SunQM-4: Using full-QM deduction and {N,n} QM's non-Born probability density 3D map to build a complete Solar system with orbital movement

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Abstract

In SunQM-3 series, we studied Solar {N,n} QM within the boundary of the traditional Schrodinger equation/solution and Born's rule. In SunQM-4 series, we start to relax that boundary. In the current paper, for a planet in nLL QM state doing circular orbital movement, we deduced out (a full-QM deduced) |Φ(φ)|^2 * |T(t)|^2 for a planet's time-dependent probability density in φ-dimension. To satisfy the well-known QM rule that a matter wave's group velocity equals to 2x of its phase velocity, we have to define a non-Born calculation as |T(t)|^2 ∝ [exp(-i * $\omega_{n,ph}$ * t)]^2 where $\omega_{n,ph}$ is the phase angular frequency of a planet's matter wave in φ-dimension. To obtain a physical meaningful |Φ(φ)|^2 * |T(t)|^2, we have to define a non-Born probability (NBP) density calculation as |Φ(φ)|^2 ∝ Φ(φ), or its φ-dimensional probability density function is directly proportional to its matter wave function. Combining with SunQM-3s11's result, we built a complete Solar system with time-dependent circular orbital movement using the full-QM deduced non-Born probability density 3D map. This 3D probability density described a Solar system not only at planet's Eigen description level, but also at any level of resolution (down to proton level, or up to the whole universe level). Therefore, we propose that "Simultaneous-Multi-Eigen-Description (SMED)" is one of many nature attributes of QM. We believe that by adding the non-Born calculation to Born calculation, the QM will become more self-consistent and more complete.

Introduction

The SunQM series articles [1] ~ [16] have shown that the formation of Solar system (as well as each planet) was governed by its {N,n} QM. In SunQM-3 series [6] ~ [16], we studied Solar {N,n} QM by using the traditional Schrodinger equation/solution and Born's probability. For example, in papers SunQM-3s6, -3s7, and -3s8, it has been shown that the formation of planet's and star's (radial) internal structure is governed by the planet's or star's radial QM. In papers SunQM-3s3 and -3s9, it has been shown that the surface mass (atmosphere) movement of Sun, Jupiter, Saturn, and Earth, etc., is governed by Star's (or planet's) θφ-2D dimensional QM. In paper SunQM-3s4 and -3s10, it has been shown that the formation of either the ring structures of a planet, or the belt structures in Solar system, is also governed by the {N,n} QM (the nLL effect). In paper SunQM-3s11, we have used {N,n} QM and Schrodinger equation's solution to build a 3D probability density map for a complete Solar system with time-dependent orbital movement. In SunQM-4 series, we are going to study Solar {N,n} QM by using the method that more or less deviated from the traditional Schrodinger equation and Born's rule. In the current paper, after using a non-Born probability (NBP) calculation (where the probability is directly proportional to the wave function, not the conjugated-squared wave function), we have built a NBP density 3D map for a complete Solar system with time-dependent orbital movement. Note: for {N,n} QM nomenclature as well as the general notes for {N,n} QM model, please see SunQM-1 section VII. Note: Microsoft Excel's number format is often used in this paper, for example: x^2 = x^2, 3.4E+12 = $3.4*10^{12}$, 5.6E-9 = $5.6*10^{-9}$. Note: The reading sequence for SunQM series papers is: SunQM-1, 1s1, 1s2, 1s3, 2, 3, 3s1, 3s2, 3s6, 3s7, 3s8, 3s3, 3s9, 3s4, 3s10, 3s11, 4. Note: for all SunQM series papers, reader should check "SunQM-9s1: Updates and Q/A for SunQM series papers" for the most recent updates and corrections.

Note: The key reason to publish the new version (named version 2021) of this paper and to overwrite the previous (3/25/2020) version is that, the exponential index of eq-55a has been corrected to n/2 (from n in the old version), because according to the calculation in SunQM-6s1, only n/2 (not n) will give the 3D spherical NBP probability peak in eq-56a.

Therefore, many equations (after eq-55a) in this paper have been updated accordingly, and all of updated equations have the yellow-highlighted equation numbers. The new versions of the alternative NBP calculation methods are also added in Appendix A, Appendix B, Appendix C, and Appendix D. The rest part of this paper follows the original version closely.

I. To build a time-dependent 3D probability density for a planet (in Solar system's orbit) based on non-Born probability $|\Phi(\varphi)|^2_{NBP} \propto e^{im\varphi}$

In the SunQM-3 series papers, we have demonstrated that our Solar system can be described by Schrodinger equation

$$i\hbar \frac{\partial}{\partial t}\Psi(r,\theta,\varphi,t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(r,\theta,\varphi,t)\right]\Psi(r,\theta,\varphi,t) \qquad \text{eq-1}$$

Under certain physics condition (e.g., plane wave, or hydrogen atom, etc.), Schrodinger equation (as a linear partial differential equation) can be solved by separating the variables so that we can find solutions that are simple products of

$$\Psi(r,\theta,\varphi,t) = R(r)\,\Theta(\theta)\,\Phi(\varphi)\,T(t) \qquad \text{eq-2}$$

Because of the rotation diffusion (or RotaFusion, or RF, see SunQM-2 for details) in θφ-2D-dimension, the two functions of Ψ in θ- and φ-dimensions are usually grouped together as spherical harmonics

$$\Theta(\theta)\,\Phi(\varphi) = Y(l,m) \qquad \text{eq-3}$$

Although the traditional time-independent probability density formula for Schrodinger equation is written as

$$r^2\,|\Psi(r,\theta,\varphi,t)|^2 = r^2\,|R(n,l)|^2\,|Y(l,m)|^2 \qquad \text{eq-4}$$

where R(r) = R(n), the general time-dependent probability density formula for Schrodinger equation should be something like

$$r^2\,|\Psi(r,\theta,\varphi,t)|^2 = r^2\,|R(r)|^2\,|\Theta(\theta)|^2\,|\Phi(\varphi)|^2\,|T(t)|^2 \qquad \text{eq-5}$$

In SunQM-3s11 sections II-a, II-b and II-c, planet's probability density in r-dimension r^2 * |R(r)|^2, in θ-dimension |Θ(θ)|^2, and in φ-dimension |Φ(φ)|^2 were all deduced by using full-QM. However, for the time-dependency (of φ-dimensional orbit movement), we simply replaced φ by φ-ωt. This makes the φ-dimension's time-dependent probability |Φ(φ)|^2 * |T(t)|^2 become a semi-QM deduced function. So in the current paper, we try to deduce |Φ(φ)|^2 and |T(t)|^2 separately, and then combine them through a simple production as shown in eq-5.

I-a. Defining $|\Phi(\varphi)|^2_{NBP} \propto e^{im\varphi}$, or $e^{-im\varphi} = 1$, for a planet's φ-dimensional probability density in Solar system

In the traditional QM, for the nLL QM state (where *l*=n-1, m=n-1, see SunQM-3s1), by default, the φ-dimension's Born probability density

$$|\Phi(\varphi)|^2 \propto \left|e^{im\varphi}\right|^2 = e^{-im\varphi}e^{+im\varphi} = 1 \qquad \text{eq-6}$$

It can be explained as its +φ directional wave cancels out its own -φ directional wave. Now we try to use the general physics to explain why this is not correct for a planet's orbital movement in the {N,n} QM (because all QM has to degenerated back to classical physics when the quantum number n → ∞). For the Solar {N,n} QM, let's define that exp(+imφ) correlates to the (planet mass' or matter wave's) eastward circular orbital movement, and exp(-imφ) correlates to the (planet mass' or matter wave's) westward circular orbital movement. From the result of SunQM-3s8 Table 6, we understand that Solar {N,n} QM's mass movement (or matter wave movement) is directly correlated to the mass' orbital velocity (notice that matter wave is a wave packet, so its orbital velocity is group velocity). Inside the Sun (where most atoms are not in nLL QM state), this orbital velocity mass movement is transformed into the micro thermal movement, and it is always in RF (notice that Sun's self-spin movement is ignored here, because Sun's spin velocity is too slow in comparison to Sun atoms' thermal movement velocity). Outside the Sun in the planet region, the same orbital velocity mass movement is transformed into planet's (macro) orbital movement and this orbital movement is neither the micro thermal movement, nor in RF.

For the Solar {N,n} QM, the traditional QM probability calculation is still correct for the space region with ~100% mass occupancy (while the slow macro movement can be ignored). Because there (inside Sun, ignoring the Sun self-spin, only consider the micro thermal movement) all mass is doing RF, so (in φ-dimension) at any time, there is always equal amount of mass (or matter waves) doing the eastward exp(imφ) thermal movement as that doing the westward exp(-imφ) thermal movement, so that the φ-dimension's matter wave related Born probability |Φ(φ)|^2 has to combine these two opposite direction's matter waves in form of eq-6, and it becomes a delocalized standing wave and φ-dimensional evenly distributed probability. However, for the space region with < 1% mass occupancy in the solar system (mainly the planet region, now we have to use planet's orbital velocity), all mass (or matter waves) has only the eastward exp(+imφ) macro movement, no westward exp(-imφ) macro movement, then what is the value of exp(-imφ)? After many tries, we were forced to choose

$$e^{-im\varphi} = 1 \qquad \text{eq-7}$$

The physical meaning of eq-7 is explained in section I-h's discussion item 1). Thus for a planet in a |n*l*m> = |nLL> = |n,*l*=n-1,m=n-1> QM state, we have the φ-dimensional wave function

$$\Phi(\varphi) \propto e^{im\varphi} \qquad \text{eq-8}$$

and the φ-dimensional probability density

$$|\Phi(\varphi)|^2 \propto \left|e^{im\varphi}\right|^2 = e^{-im\varphi}e^{+im\varphi} = 1 \times e^{+im\varphi} = e^{im\varphi} \qquad \text{eq-8b}$$

So exp(-imφ) = 1 can also be explained as that for an eastward orbital moving planet's matter wave, its westward wave component is zero, or its westward wave component makes no contribution to the probability density. We named this special probability as non-Born probability (NBP). So, eq-8b can be re-written as

$$|\Phi(\varphi)|^2_{NBP} \propto e^{im\varphi} \qquad \text{eq-9a}$$

Then how to alter a complex numbered probability in eq-9a into a real numbered probability? Here let's first use a citizen-scientist level method: for the QM probability of [exp(ix)]^2 = [cos(x) + i*sin(x)]^2, we simply ignore the imaginary part, only use the real part, so that [exp(ix)]^2 = [cos(x) + i*sin(x)]^2 → [cos(x)]^2. Using this method, we have

$$|\Phi(\varphi)|^2_{NBP} \propto e^{im\varphi} = \left(e^{i\varphi/2}\right)^{2m} = \{[\cos(\varphi/2) + i\ \sin(\varphi/2)]^2\}^m \rightarrow \{[\cos(\varphi/2)]^2\}^m = \left[\frac{1+\cos(\varphi)}{2}\right]^m = \left[\frac{1+\cos(\varphi)}{2}\right]^{(n-1)} \qquad \text{eq-10}$$

where m = +*l* = n-1. The 2nd last step in eq-10 is from the general trigonometric relationship:

$$[\cos(\varphi/2)]^2 = \frac{1+\cos(\varphi)}{2} \qquad \text{eq-11}$$

In this way, we altered a complex numbered probability into a real numbered probability. The exponential index (n-1) in eq-10 has the function to make [1 + cos(φ)]/2 curve peak become narrower. Therefore eq-10 equivalent to a true wave packet description for a planet in the φ-1D-dimension.

