dt}) \mathrm{B}(\mathrm{y})
$$

In an analogous way above,
$d \mathrm{~B}(\mathrm{y})=(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{z}) \mathrm{dz}+(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t}) \mathrm{dt}=0$ $\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t}=-(\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{z})(\mathrm{dz} / \mathrm{dt})(\mathbf{I I}-10)$
Using equation of $\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{z}=-\mu(\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t})$,

$$
\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t}=\mu(\mathrm{dz} / \mathrm{dt})(\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t})(\mathbf{I I}-11)
$$

$\partial \mathrm{B}(\mathrm{y}) / \partial \mathrm{t}-\mu(\mathrm{dz} / \mathrm{dt})(\partial \mathrm{D}(\mathrm{x}) / \partial \mathrm{t})$
$=\partial(\mathrm{B}(\mathrm{y})-\mu(\mathrm{dz} / \mathrm{dt}) \mathrm{D}(\mathrm{x})) / \partial \mathrm{t}=0(\mathbf{I I}-12)$

By judiciously choosing the point: $\mathrm{B}(\mathrm{y})=0$ at a point of $y=0$, the equation () above is,

$$
\mathrm{B}(\mathrm{y})=\mu(\mathrm{dz} / \mathrm{dt}) \mathrm{D}(\mathrm{x})(\mathrm{II}-13)
$$

In the result of the process above, in scalar notation, we can get equations.

$$
\begin{aligned}
& \mathrm{D}(\mathrm{x})=\varepsilon(\mathrm{dz} / \mathrm{dt}) \mathrm{B}(\mathrm{y})(\mathbf{( I I}-14) \\
& \mathrm{B}(\mathrm{y})=\mu(\mathrm{dz} / \mathrm{dt}) \mathrm{D}(\mathrm{x})(\mathbf{I I}-15)
\end{aligned}
$$

In vector notations,

$$
\begin{array}{r}
\mathrm{D}(\mathrm{x}) \mathbf{i}=\varepsilon \mathrm{B}(\mathrm{y}) \mathbf{j} \times(\mathrm{dz} / \mathrm{dt}) \mathbf{k}(\mathbf{I I}-16) \\
\mathrm{B}(\mathrm{y}) \mathbf{j}=-\mu \mathrm{D}(\mathrm{x}) \mathbf{i} \times(\mathrm{dz} / \mathrm{dt}) \mathbf{k}(\mathbf{I I}-17)
\end{array}
$$

where bold $\mathbf{i}, \mathbf{j}, \mathbf{k}$ is, respectively, unit vector in x direction on x axis, on y axis and on z axis, and $\times$ means vector product.

$$
\begin{aligned}
& \mathrm{D}=\varepsilon(\mathrm{dz} / \mathrm{dt}) \mathrm{B}(\mathbf{I I}-18) \\
& \mathrm{B}=\mu(\mathrm{dz} / \mathrm{dt}) \mathrm{D}(\mathbf{I I}-19)
\end{aligned}
$$

Multiplying each right hands and left hands in equation (II-18) and (II-19), we can obtain an equation below.

$$
\varepsilon \mu \mathrm{Sq}(\mathrm{dz} / \mathrm{dt})=1(\mathbf{I I}-20)
$$

Next, through invariant permittivity $\varepsilon$ and permeability $\mu$, the product of $\varepsilon$ and $\mu$ is invariant, derivative of the equation above, under conditions of equation (I-0), so we get an equation below.

$$
\mathrm{d}(\mathrm{Sq}(\mathrm{dz} / \mathrm{dt}))=0(\mathbf{I I}-21)
$$

$$
\mathrm{dz} / \mathrm{dt}=0(\mathbf{I I}-22)
$$

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[^0]:    ${ }^{1}$ Self-funding researcher tackling inherent derivations theoretically from one-dimensional Maxwell's equations on the basis of exact differential equations.
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