Now let's use an alternative method to explain eq-10. First, let's define an equivalent wave function

$$\Phi(\varphi)_{\text{equivalent}} \propto e^{im\varphi/2} \qquad \text{eq-12}$$

so that

$$\left|\Phi(\varphi)_{\text{equivalent}}\right|^2 \propto \left(e^{\frac{im\varphi}{2}}\right)^2 = e^{im\varphi} \propto |\Phi(\varphi)|^2_{\text{NBP}} \qquad \text{eq-13}$$

Notice that eq-12 and eq-13 do have the real physical meaning (see SunQM-4s1 section I-b). Second, let's use eq-31-SunQM-3s11 (meaning SunQM-3s11's eq-31) to construct a new real numbered wave function, (Notice that the m (or *l*) in eq-31-SunQM-3s11 is replaced by m/2 (or *l*/2) here)

$$Y\left(\frac{l}{2} = n-1, \pm\frac{m}{2} = n-1\right) = \cos(m\varphi/2)\,[\sin(\theta)]^{l/2} \qquad \text{eq-14}$$

where *l*/2 = m/2 = n-1, and its probability function

$$\left|Y\left(\frac{l}{2} = n-1, \pm\frac{m}{2} = n-1\right)\right|^2 = [\cos(m\varphi/2)]^2\,[\sin(\theta)]^m \qquad \text{eq-15}$$

Third, let's use a linear combination of eq-15 and follow the method from eq-33-SunQM-3s11 through eq-43-SunQM-3s11 (notice that the m → m/2 is replaced by φ → φ/2 here, and we can NOT do both at the same time), we obtain

$$\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(m+\delta)\varphi/2]\right\}^2 \to \cos(\varphi/2)^{2m} \qquad \text{eq-16}$$

or, the left side of eq-16 is infinitely approaching to the right side of eq-16 when the integer δ (<< m) value is high enough. Eq-16 bridges the complex numbered probability of exp(ix) = cos(x) + i*sin(x) to its corresponding real numbered probability [cos(x/2)]^2 in eq-10. For better understanding, please read SunQM-3s11 section III-c, and SunQM-4s1. Note: if using the version-c NBP calculation formula in Appendix C, then eq-14 through eq-16 can be explained by directly using eq-35-SunQM-3s11 and SunQM-3s11's Figure 1. That is why the version-c NBP calculation formula is our favorite (even though we are using version-a NBP calculation formula in the main paper, check Appendix C for details).

I-b. How a planet's n in {N,n//6} QM relates to the n in Bohr-de Broglie's 1D circular orbit QM in the formula nλ = 2πr?

This is a pre-required knowledge for section I-c's study. In the Bohr-de Broglie's 1D circular orbit QM (abbreviated as "circular 1D QM")'s formula nλ = 2πr, let's rename the quantum number n to be j, so it becomes

$$j\,\lambda = 2\pi r \qquad \text{eq-17}$$

where j= 1, 2, ... is a positive integer number (at least for now). Using Earth-correlated {1,5//6} orbit as the example, it has orbit r = 1.565E+11 m. So using circular 1D QM's eq-17, at j=1, $\lambda = 2\pi r/j$ = 9.84E+11 m. At j=5, $\lambda = 2\pi r/5$ = 1.97E+11 m. At j=30, $\lambda = 2\pi r/30$ = 3.28E+10 m. Now comparing to {N,n//q} QM's orbit λ calculation (in SunQM-2 Table 1), we see that {1,5//6} with r_1 at {1,1//6} and n=5 has λ= 1.97E+11 m, exactly equals to circular 1D QM's j=5. Also, we see that {0,30//6} with r_1 at {0,1//6} and n = 5^6^1 = 30 has λ= 3.28E+10 m, exactly equals to circular 1D QM's j=30. It is obvious that at least for |nLL> QM state (therefore for all planets or even for all belts in Solar system), {N,n//q} QM's quantum number n is a special set of quantum number j in circular 1D QM which satisfy

$$j = n\, q^{(1-N)} \qquad \text{eq-18}$$

if we set r_1 at {N,1//q}. Notice that (in eq-18) j is φ-1D-dimension-only quantum number, while n is rθφ 3D-dimension quantum number (see SunQM-4s1 section I-a for more detailed figure explanation). For example: if we set r_1 at {N=1,1//6}, then a {1,5//6} orbit has circular 1D QM's j=5*6^(1-1)=5. If we set r_1 at {N=0,1//6}, then a {1,5//6} = {0,30//6} orbit has circular 1D QM's j = 5*6^(1-0) = 30. If we set r_1 at {N=2,1//6}, then a {1,5//6} = {2,5/6//6} orbit has circular 1D QM's j = 5*6^(1-2) = 5/6, (notice that j=5/6 is not a integer, but a pseudo-integer number for {N,n//q} QM, because in {N,n//q} QM, all n/q^N numbers are treated as pseudo-integer number). Hence, we can move r_1 around to change the circular 1D QM's j. In this way, we also unified the {N,n//q} QM with the Bohr-de Broglie's 1D circular orbital movement QM (at least for the nLL QM state, like all planets in the Solar system).

Statement-1: In circular 1D QM, the ground state is j=1, so it correlates to r_1 at {N,n//q} orbit in {N,n//q} QM. For example, circular 1D QM's j=1 ground state for {1,5//6} orbit is r_1 at {1,5//6}. This knowledge is crucial in the next section's full-QM deduction.

I-c. For a circular orbital moving planet, deduce its relationship between $E_{\varphi,j}/\hbar_{gen}$, r_n, $v_{n,gr}$, $v_{n,ph}$, $\omega_{n,gr}$, and $\omega_{n,ph}$

Here we use (de Broglie's) free particle plane wave (which is forced to do the circular orbital moving in φ-1D-dimension) to describe a planet's orbital movement (see SunQM-3s11 section III-c, and SunQM-4s1 for why this is valid). The standard wave function of a plane wave (as Schrodinger equation's solution) is [17]

$$\Psi_k(x,t) = Ae^{\left[i\left(kx-\frac{\hbar k^2}{2m'}\right)t\right]} = Ae^{\left[i\left(kx-\frac{E}{\hbar}t\right)\right]} = Ae^{[i(px-Et)/\hbar]} \qquad \text{eq-19}$$

where k is the wave number, ℏ is the Planck constant divided by 2π, m' is the mass of a particle/planet (not a quantum number), and the momentum p = ℏ * k. Here we only interest in the time-dependency (of a planet's orbital movement) in φ-1D-dimension, so E is the energy only in φ-1D-dimension. Therefore, we need to use quantum number j (rather than quantum number n). It is

$$E_{\varphi,j} = K_{\varphi,j} + V_{\varphi,j} = K_{\varphi,j} = \frac{1}{2}m'{v_j}^2 \qquad \text{eq-20}$$

because $V_{\varphi,j} \equiv 0$ in φ-1D-dimension (for a point-central gravity-force field). Notice that it should not be

$$E_n = E_{r\theta\varphi,n} = \frac{-1}{2}m'{v_n}^2 \qquad \text{eq-21}$$

because eq-21 includes the $E_{r,n}$'s contribution (see SunQM-4s4 section VII for more detailed explanation). Then according to eq-19, the time portion (or the time dependency) of a plane wave's wave function is

$$T(t) \propto e^{-i\frac{E_{\varphi,j}}{\hbar}t} \qquad \text{eq-22}$$

According to SunQM-2's result, we should use $\hbar_{gen}$ (the general Planck constant divided by 2π) to replace $\hbar$ in the Solar {N,n//q} QM. So eq-22 become

$$T(t) \propto e^{-i\frac{E_{\varphi,j}}{\hbar_{gen}}t} \qquad \text{eq-23}$$

Also according to SunQM-2 's result, the wave form of eq-20 is

$$E_{\varphi,j} = Hm'f_{j,ph} = h_{gen}f_{j,ph} \qquad \text{eq-24}$$

Where H is the pseudo-Planck constant, $f_{j,ph}$ is the planet matter wave's circular orbital moving (phase) frequency, and m' is the mass of the planet. Again notice that $E_n = E_{n,r\theta\varphi}$ = -H * m' * $f_{n,ph}$ is E_n in rθφ 3D-dimension, and $E_{\varphi,j}$ = H * m' * $f_{j,ph}$ is E_j in φ-1D-dimension. Also notices that values of h_{gen}, H, $f_{n,ph}$ depend on where you choose r_1, while values of E_n, r_n, v_n (= $v_{n,gr}$) are r_1 position independent value (see SunQM-2 for explanation). Statement-2: Use φ-1D-dimension's E_φ, with $V_\varphi \equiv 0$, then $E_\varphi = K_\varphi$. So the $\omega_{n,ph}$ is only for φ-1D-dimension.

So, a planet matter wave's (not the planet's) time portion is

$$T(t) \propto e^{-i\frac{E_{\varphi,j}}{\hbar_{gen}}t} = e^{-i\frac{h_{gen}f_{j,ph}}{\hbar_{gen}}t} = e^{-i2\pi f_{j,ph}t} \qquad \text{eq-25}$$

where

$$2\pi f_{j,ph} = \omega_{j,ph} \qquad \text{eq-26}$$

where $\omega_{j,ph}$ is the phase angular frequency/velocity. Notice that eq-26 is the planet matter wave's angular (phase) frequency/velocity (because its meaning is more like angular velocity, although in Giancoli's text book it is called angular frequency), it does not equal to the planet's circular orbit moving angular (group) frequency/velocity. Later on we will see that the former one is related to matter wave packet's phase velocity, and the later one is related to matter wave packet's group velocity (for better understanding, see SunQM-3s11's Figure 1, and SunQM-4s1). Also notice that for a circular moving planet matter wave's $f_{j,ph}$ (or $\omega_{j,ph}$), the quantum number j (in φ-1D-dimension) is inter-changeable with the quantum number n (in 3D-dimension) through eq-18. It is much convenient to use n rather than j because we can directly compare with the previous results (e.g., in SunQM-2 Table 1) for $f_{n,ph}$, $\omega_{n,ph}$, (except for E because eq-20 does not equal to eq-21). For example, after re-write eq-26 as

$$2\pi f_{n,ph} = \omega_{n,ph} \qquad \text{eq-27}$$

then we can directly use SunQM-2 Table 1 column 15 to show that Earth's matter wave orbit frequency $f_{n,ph}$ at n=30 (or r_1 at {0,1}) is 4.44E-7 (cycle/sec = Hz), so its 2π * $f_{n,ph}$ = 2.79E-6 arc/s. It does not equal to Earth planet orbit's ω = ω_{gr} = 2π / (365.2 days) = 1.99E-7 arc/s. Similarly, after switch j for n, eq-25 can be re-written as

$$T(t) \propto e^{-i\frac{E_{\varphi,j}}{\hbar_{gen}}t} = e^{-i2\pi f_{n,ph}t} \qquad \text{eq-28}$$

Now let's use particle QM's method to deduce the relationship between $E_{\varphi,j}/\hbar_{gen}$, r_n, $v_{n,gr}$, $v_{n,ph}$, $\omega_{n,gr}$, and $\omega_{n,ph}$. From SunQM-2 Table 1, H = h/m' = 2π * sqrt(G*M*r_1), n = sqrt(r_n/r_1), combining two we obtain H*n = 2π *sqrt(G*M*r_n) for the G-central-forced circular movement. So H is a n related value, and should be written as H_n

$$H_n = \frac{2\pi}{n}\sqrt{GMr_n} \qquad \text{eq-29}$$

Then, using Newton's $F = m' * a = m' * v_n^2 / r_n$, $F = G*M*m' / r_n^2$, $m'*v^2/r_n = G*M*m'/r_n^2$, $r_n *v_n^2 = G*M$, $r_n *v_n = sqrt(G*M*r_n)$, we have

$$\frac{E_{\varphi,j}}{\hbar_{gen}} = \frac{\frac{1}{2}m' v_n^2}{\frac{H_n m'}{2\pi}} = \frac{\pi v_n^2}{H_n} = \frac{\pi v_n^2}{\frac{2\pi\sqrt{GMr_n}}{n}} = \frac{n}{2}\frac{v_n^2}{v_n r_n} = \left(\frac{n}{2}\right)\frac{v_n}{r_n} \quad \text{eq-30}$$

Combining eq-28 and eq-30, we have

$$2\pi f_{n.ph} = \left(\frac{n}{2}\right)\frac{v_n}{r_n} = \omega_{n,ph} \quad \text{eq-31}$$

Here we define $\omega_{n,ph}$ as the planet/particle's matter wave's orbital angular (phase) frequency/velocity, and $f_{n,ph}$ is the planet/particle's matter wave's orbital (phase) frequency. Then, eq-19 can be written as

$$\Psi_k(x,t) = Ae^{\left[i\left(kx - \frac{E}{\hbar}t\right)\right]} = Ae^{i(kx - \omega_{n.ph}t)} = Ae^{ikx}e^{-i\omega_{n,ph}t} \quad \text{eq-32}$$

where

$$T(t) \propto e^{-i\omega_{n,ph}t} \quad \text{eq-33}$$

Now let's define

$$\frac{v_n}{r_n} = \omega_n = \omega_{n,gr} \quad \text{eq-34}$$

Since r_n and v_n (= $v_{n,gr}$) is planet/particle's orbit r and orbit v, so we know that $\omega_n = v_n/r_n$ must equal to planet/particle's orbital (group) angular frequency/velocity. Notices that in eq-31 and eq-34, values of n, $\omega_{n,ph}$, and $f_{n,ph}$ depend on where we choose r_1, while values of r_n, $v_n = v_{n,gr}$, $\omega_n = \omega_{n,gr}$ are r_1 position independent values.

QM text books tell us that a particle's matter wave is actually a wave packet that has group velocity (v_{gr}) and phase velocity (v_{ph}). Our results in SunQM-3s11 section III-c also revealed that a planet's wave function in the φ-dimension is composed by a group of wave functions (which are the Schrodinger equation's solution) that further forms a wave packet (or group wave) out of the phase waves (see SunQM-3s11 Figure 1, and SunQM-4s1). According to text books [18, 19], the relationship between the classical particle velocity and matter wave v_{gr} and v_{ph} is:

$$v_{classical} = v_{gr} = 2v_{ph} \quad \text{eq-35}$$

Notice that eq-35 equivalent to

$$v_{classical} = v_n = v_{n,gr} = v_{gr} = 2v_{ph} = 2v_{n,ph} \quad \text{eq-36}$$

For any wave (including matter wave), it always has

$$v_{ph} = \lambda f_{ph} \quad \text{eq-37}$$

where λ is the wavelength and f_{ph} is the wave (phase) frequency. Using eq-36 or $v_{n,ph} = v_n/2$, we can re-write eq-31 as

$$\omega_{n,ph} = 2\pi f_{n,ph} = \left(\frac{n}{2}\right)\frac{v_n}{r_n} = \frac{n\, v_{n,ph}}{r_n} \quad \text{eq-38}$$

where $v_{n,ph}$ is the phase velocity of planet/particle's orbit velocity. Eq-38 clearly shows the physical meaning of $\omega_{n,ph}$ and its relationship with $v_{n,ph}$.

For a planet/particle moving in circular orbit with quantum number n, eq-34 is always correct, or

$$\omega_n = \omega_{n,gr} = \frac{v_{n,gr}}{r_n} = \frac{v_n}{r_n} \qquad \text{eq-39}$$

where $\omega_{n,gr}$ is the group angular frequency/velocity of planet/particle's orbital movement. Eq-39 clearly shows the physical meaning of $\omega_{n,gr}$ and its relationship with $v_{n,gr}$. Notice here that values of planet/particle's $\omega_n = \omega_{n,gr}$, $v_n = v_{n,gr}$, and r_n are r_1 position independent. Using eq-39 (or eq-34), we can re-write eq-31 as

$$\boldsymbol{\omega_{n,ph}} = 2\pi f_{n,ph} = \left(\frac{n}{2}\right)\frac{v_n}{r_n} = \left(\frac{n}{2}\right)\omega_{n,gr} = \boldsymbol{\left(\frac{n}{2}\right)\omega_n} \qquad \textbf{eq-40}$$

Notice here that values of n, $f_{n,ph}$, and $\omega_{n,ph}$ depend on where we choose r_1, while values of r_n, v_n and ω_n are r_1 position independent. So eq-40 tells us that planet matter wave's φ-angular phase velocity $\omega_{n,ph}$ equals to planet circular moving φ-angular (group) velocity ω_n times n/2. For example, after eq-27, we showed that Earth's matter wave orbit (phase) frequency $f_{n,ph}$ at n=30 (or r_1 at {0,1}) is 4.44E-7 (cycle/sec = Hz), so its $\omega_{n,ph}$ =2π * f_n = 2.79E-6 arc/s. Then, according to eq-40, ω_n = $\omega_{n,ph}$ *2 /n = 2.79E-6 *2/30 = 1.86E-7 arc/s, which (closely) equals to Earth planet orbit's ω = 2π/(365.2 days) = 1.99E-7 arc/s (Note: the difference comes from that Earth's orbit is a little bit deviated from {1,5//6}).

According to eq-35, my (and probably most readers') original thought was that a planet matter wave's angular (group) frequency/velocity must equal to 2x of planet's (phase) angular frequency/velocity, or

$$\omega_{n,ph} = \frac{\omega_n}{2} \qquad \text{eq-41}$$

But eq-41 is correct only under a special condition. This is because $\omega_{n,ph}$ is a r_1 dependent value while ω_n is a r_1 independent value. Only if we choose the r_1 position at planet's orbit-r (that is, when forcing $r_1 = r_n$), then eq-41 will exist (see statement-1 in section I-b). This is exactly the same meaning as that in eq-40-SunQM-3s10's deduction where we chose k = 2π/λ = 1, or λ= 2π, or equivalent to in eq-17, we choose j=1. I emphasize it here because it is easily to be confused, and it had costed me many weeks to figure out. (Note: Also see SunQM-6s1 section I-a for the complete set of equations).

Wiki "Spin" mentioned that "*The conventional definition of the spin quantum number, s, is s = n/2, where n can be any non-negative integer. Hence the allowed values of s are 0, ½, 1, 3/2, 2, etc.*". Eq-40 may explain the origin of s=n/2, where the spin quantum number s is equivalent to the phase angular frequency/velocity $\omega_{n,ph}$ in eq-40. Also, does eq-41 mean the "Mobius strip" (see wiki "Mobius strip")?

I-d. For a planet doing circular orbital movement, the time-dependency of probability density $|\mathbf{T(t)}|^2_{\mathbf{NBP}} \propto \left|\mathbf{e}^{-i\omega_{n,ph}t}\right|^2 = \left(\mathbf{e}^{-i\omega_{n,ph}t}\right)^2 = \mathbf{e}^{-i\omega_{n=1,gr}t} \neq \mathbf{1}$

So, with eq-28, eq-30, eq-31, and eq-40, we now have a planet matter wave's (not planet's) time portion as:

$$T(t) \propto e^{-i\frac{E_{\varphi,j}}{\hbar_{gen}}t} = e^{-i\omega_{n,ph}t} = e^{-i\left(\frac{n}{2}\right)\omega_n t} \qquad \text{eq-42}$$

where ω_n is the planet's true orbital (or group) angular frequency/velocity. Because we used eq-20 for $\omega_{n,ph}$ deduction, eq-42 must be the time portion of a planet's matter wave Φ(φ) that doing circular movement only in φ-dimension. So we need to combine Φ(φ) and T(t) together. Using eq-8 and eq-42, we have

$$\Phi(\varphi)\, T(t) \rightarrow e^{im\varphi}e^{-i\omega_{n,ph}t} = e^{i\left[m\varphi-\left(\frac{n}{2}\right)\omega_n t\right]} \qquad \text{eq-43}$$

The classical physics tells us that for a wave doing circular moving in φ-dimension with the angular frequency/velocity ω, we only need to replace φ by φ - ωt, just like we did in SunQM-3s11's eq-44. Then the "Correspondence principle" of QM tells us that the QM result has to fit to the classical physics result at n >> 1. The only way to achieve above task is to violate the traditional QM's rule of |T(t)|^2 ∝ |exp(-i*$\omega_{n,ph}$*t)|^2 = exp(-i*$\omega_{n,ph}$*t) * exp(+i*$\omega_{n,ph}$*t) = 1, and define a non-Born probability (NBP) of |T(t)|^2 as

$$|T(t)|^2_{NBP} \propto \left|e^{-i\omega_{n,ph}t}\right|^2 = e^{-i\omega_{n,ph}t}e^{-i\omega_{n,ph}t} = e^{-i2\omega_{n,ph}t} = e^{-in\omega_{n,gr}t} = e^{-in\omega_n t} \qquad \text{eq-44}$$

Then |Φ(φ) $|_{NBP}$ ^2 * |T(t)$|_{NBP}$ ^2 represents the NBP density (of a planet's matter wave packet) that is doing orbital movement in Solar system's φ-dimension, which equivalent to this planet is doing orbital movement in Solar system's φ-dimension. At n=1 (meaning j=1 in φ-dimension), eq-44 can also be written as

$$|T(t)|^2_{NBP} \propto \left(e^{-i\omega_{n,ph}t}\right)^2 = e^{-i\omega_{n=1,gr}t} \neq 1 \qquad \text{eq-45}$$

Using eq-9a and eq-44, we have

$$|\Phi(\varphi)|^2_{NBP}\,|T(t)|^2_{NBP} \propto e^{im\varphi}e^{-in\omega_n t} = e^{\left\{\frac{i\left[\varphi-\left(\frac{n}{m}\right)\omega_n t\right]}{2}\right\}2m} \rightarrow \left\{\left[\cos\left(\frac{\varphi-\frac{n}{m}\omega_n t}{2}\right)\right]^2\right\}^m = \left[\frac{1+\cos\left(\varphi-\frac{n}{m}\omega_n t\right)}{2}\right]^m$$

eq-46

where m = n-1 because the whole deduction is based on that the planet is in |n*l*m> = |nLL> = |n, n-1,n-1> QM state. Notice that in eq-46 we again used eq-10 type (citizen-scientist level) method to alter a complex numbered probability to a real numbered probability. Readers should be able to use the alternative math (as shown in eq-12 through eq-16) to obtain the same result. The last step in eq-46 is from the general trigonometric relationship in eq-11. Note: for the step-by-step explanation of the physical meaning of each item in eq-46, see SunQM-4s1.

So **the final full-QM deduced planet's time-dependent φ-dimensional (non-Born) probability density is**

$$\boldsymbol{|\Phi(\varphi)|^2_{NBP}\,|T(t)|^2_{NBP} \propto \left[\frac{1+\cos\left(\varphi-\frac{n}{n-1}\omega_n t\right)}{2}\right]^{n-1}} \qquad \textbf{eq-47a}$$

Then, at n >> 1, eq-47a becomes

$$|\Phi(\varphi)|^2_{NBP}\,|T(t)|^2_{NBP} \propto \left[\frac{1+\cos(\varphi-\omega_n t)}{2}\right]^n \qquad \text{eq-48}$$

I-e. Further modifying of θ-dimensional NBP density formula $|\Theta(\theta)|^2_{NBP}$ according to $|\Phi(\varphi)|^2_{NBP}\,|T(t)|^2_{NBP}$

From eq-25-SunQM-3s11, we know that for a nLL QM state where m = +*l* = n-1, we can write

$$\Theta(\theta) \propto \sin(\theta)^l = \sin(\theta)^{(n-1)} \qquad \text{eq-49}$$

Because the definition of eq-9a, and because the projection of a ball-shaped planet in Solar system's θφ-2D dimension has to be a circular shape, we have to define the $|\Theta(\theta)|^2_{NBP}$ as

$$|\Theta(\theta)|^2_{\mathrm{NBP}} \propto \sin(\theta)^{l} = \sin(\theta)^{(n-1)} \qquad \text{eq-50}$$

Notice that it is different than |Θ(θ)|^2 ∝ [sin(θ)]^[2(n-1)] in eq-26-SunQM-3s11. Then, because eq-50 may have negative values under certain condition, and the probability density has to be a positive value, we need to alter eq-50 to be a forever positive form. Thus we used the same trick (a citizen-scientist level method) as that we have used in eq-10 for the transformation of eq-50:

$$|\Theta(\theta)|^2_{\mathrm{NBP}} \propto \sin(\theta)^{(n-1)} = \left[\cos\left(\frac{\pi}{2}-\theta\right)\right]^{(n-1)} \to \left[e^{i\left(\frac{\pi}{2}-\theta\right)}\right]^{(n-1)} = \left[e^{i\left(\frac{\pi}{2}-\theta\right)/2}\right]^{2(n-1)} = \left\{\cos\left[\left(\frac{\pi}{2}-\theta\right)/2\right] + i\ \sin\left[\left(\frac{\pi}{2}-\theta\right)/2\right]\right\}^{2(n-1)} \to \left\{\cos\left[\left(\frac{\pi}{2}-\theta\right)/2\right]\right\}^{2(n-1)} = \left\{\left\{\cos\left[\left(\frac{\pi}{2}-\theta\right)/2\right]\right\}^{2}\right\}^{(n-1)} = \left\{\left[1+\cos\left(\frac{\pi}{2}-\theta\right)\right]/2\right\}^{(n-1)} = \left\{\frac{[1+\sin(\theta)]}{2}\right\}^{(n-1)}$$

eq-51

or

$$|\Theta(\theta)|^2_{\mathrm{NBP}} \propto \left[\frac{1+\sin(\theta)}{2}\right]^{(n-1)} \qquad \text{eq-52a}$$

Thus eq-52a is the final full-QM deduced θ-dimensional (non-Born) probability density for an orbital moving planet. Notice that the difference between eq-52a and eq-50 is that we simply lifted sin(θ) curve upward to [1 + sin(θ)] so that its original minimum (= -1) value now equals to zero (or there is no negative value), and then normalized its maximum to 1 (by dividing 2 for [1 + sin(θ)]). That is the meaning of eq-11, and this is the standard formula of non-Born probability (NBP). In SunQM-4s1, we will see that NBP applies to many different formed QM.

Combining eq-47a and eq-52a, we have the final time-dependent NBP density for a planet in θφ-2D dimension of Solar system:

$$|Y(l,m)|^2_{\mathrm{NBP}} = |\Theta(\theta)|^2_{\mathrm{NBP}}\ |\Phi(\varphi)|^2_{\mathrm{NBP}} |T(t)|^2_{\mathrm{NBP}} \propto \left[\frac{1+\sin(\theta)}{2}\right]^{(n-1)} \left[\frac{1+\cos\left(\varphi-\frac{n}{n-1}\omega_n t\right)}{2}\right]^{n-1} \qquad \text{eq-53}$$

At n >> 1, n/(n-1) ≈ 1, eq-53 become

$$|Y(l,m)|^2_{\mathrm{NBP}} \propto \left[\frac{1+\sin(\theta)}{2}\right]^{n} \left[\frac{1+\cos(\varphi-\omega_n t)}{2}\right]^{n} \qquad \text{Eq-54}$$

Notice that eq-53 and eq-54 are valid only for nLL QM state. Figure 1 shows a 3D plot of a probability density peak (generated by using eq-54) moving in φθ-2D-dimension.

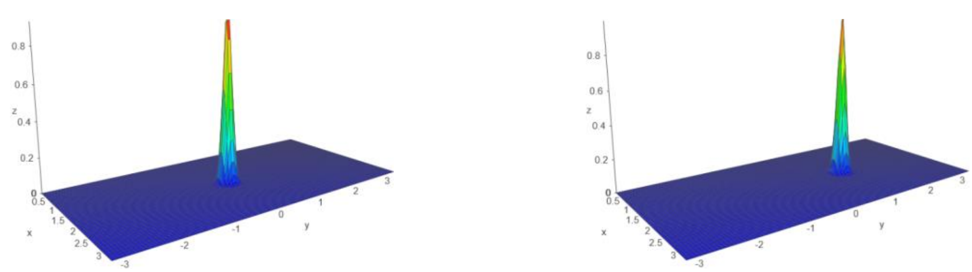

Figure 1(a, b). 3D plot of a probability density peak (generated by using eq-54 at n=256) moving in θφ-2D-dimension. Figure 1a (left), with ω * t =0. Figure 1b (right), with ω * t =1. Using "3D Surface Plotter - An online tool to create 3D plots of surfaces" at: https://academo.org/demos/3d-surface-plotter/

I-f. Further modification of r-dimensional NBP density formula $\mathbf{r^2|R(n, \mathit{l} = n-1)|^2_{NBP}}$ according to $\mathbf{|\Theta(\theta)|^2_{NBP}\ |\Phi(\varphi)|^2_{NBP}|T(t)|^2_{NBP}}$

For the same reason, because a planet has roughly same diameters in all 3D (r, θ, φ), the traditional QM's r-dimensional probability density formula r^2 *|R(n,*l*=n-1)|^2 ∝ [r/r_n * exp(1 - r/r_n)]^(2*n) (see eq-20-SunQM-3s11) will produce a planet with 50% of the diameter in r-dimension than that in θ or φ-dimension. Therefore, by comparing to eq-53, we are forced to define

$$r^2|R(n, l = n-1)|^2_{NBP} \propto \left[\frac{r}{r_n} e^{\left(1-\frac{r}{r_n}\right)}\right]^{n/2} \qquad \text{eq-55a}$$

Since the result of eq-55a is always greater than zero, we do not need to make further modification (like what we did in eq-51). Note: This is the key correction to the previous (3/25/2020) version, where the exponential index was n, now it is corrected to n/2, because according to the calculation in Appendix A's Table 1, only the later one (not the former one) gives the 3D spherical NBP peak in eq-56a.

I-g. The final full-QM deduced 3D NBP density for an orbital moving planet in Solar system

Combining eq-55a to eq-53, we obtain a planet's NBP density

$$\mathbf{r^2\left|\Psi(r,\theta,\varphi,t)_{planet}\right|^2_{NBP} = r^2|R(r)|^2_{NBP}|\Theta(\theta)|^2_{NBP}|\Phi(\varphi)|^2_{NBP}|T(t)|^2_{NBP}} \propto \left[\frac{r}{r_n} e^{\left(1-\frac{r}{r_n}\right)}\right]^{n/2} \left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(\varphi-\frac{n}{n-1}\omega_n t)}{2}\right]\right\}^{(n-1)}$$

eq-56a

where r_n is planet's orbital radius, and ω_n is planet's orbital angular frequency/velocity. **Eq-56a is the final full-QM deduced 3D (non-Born) probability density for a planet** (or for any particle in nLL QM state), and it is valid for both base n and multiplier n'. For a circular orbital moving planet (which is in nLL QM state), we should use its Eigen n' for the exponential index (for Eigen n' concept, see SunQM-3s10 section-IV. Also see SunQM-3s11 Table 1, where planet's Eigen $n'_r = n'_\theta = n'_\varphi$). Then eq-56a is an Eigen description for a circular orbital moving planet. (Note: This is the version-a of NBP formula, see the calculation in Table 1 of Appendix A. Also see Appendix B, Appendix C and Appendix D for the version-b, version-c and version-d of NBP formula. We believe that version-c is the most correct NBP formula, see Appendix C for the detailed explanation).

At n >> 1, n/(n-1) ≈ 1, eq-56a become

$$r^2\left|\Psi(r,\theta,\varphi,t)_{planet}\right|^2_{NBP} \propto \left[\frac{r}{r_n} e^{\left(1-\frac{r}{r_n}\right)}\right]^{n/2} \left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(\varphi-\omega_n t)}{2}\right]\right\}^{n} \qquad \textbf{eq-57}$$

Both eq-56a and eq-57 are not only valid for a planet, but should also be valid for any object that is in nLL QM state and doing orbital movement (even for an electron). It is valid for both base n and multiplier n'.

I-h. More discussions on eq-56a and its deduction

1) For eq-7, we have explained it as exp(+imφ) correlates to planet's eastward orbital rotation's $\omega_{n,ph}$, while exp(-imφ) correlates to the westward orbital rotation's $\omega_{n,ph}$. Because for an eastward rotational planet, it can be thought as the westward orbital rotation's $\omega_{n,ph}$ =0, so exp(-imφ) ∝ exp[i*$\omega_{n,ph,west}$*t) = exp[i*0*t) = 1. So we have a good physical meaning for the φ-dimension's non-Born probability calculation in eq-9. For the physical meaning of θ-dimension's eq-50 and r-dimension's eq-55a, see SunQM-4s1.

2) When looking into the formula of eq-56a, we are amazed by how simple the formula is and how straightforward the meaning it is: r^2 *|R(n,*l*=n-1)|$_{NBP}$^2 ∝ [r/r_n * exp(1 - r/r_n)]^(n/2) produces an exponential rising curve times an exponential declining curve, with the peak always at r = r_n, and the higher the n, the narrower the peak. |Θ(θ)|$_{NBP}$^2 ∝ {[1 + sin(θ)]/2}^(n-1) produces a peak at θ = π/2, and the higher the n, the narrower the peak. At n >> 1, |Φ(φ)|$_{NBP}$^2 * |T(t)|$_{NBP}$ ^2 ∝ {[1 + cos(φ - ω_n *t)]/2}^n produces a peak at φ = ω_n * t, and the higher the n, the narrower the peak.

3) Comparing to the classical physics in SunQM-3s11's eq-45, the QM formed eq-56a has the angular frequency/velocity n/(n-1) * ω_n = ω_n + 1/(n-1)*ω_n > ω_n. So there should be a positive precession for all circular orbital moving planets (if the time-dependent non-Born probability density is correct). At n → 1, eq-47a's angular frequency/velocity n/(n-1)*ωn → ∞. In one of the future SunQM papers, we will use this character to explain how RF is increased when a free particle is gradually trapped in a central (gravity or electric) force field. In another one of the future SunQM papers, we will explore several possible applications of this positive precession in the Solar {N,n} QM.

II. To build a full-QM deduced NBP density 3D map to describe the whole Solar system with time-dependent circular orbital movement

II-a. Full-QM deduced (Asteroid or the cold-KBO) belt's 3D probability density r^2 *|Ψ(r,θ,φ,t)$_{Belt}$|^2

As mentioned before, eq-56a is valid for any object (in belt) that is in nLL QM state of the Solar {N,n} QM structure and doing orbital movement. But if the whole belt (or all mass of this belt) that is in nLL QM state (like Asteroid belt or the cold-KBO, see SunQM-3s10), then this belt's orbital rotation (in φ-dimension) can be described by using the alternated eq-46

$$|\Phi(\varphi)_{Belt}|^2_{NBP}|T(t)|^2_{NBP} \propto \left(e^{im\varphi/2}\right)^2\left[e^{-i\left(\frac{n}{2}\right)\omega_n t}\right]^2 = e^{\left\{\frac{im\left[\varphi-\left(\frac{n}{m}\right)\omega_n t\right]}{2}\right\}2} \to \left\{\cos\left[\frac{m\left(\varphi-\frac{n}{m}\omega_n t\right)}{2}\right]\right\}^2 = \frac{1+\cos\left[m\left(\varphi-\frac{n}{m}\omega_n t\right)\right]}{2} = \frac{1+\cos\left[(n-1)\left(\varphi-\frac{n}{(n-1)}\omega_n t\right)\right]}{2}$$

Eq-58

where $\omega_n = \omega_{rotation} = v_n / r_n$ (same as eq-39), and m=n-1. When n >> 1, m= n-1 ≈n, eq-58 can be simplified as:

$$|\Phi(\varphi)_{Belt}|^2_{NBP}|T(t)|^2_{NBP} \propto \frac{1+\cos[n(\varphi-\omega_n t)]}{2}$$ eq-59

Eq-59 is valid for both base n and multiplier n'. Figure 2 shows the plot of eq-59 with n=1 and n=8. We can see that at n=1, eq-59 in the form of [1 + cos(φ - $\omega_{rotation}$*t)] produces one probability density peak traveling eastward with $\omega_{rotation}$.
At n=8, eq-59 in the form of {1 + cos[8*(φ - $\omega_{rotation}$*t)]} produces eight probability density peaks traveling eastward with $\omega_{rotation}$. Therefore, eq-59 is perfect for describing a belt that is made of n pieces of equal mass, equal size and equal distance, and each piece is doing orbital movement with $\omega_{rotation}$. Then, after combining eq-58 with eq-52a and eq-55a, we have r^2 *|Ψ(r,θ,φ,t)|$_{NBP}$^2 ∝ r^2 *|R(r)|$_{NBP}$ ^2 * |Θ(θ)|$_{NBP}$ ^2 * |Φ(φ)$_{Belt}$|$_{NBP}$ ^2 * |T(t)|$_{NBP}$ ^2 for a belt as

$$r^2|\Psi(r,\theta,\varphi,t)_{belt}|^2_{NBP} \propto \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right]^{n/2} \left[\frac{1+\sin\theta}{2}\right]^{(n-1)} \left\{\frac{1+\cos\left[(n-1)\left(\varphi-\frac{n}{(n-1)}\omega_n t\right)\right]}{2}\right\} \quad \text{eq-60}$$

at n >> 1, m = n-1 ≈ n, eq-58 can be simplified as:

$$r^2|\Psi(r,\theta,\varphi,t)_{belt}|^2_{NBP} \propto \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right]^{n/2} \left[\frac{1+\sin\theta}{2}\right]^{n} \left\{\frac{1+\cos[n(\varphi-\omega_n t)]}{2}\right\} \quad \text{eq-61}$$

where n >> 1. **Eq-60 (and eq-61) is the final full-QM deduced 3D probability density for a φ-rotating belt**. It can be used for Asteroid belt (with n= 48) and the cold-KBO (with n= 192).

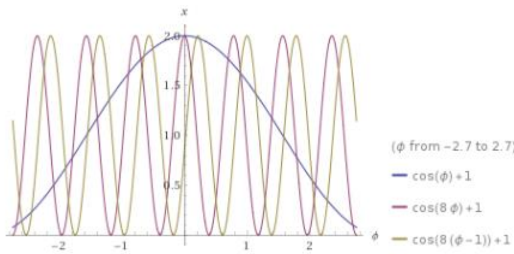

Figure 2. Plot eq-59 at n=1 with ω_n*t = 0, at n=8 with ω_n*t = 0, and at n=8 with ω_n*t = 1.

II-b. QM deduced $|\Phi(\varphi)|^2_{NBP}|T(t)|^2_{NBP}$ orbital rotation for Oort cloud

For Oort cloud's mass, if it is in |nLL> QM state, then the orbital movement of probability density 3D map can be described by either eq-60 (or eq-61). If it is not in |nLL> QM state, then its probability density's φ-dimensional macro movement is more complicated, but still can roughly be described by eq-60 (or eq-61). For simplicity, let's use eq-61. Then, combining eq-61 to SunQM-3s11's eq-17, we may have a QM deduced orbital rotational Oort cloud's probability density function looks like

$$r^2\,|\Psi(r,\theta,\phi,t)_{OortCloud}|^2_{NBP} \propto r^2 \left\{|d_1\, R(1\times 5.33\times 6^3, l)\, Y(l,m)|^2 \left[\frac{1+\cos[n(\varphi-\omega_{4,1}t)]}{2}\right] + |d_2\, R(2\times 5.33\times 6^3, l)\, Y(l,m)|^2 \left[\frac{1+\cos[n(\varphi-\omega_{4,2}t)]}{2}\right] + |d_3\, R(3\times 5.33\times 6^3, l)\, Y(l,m)|^2 \left[\frac{1+\cos[n(\varphi-\omega_{4,3}t)]}{2}\right] + |d_4\, R(4\times 5.33\times 6^3, l)\, Y(l,m)|^2 \left[\frac{1+\cos[n(\varphi-\omega_{4,4}t)]}{2}\right] + |d_5\, R(5\times 5.33\times 6^3, l)\, Y(l,m)|^2 \left[\frac{1+\cos[n(\varphi-\omega_{4,5}t)]}{2}\right]\right\}$$

eq-62

where l = 0 ... n-1, m = -l ... +l, and d_1 … d_5 are the linear combination coefficients, and $\omega_{4,1}$, $\omega_{4,2}$, $\omega_{4,3}$, $\omega_{4,4}$, and $\omega_{4,5}$ are the (averaged) angular frequencies/velocities for the orbital moving mass in {4,n=1..5//6}o orbital spaces. Notice that n in eq-62 can be any value (as long as >> 1), here we can choose n=6^4=1296. (Note: eq-62 is only a "citizen scientist leveled" expression, it may not be a solid mathematic expression).

II-c. QM deduced $|\Phi(\varphi)|^2_{NBP}|T(t)|^2_{NBP}$ for a self-spinning Sun

Sun's self-spinning can also be roughly described by eq-59. At the lowest resolution, let's suppose the Sun spins like a solid ball with a single angular frequency/velocity $\omega_{SunSpin}$. By combining eq-5-SunQM-3s11 and eq-61, we may have a QM deduced Sun's probability density function (with self-spin) looks like

$$r^2\,|\Psi(r,\theta,\phi,t)_{Sun}|^2_{NBP} \propto [eq-5-SunQM-3s11]\left[\frac{1+\cos[n(\varphi-\omega_{SunSpin}t)]}{2}\right] \qquad \text{eq-63}$$

At a higher resolution, according to Solar {N,n} QM theory, Sun core spins faster than its outer shell. Let's assume that Sun's {-1,1} inner core has $\omega_{SunSpin1}$, Sun's {-1,n=1..5}o orbit shells has $\omega_{SunSpin2}$, and Sun's {0,1}o orbit shell has $\omega_{SunSpin3}$, with $\omega_{SunSpin1} > \omega_{SunSpin2} > \omega_{SunSpin3}$. Since eq-5-SunQM-3s11 only includes Sun's {-1,n=1..5}o orbit shells and {0,1}o orbit shell for the enough accuracy, so we don't need to consider $\omega_{SunSpin1}$. Then, at a higher resolution, we may have a QM deduced self-spinning Sun's probability density function looks like

$$r^2\,|\Psi(r,\theta,\phi,t)_{Sun}|^2_{NBP} \propto r^2\left[\left|a_1R\left(\tfrac{1}{6},0\right)Y(0,0)\right|^2+\left|a_2R\left(\tfrac{2}{6},l=0..1\right)Y(l,m)\right|^2+\left|a_3R\left(\tfrac{3}{6},l=0..2\right)Y(l,m)\right|^2+\left|a_4R\left(\tfrac{4}{6},l=0..3\right)Y(l,m)\right|^2+\left|a_5R\left(\tfrac{5}{6},l=0..4\right)Y(l,m)\right|^2\right]\left\{\frac{1+\cos[n(\varphi-\omega_{SunSpin2}t)]}{2}\right\}+ r^2\left[\,|a_6R(1,0)\,Y(0,0)|^2\right]\left\{\frac{1+\cos[n(\varphi-\omega_{SunSpin3}t)]}{2}\right\} \qquad \text{eq-64}$$

where l = 0 ... n-1, m = -l … +l, and a_1 … a_6 are the linear combination coefficients. Notice that n in eq-64 can be any value (as long as >> 1), here we can choose n=6^4=1296. (Note: eq-63 and eq-64 are only a "citizen scientist leveled" expression, they may not be a solid mathematic expression).

II-d. To build a full-QM deduced non-Born probability density 3D map to describe the whole Solar system with time-dependent circular orbital movement

It has the similar form as that in SunQM-3s11's eq-62, except that the probability density formulas of planet (or belt) are now following eq-56a (or eq-60). The Eigen n' (from Appendix A's Table 1) is used in eq-56a's exponential index for all planets so that it is the Eigen description of the 3D probability density for the orbital moving planets (meaning besides the orbit r_n, it also describes the planet's size). Because all planets have extremely high Eigen n' value (> 1E+9), so we can use eq-57 (instead of eq-56a). However, for n/(n-1) * ω_n in eq-56a, we don't know which n we should use. For example, for Earth at {1,5//6}, if we use Earth planet's Eigen n' = 1.81E+9, then n/(n-1) =1, or eq-57 is accurate. But if we use n' = 5*6^1=30, then n/(n-1) = 30/(30-1) =1.03 will produce huge amount of positive procession. For this uncertainty, we use eq-57 but still retain the n/(n-1) * ω_n in eq-56a. So the actual 3D probability density formula for all eight planets is:

$$r^2\left|\Psi(r,\theta,\varphi,t)_{planet}\right|^2_{NBP} \propto \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right]^{n'/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(\varphi-\frac{n}{n-1}\omega_n t)}{2}\right]\right\}^{n'} \qquad \text{eq-65}$$

where n' is planet's Eigen n' (>1E+9), and n is one of planet's multiplier n (uncertain which one).

Here are eight known planets' full-QM deduced non-Born probability density formulas (all r_n, n' are obtained from the Table 1 in Appendix A, all ω_n and initial φ are obtained from SunQM-3s11's Table 1, and the initial φ position is set on Aug. 14, 2019):

$$r^2\left|\Psi(r,\theta,\varphi,t)_{Mercury}\right|^2_{NBP} \approx \left[\frac{r}{5.64\times10^{10}}e^{\left(1-\frac{r}{5.64\times10^{10}}\right)}\right]^{6.53\times10^9/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(1.08-\frac{n}{n-1}8.61\times10^{-7}t)}{2}\right]\right\}^{6.53\times10^9} \qquad \text{eq-66}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Venus}|^2_{NBP} \approx \left[\frac{r}{1.00\times10^{11}}e^{\left(1-\frac{r}{1.00\times10^{11}}\right)}\right]^{8.71\times10^9/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(3.14-\frac{n}{n-1}3.63\times10^{-7}t)}{2}\right]\right\}^{8.71\times10^9} \quad \text{eq-67}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Earth}|^2_{NBP} \approx \left[\frac{r}{1.57\times10^{11}}e^{\left(1-\frac{r}{1.57\times10^{11}}\right)}\right]^{1.09\times10^{10}/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(0-\frac{n}{n-1}1.86\times10^{-7}t)}{2}\right]\right\}^{1.09\times10^{10}} \quad \text{eq-68}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Mars}|^2_{NBP} \approx \left[\frac{r}{2.25\times10^{11}}e^{\left(1-\frac{r}{2.25\times10^{11}}\right)}\right]^{7.84\times10^{10}/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(3.33-\frac{n}{n-1}1.08\times10^{-7}t)}{2}\right]\right\}^{7.84\times10^{10}} \quad \text{eq-69}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Jupiter}|^2_{NBP} \approx \left[\frac{r}{7.12\times10^{11}}e^{\left(1-\frac{r}{7.12\times10^{11}}\right)}\right]^{7.26\times10^8/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(5.31-\frac{n}{n-1}1.92\times10^{-8}t)}{2}\right]\right\}^{7.26\times10^8} \quad \text{eq-70}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Saturn}|^2_{NBP} \approx \left[\frac{r}{1.60\times10^{12}}e^{\left(1-\frac{r}{1.60\times10^{12}}\right)}\right]^{6.53\times10^9/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(5.74-\frac{n}{n-1}5.96\times10^{-9}t)}{2}\right]\right\}^{6.53\times10^9} \quad \text{eq-71}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Uranus}|^2_{NBP} \approx \left[\frac{r}{2.85\times10^{12}}e^{\left(1-\frac{r}{2.85\times10^{12}}\right)}\right]^{3.13\times10^{11}/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(1.27-\frac{n}{n-1}2.40\times10^{-9}t)}{2}\right]\right\}^{3.13\times10^{11}} \quad \text{eq-72}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Neptune}|^2_{NBP} \approx \left[\frac{r}{4.45\times10^{12}}e^{\left(1-\frac{r}{4.45\times10^{12}}\right)}\right]^{3.92\times10^{11}/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(0.52-\frac{n}{n-1}1.23\times10^{-9}t)}{2}\right]\right\}^{3.92\times10^{11}} \quad \text{eq-73}$$

Following eq-60, we have the full-QM deduced non-Born probability density formulas (in Eigen description) for the two belts:

$$r^2\,|\Psi(r,\theta,\varphi,t)_{AsteroidBelt}|^2_{NBP} \approx \left[\frac{r}{4.01\times10^{11}}e^{\left(1-\frac{r}{4.01\times10^{11}}\right)}\right]^{48/2}\left[\frac{1+\sin\theta}{2}\right]^{47}\left\{\frac{1+\cos\left[n\left(\phi-\frac{n}{n-1}4.54\times10^{-8}t\right)\right]}{2}\right\} \quad \text{eq-74}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Cold-KBO}|^2_{NBP} \approx \left[\frac{r}{6.40\times10^{12}}e^{\left(1-\frac{r}{6.40\times10^{12}}\right)}\right]^{192/2}\left[\frac{1+\sin\theta}{2}\right]^{191}\left\{\frac{1+\cos\left[n\left(\phi-\frac{n}{n-1}7.11\times10^{-10}t\right)\right]}{2}\right\} \quad \text{eq-75}$$

where Asteroid belt's orbital rotation ω_n = 4.54E-8 arc/s, and Kuiper belt's ω_n = 7.11E-10 arc/s are obtained from SunQM-3s11's Table 1. Notice that we are also not sure which multiplier n' (n' >> 1) is the correct one for n/(n-1) in eq-74 and eq-75.

For the four undiscovered {3,n=2..5//6} planets/belts, here we treat them as planets, so they have the same 3D probability density formula as eq-65:

$$r^2\,|\Psi(r,\theta,\varphi,t)_{\{3,2\}Planet}|^2_{NBP} \approx \left[\frac{r}{2.56\times10^{13}}e^{\left(1-\frac{r}{2.56\times10^{13}}\right)}\right]^{3.39\times10^{13}/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(\varphi_{\{3,2\}}-\frac{n}{n-1}8.89\times10^{-11}t)}{2}\right]\right\}^{3.39\times10^{13}} \quad \text{eq-76}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{\{3,3\}Planet}|^2_{NBP} \approx \left[\frac{r}{5.76\times10^{13}}e^{\left(1-\frac{r}{5.76\times10^{13}}\right)}\right]^{3.05\times10^{14}/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(\varphi_{\{3,3\}}-\frac{n}{n-1}2.63\times10^{-11}t)}{2}\right]\right\}^{3.05\times10^{14}} \quad \text{eq-77}$$

$$r^2\,|\Psi(r,\theta,\varphi,t)_{\{3,4\}Planet}|^2_{NBP} \approx \left[\frac{r}{1.02\times10^{14}}e^{\left(1-\frac{r}{1.02\times10^{14}}\right)}\right]^{4.06\times10^{14}/2}\left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(\varphi_{\{3,4\}}-\frac{n}{n-1}1.11\times10^{-11}t)}{2}\right]\right\}^{4.06\times10^{14}} \quad \text{eq-78}$$

$$r^2 \left|\Psi(r,\theta,\varphi,t)_{\{3,5\}Planet}\right|^2_{NBP} \approx \left[\frac{r}{1.60\times10^{14}} e^{\left(1-\frac{r}{1.60\times10^{14}}\right)}\right]^{3.05\times10^{15}/2} \left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(\varphi_{\{3,5\}}-\frac{n}{n-1}5.69\times10^{-12}t)}{2}\right]\right\}^{3.05\times10^{15}} \quad \text{eq-79}$$

where $\varphi_{\{N=3,n=2..5\}}$ are the (unknown) initial φ-positions for the four undiscovered planets.

Now we can use a matrix production to constitute a complete probability density function for Solar {N,n} QM structure (as shown in eq-80). Notice that in comparison with SunQM-3s11's eq-62, eq-80 not only produces a 3D map of probability density peaks for a complete Solar system (which including a Sun, eight known planets, two known belts, four undiscovered planets, and Oort cloud (all in circular orbital movement)), but also includes Sun's self-spin, and may even reflect planet's orbital precession movement. Notice that in eq-80, the coefficient matrix is a diagonal only matrix, all non-diagonal cells have values of zero. The (most right) vector space column is composed by the probability density functions (in bold) with the equation numbers of eq-64, eq-66 through eq-79, and eq-62.

$$\mathbf{r^2\,|\Psi(r,\theta,\phi,t)_{SolarSystem}|^2 =} \begin{bmatrix} a^2 &&&&&&&&&&&&&&& \\ & c_1^2 &&&&&&&&&&&&&& \\ && c_2^2 &&&&&&&&&&&&& \\ &&& c_3^2 &&&&&&&&&&&& \\ &&&& c_4^2 &&&&&&&&&&& \\ &&&&& c_5^2 &&&&&&&&&& \\ &&&&&& c_6^2 &&&&&&&&& \\ &&&&&&& c_7^2 &&&&&&&& \\ &&&&&&&& c_8^2 &&&&&&& \\ &&&&&&&&& c_9^2 &&&&&& \\ &&&&&&&&&& c_{10}^2 &&&&& \\ &&&&&&&&&&& c_{11}^2 &&&& \\ &&&&&&&&&&&& c_{12}^2 &&& \\ &&&&&&&&&&&&& c_{13}^2 && \\ &&&&&&&&&&&&&& c_{14}^2 & \\ &&&&&&&&&&&&&&& c_{15}^2 \end{bmatrix} \begin{bmatrix} \text{Sun, eq-64} \\ r^2\,|\Psi(r,\theta,\phi,t)_{Mercury}|^2\text{, eq-66} \\ r^2\,|\Psi(r,\theta,\phi,t)_{Venus}|^2\text{, eq-67} \\ r^2\,|\Psi(r,\theta,\phi,t)_{Earth}|^2\text{, eq-68} \\ r^2\,|\Psi(r,\theta,\phi,t)_{Mars}|^2\text{, eq-69} \\ r^2\,|\Psi(r,\theta,\phi,t)_{Jupiter}|^2\text{, eq-70} \\ r^2\,|\Psi(r,\theta,\phi,t)_{Saturn}|^2\text{, eq-71} \\ r^2\,|\Psi(r,\theta,\phi,t)_{Uranus}|^2\text{, eq-72} \\ r^2\,|\Psi(r,\theta,\phi,t)_{Neptune}|^2\text{, eq-73} \\ r^2\,|\Psi(r,\theta,\phi,t)_{AsteoridBelt}|^2\text{, eq-74} \\ r^2\,|\Psi(r,\theta,\phi,t)_{KuiperBelt}|^2\text{, eq-75} \\ r^2\,|\Psi(r,\theta,\phi,t)_{\{3,2\}Planet}|^2\text{, eq-76} \\ r^2\,|\Psi(r,\theta,\phi,t)_{\{3,3\}Planet}|^2\text{, eq-77} \\ r^2\,|\Psi(r,\theta,\phi,t)_{\{3,4\}Planet}|^2\text{, eq-78} \\ r^2\,|\Psi(r,\theta,\phi,t)_{\{3,5\}Planet}|^2\text{, eq-79} \\ \text{Oort cloud, eq-62} \end{bmatrix}$$

eq-80

Similar as that in SunQM-3s11, here we name eq-80 as the "Solar $r^2|R(n,l)|^2\ |Y(l,m)|^2$ master matrix formula". Eq-80 is the Eigen description (at planet or belt level) of our Solar system using Schrodinger equation's solution. It can also be written as the integration form:

$$\text{Mass}(r,\theta,\varphi,t) = 1.99\times10^{30}(\text{kg}) = \int_0^{1500\times1.49\times10^{11}} \int_0^{\pi} \int_0^{2\pi} [\text{eq}-80]\ \times \sin(\theta)\,dr\,d\theta\,d\varphi \quad \text{eq-81}$$

Therefore, each coefficient in eq-80's diagonal matrix can be obtained because each item (Sun, planet, belt, cloud)'s integration should equal to this item's mass.

III. More discussion on the Solar $r^2|R(n,l)|^2\ |Y(l,m)|^2$ master matrix formula eq-80

1) The calculated r-dimensional probability density distribution of eq-80 is shown in Figure 3. Notice that the previous low-resolution diagram of probability density r-distribution (in SunQM-3s1 Figure 3 where all eight planets' Born probability density peak widths were very broad) is now updated to a high-resolution diagram (in which all eight planets' NBP probability density peak widths are close to planets' true diameters). Check SunQM-3s11 section V for detailed explanation.

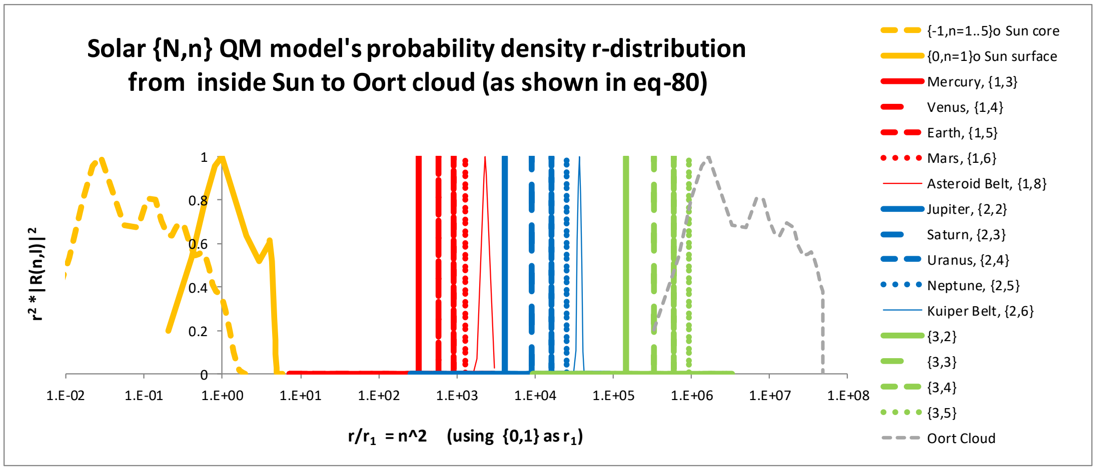

Figure 3. NBP Probability density distribution in r-dimension calculated from eq-80 (where all eight planets' probability density peak widths are close to planets' diameters).

2) The real time-dependency of eight planets in θ=π/2 plane (or x-y plane in Solar system) is illustrated in Figure 4 with the initial φ-dimensional positions set on Aug. 14, 2019. Check SunQM-3s11 section V for detailed explanation.

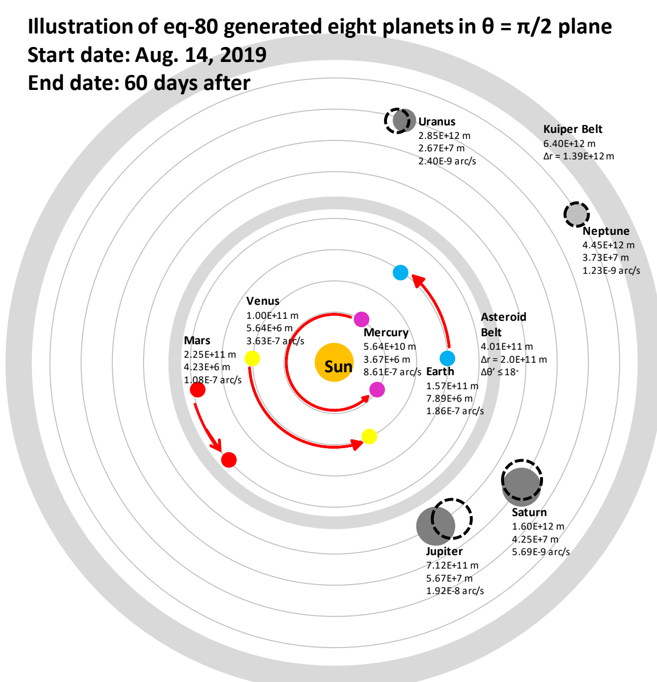

Figure 4. Illustration of eq-80: eight planets in θ=π/2 plane with initial φ positions on Aug. 14, 2019, and the φ-ωt positions after 60 days of orbital movement.

3) Can we use 3D probability density $r^2\ |R(n,l)|^2\ |Y(l,m)|^2$ map to calculate out the φ-positions of the four undiscovered planets at {3,n=2..5//6} orbits? (see SunQM-3s11 section VIII for detailed discussion).

4) A prediction that all mass entities (from the whole universe to a single quark) can be described by Schrodinger equation and solution (see SunQM-3s11 section IX for detailed discussion).

5) A (kind of) wrap-up discussion on the phase-1 study of Solar {N,n} QM modeling (see SunQM-3s11 section X for details).

IV. The non-Born probability density method is valid (at least) for the macro movement in {N,n} QM

This study revealed that at least for the macro movement in {N,n} QM, the probability density function can be directly proportional to its matter wave function (here we named it as the non-Born probability (NBP) density, also see the discussion in Appendix C). More detailed explanations and evidences have been moved to the next paper SunQM-4s1. Although the NBP method was developed in 2019, I did not dare to publish it until I finished SunQM-4s1.

V. Eq-80 describes a Solar system not only at planet's Eigen description level, but also at any level of resolution (down to proton level, or up to the whole universe level)! Therefore "Simultaneous-Multi-Eigen-Description (SMED)" is one of many nature attributes of QM (same as in the 3/25/2020 version)

SunQM-3s10's Figure 3 revealed that a Kuiper belt at {2,6} can be described by {N,n} QM at different levels of resolution. In current paper, with the new analysis result, let's further expand this idea with more examples.
Example-1:
In SunQM-3s11 section-VII, we had an example to show that the cold-KBO at orbit {3,1//6} can be Eigen described as {0,n=6*6*5.33//6} = {0,192//6}, or n=192, or in |192,191,192> QM state. with the radial wave function r^2 * |R(n=6^3,*l*=n-1)|^2 with r_1 at {0,1//6}. It can also be described with a linear combination of radial wave functions r^2 * |R(n=6^18,*l*=n-1)|^2 with r_1 at {-15,1//6}, which means many proton's matter wave packets are running in the cold-KBO's matter wave resonance chamber, or, the cold-KBO is made of many protons (and neutrons). Furthermore, it can also be described by radial wave function r^2 * |R(n=6^(-5),*l*=n-1)|^2 with r_1 at {8,1//6}, which means the Milky way's matter wave packet is also running in the cold-KBO's matter wave resonance chamber.
Example-2:
Figure 5 demonstrated that a planet in Solar system can be described by{N,n} QM's r-dimensional probability density at different levels of resolution. At low resolution level, this planet can be described by a single r^2 * |R(n,*l*)|^2 curve which describes the whole Solar system's {N=1..4,n//6} region (as shown in Figure 5's thick grey line). At median resolution level, this planet can be described by a single r^2 * |R(n,*l*)|^2 curve that describes only one N super-shell in Solar system's {N=1..4,n//6} region (as shown in Figure 5's dot line) in which this planet is located. At median-high resolution level, this planet can be described by a single r^2 * |R(n,*l*)|^2 curve that describes only one n shell in Solar system's {N=1..4,n//6} region (as shown in Figure 5's grey thin line) and this curve covers the whole {N,n//6}o orbit shell (r-dimensional) space from r_n to r_{n+1}. At high resolution level, this planet can be described by a single r^2 * |R(n,*l*)|^2 curve at the Eigen n that describes not only the orbit r of this planet, but also the body size of this planet (as shown in Figure 5's solid thin line). Furthermore, at a very high resolution level, this planet can be described by a combination of many r^2 * |R(n,*l*)|^2 curves at the Eigen n that describing for a proton/neutron. So while each single r^2 * |R(n,*l*)|^2 describes not only the size of a single proton/neutron, but also the orbit r of this proton/neutron located (in Solar system and inside this planet), the combined r^2 * |R(n,*l*)|^2 describes not only the orbit r, but also the body size of this planet (not shown in Figure 5). This example explains a planet in Solar system can be described by {N,n} QM's at different levels of resolution.
Example-3:
Here let us show another similar example: Earth at orbit {1,5//6} can be Eigen described as {-11,n=5*6^12//6} = {-11,1.09E+10//6}, or n=1.09E+10, or in |1.09E+10,n-1,n-1> QM state, with the radial wave function r^2 * |R(n=5*6^12=1.09E+10,*l*=n-1)|^2 with r_1 at {-11,1//6}, (see Appendix C's Table 3). It can also be described with a linear combination of radial wave functions r^2 * |R(n=5*6^16,*l*)|^2 with r_1 at {-15,1//6}, (notice that here *l* is no longer limited to be n-1, but includes n-2, n-3, …), which means many proton's matter wave packets are running in Earth's matter wave resonance chamber, or, the Earth is made of many protons (and neutrons). Furthermore, it can also be described by radial

wave function r^2 * |R(n=5*6^(-7),*l*=n-1)|^2 with r_1 at {8,1//6}, which means the Milky way's matter wave packet is also running in the Earth's matter wave resonance chamber.
Example-4:
Suppose Sun at size of {0,2//6} contains (Sun's mass / proton's mass = 1.9885E+30 kg / 1.67E-27 kg) ~1.19E+57 of proton/neutrons. Also suppose our universe has a size of {13,1//6} (see SunQM-1s2 Table 1). Then Sun can be Eigen described as {0,2//6}, or in |1,0,0> QM state with the radial wave function r^2 * |R(n=1,*l*=0)|^2 with r_1 at {0,1//6}, (see SunQM-3s11 Table 1). It can also be described with a linear combination of radial wave functions r^2 * |R(n=1*6^15,*l*)|^2 with r_1 at {-15,1//6}, (notice that here *l* is no longer limited to be n-1, but includes n-2, n-3, …) which means ~1.19E+57 of proton/neutron's matter wave packets are running in Sun's matter wave resonance chamber, or, the Sun is made of ~1.19E+57 protons/neutrons. Furthermore, it can also be described by radial wave function r^2 * |R(n=1*6^(-13),*l*=n-1)|^2 with r_1 at {13,1//6}, which means that our universe's matter wave packet is also running inside the Sun's matter wave resonance chamber.

We can expand this explanation to a new concept: any object in our universe can be described by {N,n} QM at different levels of resolution. So based on the {N,n//6} QM structure model, eq-80 describes a solar system not only at the Eigen description (of a planet or a belt) level, but can be (simultaneously) at any possible levels of resolution (down to atom level, proton level, or up to galaxy level, even to the whole universe level)! In other words, the Eigen level is changeable in eq-80. In the current eq-80, the Eigen level is set to at Solar system's planet/belt level. We can reset the Eigen level down to the hydrogen atom level (by setting r_1 to the size of a H-atom at {-12,1//6}), or to proton/neutron level (by setting r_1 to the size of a proton at {-15,1//6}), or to quark level (by setting r_1 to the size of a quark at {-17,1//6}), or up to galaxy level (by setting r_1 to the size of a galaxy at {8,1//6}), or even to the whole universe level (by setting r_1 to the size of our universe at {13,1//6} or whatever the size it should be). When eq-80's Eigen level is set at H-atom level, then it describes a Solar system's {N,n} QM structure based on H-atom (at least ideally). When eq-80's Eigen level is set at the whole universe level, then it describes a Solar system's {N,n} QM structure based on our whole universe (also at least ideally).

The same concept has been put forward in SunQM-2 section IV-c and section IV-d in a different expression. It says that all objects in our universe (universe, galaxies, stars, atoms, protons, quarks, etc.) are both matter wave resonance chambers (MWRC) and matter wave packets (MWP) at the same time. For example, a star's MWP runs in star's MWRC in a bended pathway, and get resonanced and amplified. At the same time, this star's MWP also runs in a galaxy's MWRC, and in universe's MWRC in a more straight-line pathway, and get resonanced and amplified (in a different weight). Meanwhile, this star's MWP also runs in a proton's MWRC, and in a quark's MWRC in a very bended pathway, and get resonanced and amplified (at a different weight). Vice versa, all MWP of universe, galaxies, stars, atoms, protons, quarks, etc. run in this star's MWRC at the same time (in bended or straight pathways), and get amplified at a different weight.

Another similar concept but different expression has been put forward in SunQM-3s10 section IV: "*It is like that a small piece of hologram still give the whole picture information (although with low resolution), the large piece give the more complete information, or like a SVD matrix converted picture for digital transportation, a smaller file still contains the whole picture, although with low resolution. So both Fourier transformation and matrix conversion are good tools for this kind of analysis*". This is why both wave mechanics and matrix mechanics were developed in parallel in the early quantum mechanics development. Let's name this phenomenon as the "Simultaneous-Multi-Eigen-Description", or "SMED". Here we need to emphasis that the SMED character is the nature attribute of both wave mechanics and matrix mechanics, therefore "Simultaneous-Multi-Eigen-Description (SMED)" is one of many nature attributes of quantum mechanics! Some of the other known nature attributes of QM are: the particle-wave duality, uncertainty principle, rotation diffusion (or RotaFusion, or RF), etc.

In SunQM-2 section II-c, the same SMED concept is expressed in a completely different angle: "… h_{Planck} is a unit (which happened to be obtained from the work dealing with hydrogen atom's electron), and n is the quantum number for that unit and n is obtained through QM calculation. If we dealing with planet (or galaxy, or proton, or quark, etc.), we need only to scale up (or down) the unit, and then we can obtain the meaningful n quantum number through the same QM calculation! … in old way, we keep unit constant, and change formula or variable input value for calculation. In the new way: we keep QM formula (of hydrogen atom) unchanged, but only change $h_{general}$ unit accordingly! In other words, $h_{general}$ is not a constant at all, it is a variable unit, which make us able to use the same QM formula for hydrogen atom to calculate the orbit movement for all kinds of attractive-central-force system …".

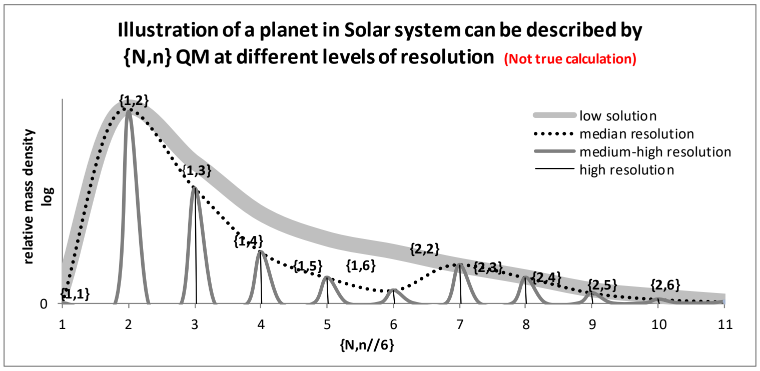

Figure 5. Illustration of a planet in a Solar system can be described by {N,n} QM at different levels of resolution (copied from SunQM-3s11's Figure 5a with some modifications). Note: these curves are artificially drawn to mimic the r^2|R(n,*l*)|^2 calculated curve, they are not the true calculated result.

Conclusion

In this paper, we developed a non-Born probability density calculation for a planet in nLL QM state doing circular orbital movement: its φ-dimensional probability density function is directly proportional to its matter wave function |Φ(φ)|^2 ∝ Φ(φ) ∝ exp(imφ), and its time-dependency is also a non-Born calculation |T(t)|^2 ∝ [exp(-i * $\omega_{n,phase}$ * t)]^2. Then we calculated out (a full-QM deduced) |Φ(φ)|^2 * |T(t)|^2 for a planet's time-dependent probability density in φ-dimension. We believe that by adding the non-Born calculation to Born calculation, the QM will become more self-consistent and more complete.

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SunQM-5: A new version of QM based on interior {N,n}, multiplier n', |R(n,*l*)|^2 * |Y(*l*,m)|^2 guided mass occupancy, and RF, and its application from string to universe (drafted in April 2018).
SunQM-5s1: White dwarf, neutron star, and black hole re-analyzed by using the internal {N,n} QM (drafted in April 2018).
SunQM-6s1: Using Bohr atom, {N,n} QM field theory, and non-Born probability to describe a photon's emission and propagation.
SunQM-7: Can we use {N,n} QM and Simultaneous-Multi-Eigen-Description (SMED) to describe our universe?
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Appendix A. Version-a of NBP formula (for nLL QM state only)

See eq-9a, eq-47a, eq-52a, eq-55a, and eq-56a in the paper. The major difference between version-a to other versions is that they have difference 3D NBP peak width (with the ratio of version-a to version-b to version-c to version-d = 1 : 0.707 : 0.707 : 0.5). The similar standard deviations in Table 1, Table 2 and Table3 suggests that eq-56a, eq-56c, and eq-56d have the similar accuracy in describing all eight known planets.

Table 1. Using eq-56a to calculate a planet's Eigen n' (in both r- and θ-dimension, see SunQM-3s11's Table 1 for details).

NASA's data of planets			{N,n} model			set total n=1 at Sun core		Determine planet r-dimensional n' & w								Determine planet θ-dimensional n' & w				
						calc model n, r_n, v_n										Note: w(ϕ) = w(θ), and $n'_\phi = n'_\theta$				
	Sun's body-r or planets' orbit-r_n	planet's body-r, b=	N	n	q	total n from Sun core	$r_n= r_1$*n^2	n' =2* ln(0.01) / [ln(1 + b /r_n)-(b /r_n)]	w= log(n' / n) / log(6)	round up w	n'_r = n*q^w	b(r) =	[(1+b/r_n)*exp(-b r_n)]^n' = 0.01	$r_1 = r_{n'}$ / (n*q^w)^2	r_1 at {N,1//6}	w(θ) =	n'_θ = n*q^w	b(θ) = r*sin(acos{2*0.01^[1/(n-1)]-1})	b(r) / b(θ)	planet's body-r /b(θ)
unit	m	m					m					m		m				m	m/m	m/m
Sun core	1.74E+08		0	1	6	1	1.74E+08													
SUN	6.96E+08		0	2	6	2	6.96E+08													
Mercury	5.79E+10	2.44E+06	1	3	6	18	5.64E+10	9.83E+09	12.23	12	6.53E+09	2.99E+06	0.010	1.32E-09	{-11,1//6}	12	6.53E+09	2.99E+06	1.00	0.82
Venus	1.08E+11	6.05E+06	1	4	6	24	1.00E+11	5.05E+09	11.70	12	8.71E+09	4.61E+06	0.010	1.32E-09	{-11,1//6}	12	8.71E+09	4.61E+06	1.00	1.31
Earth	1.49E+11	6.38E+06	1	5	6	30	1.57E+11	1.11E+10	12.01	12	1.09E+10	6.44E+06	0.010	1.32E-09	{-11,1//6}	12	1.09E+10	6.44E+06	1.00	0.99
Mars	2.28E+11	3.40E+06	1	6	6	36	2.25E+11	8.12E+10	13.02	13	7.84E+10	3.46E+06	0.010	3.67E-11	{-12,1//6}	13	7.84E+10	3.46E+06	1.00	0.98
Asteroid belt	4.02E+11		1	8	6	48	4.01E+11													
Jupiter	7.78E+11	7.15E+07	2	2	5.33	64.0	7.12E+11	1.82E+09	11.45	11	7.26E+08	1.13E+08	0.010	1.35E-06	{-9,1//6}	11	7.26E+08	1.13E+08	1.00	0.63
Saturn	1.43E+12	6.03E+07	2	3	5.33	95.9	1.60E+12	1.30E+10	12.32	12	6.53E+09	8.50E+07	0.010	3.75E-08	{-10,1//6}	12	6.53E+09	8.50E+07	1.00	0.71
Uranus	2.97E+12	2.56E+07	2	4	5.33	127.9	2.85E+12	2.28E+11	13.76	14	3.13E+11	2.18E+07	0.010	2.90E-11	{-12,1//6}	14	3.13E+11	2.18E+07	1.00	1.17
Neptune	4.51E+12	2.48E+07	2	5	5.33	159.9	4.45E+12	5.94E+11	14.17	14	3.92E+11	3.05E+07	0.010	2.90E-11	{-12,1//6}	14	3.92E+11	3.05E+07	1.00	0.81
Kuiper belt	5.91E+12		2	6	5.33	191.9	6.40E+12													
{3,2}		2.18E+07	3	2	6	383.8	2.56E+13	2.54E+13	16.77	17	3.39E+13	1.89E+07	0.010	2.24E-14	{-14,1//6}	17	3.39E+13	1.89E+07		
{3,3}		1.80E+07	3	3	6	575.6	5.76E+13	1.89E+14	17.67	18	3.05E+14	1.42E+07	0.010	6.21E-16	{-15,1//6}	18	3.05E+14	1.42E+07		
{3,4}		1.59E+07	3	4	6	767.5	1.02E+14	7.67E+14	18.29	18	4.06E+14	2.18E+07	0.010	6.21E-16	{-15,1//6}	18	4.06E+14	2.18E+07		
{3,5}		1.42E+07	3	5	6	959.4	1.60E+14	2.27E+15	18.77	19	3.05E+15	1.26E+07	0.010	1.72E-17	{-16,1//6}	19	3.05E+15	1.26E+07		
Average																			1.00	0.93
StdDev																			0.00	0.23

Appendix B. Version-b of NBP formula (for nLL QM state only)

An alternative way (named as version-b) to produce a 3D spherical NBP peak for eq-56a (version-a) is

$$r^2\left|\Psi(r,\theta,\varphi,t)_{planet}\right|^2_{NBP} = r^2|R(r)|^2_{NBP}|\Theta(\theta)|^2_{NBP}\,|\Phi(\varphi)|^2_{NBP}|T(t)|^2_{NBP} \propto \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right]^n \left\{\left[\frac{1+\sin\theta}{2}\right]\left[\frac{1+\cos(\varphi-\frac{n}{n-1}\omega_n t)}{2}\right]\right\}^{2(n-1)}$$

eq-56b

In comparison with eq-56a, eq-56b narrows the 3D NBP peak by $\frac{1}{\sqrt{2}} = 0.707$ (calculation not shown here). However, the version-b is disfavored (in general) because the"2x" exponential index is not come from the mathematical deduction, only come from a make-up to make the 3D NBP peak spherical.

Appendix C. Version-c of NBP formula (for nLL QM state only, **the most favorite form**)

The 3rd way (named as version-c) to produce a 3D spherical NBP peak for eq-56a (version-a) is

$$r^2\left|\Psi(r,\theta,\varphi,t)_{planet}\right|^2_{NBP} = r^2|R(r)|^2_{NBP}|\Theta(\theta)|^2_{NBP}\,|\Phi(\varphi)|^2_{NBP}|T(t)|^2_{NBP}$$
$$\propto \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right]^n [\sin(\theta)]^{n-1}\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos\left[(n-1+\delta)\left(\varphi-\frac{n}{n-1}\omega_n t\right)\right]\right\}$$

eq-56cc

Although eq-56cc is accurately correct (it produces only one peak at $\left(\varphi-\frac{n}{n-1}\omega_n t\right) = 0$, no second peak at $\left(\varphi-\frac{n}{n-1}\omega_n t\right) = \pm\pi$), it is practically difficult to use (because of the SUM). Using the wave packet in Figure 1 of SunQM-3s11 (and eq-35-SunQM-3s11, listed below as eq-16c), we can simplify eq-56cc as

$$\mathbf{r^2\left|\Psi(r,\theta,\varphi,t)_{planet}\right|^2_{NBP} \propto \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right]^n \left[\sin(\theta)\cos\left(\varphi-\frac{n}{n-1}\omega_n t\right)\right]^{n-1}} \quad \textbf{eq-56c}$$

with the limitation that $\frac{-\pi}{2} \le \left(\varphi-\frac{n}{n-1}\omega_n t\right) \le \frac{\pi}{2}$, and manually set NBP=0 for both $\left(\varphi-\frac{n}{n-1}\omega_n t\right) > \frac{\pi}{2}$ and $\left(\varphi-\frac{n}{n-1}\omega_n t\right) < \frac{-\pi}{2}$ (to manually eliminate the extra peak of $\cos(\varphi)^m$ at φ=±π). Also pay attention that t is in φ-dimension only.

$r^2|R(n,l=n-1)|^2_{NBP} \propto \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right]^n$ eq-55c

$|\Theta(\theta)|^2_{NBP} \propto \sin(\theta)^{(n-1)}$ eq-52c

$|\Phi(\varphi)|^2_{NBP}|T(t)|^2_{NBP} \propto \left[\cos\left(\varphi-\frac{n}{n-1}\omega_n t\right)\right]^{(n-1)}$ eq-47c

$|\Phi(\varphi)|^2_{NBP} \propto e^{im\varphi} = [\cos(\varphi)+i\,\sin(\varphi)]^m \to [\cos(\varphi)]^m = [\cos(\varphi)]^{n-1}$ eq-10c

$\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(m+\delta)\varphi] \to \cos(\varphi)^m$, and ignore the extra peak of $\cos(\varphi)^m$ at φ=π eq-16c

In comparison with eq-56a, eq-56c narrows the 3D NBP peak by $\frac{1}{\sqrt{2}} = 0.707$ (see Table 2 below for detailed calculation). We favor the version-c the most, then version-d, version-a, and version-b. One major advantage of version-c is that it directly uses the wave function as its NBP function, and by doing so it significantly simplified the NBP calculation (see SunQM-4s2). However, it introduced two new problems at the same time: now NBP's minimum is a negative value, and $\cos(\varphi)^m$ contains an extra peak at φ=π (that should not be there). To solve (or avoid) this problem, we need to use the wave packet in Figure 1 of SunQM-3s11 to represent the NBP peak. Then, eq-12 through eq-16 (in the current paper) can be explained much easier. (Note: this may be the only correct way among version-a, -b, -c and -d).

The version-c of the NBP density formula for the orbital moving Earth in Solar system is

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Earth}|^2_{NBP} \approx \left[\frac{r}{1.57\times10^{11}}\,e^{\left(1-\frac{r}{1.57\times10^{11}}\right)}\right]^{1.09\times10^{10}} \left[\sin(\theta)\cos\!\left(0-\frac{n}{n-1}1.86\times10^{-7}t\right)\right]^{1.09\times10^{10}} \qquad \text{eq-68c}$$

The more accurate but practically less useful form is

$$r^2\,|\Psi(r,\theta,\varphi,t)_{Earth}|^2_{NBP} \approx \left[\frac{r}{1.57\times10^{11}}\,e^{\left(1-\frac{r}{1.57\times10^{11}}\right)}\right]^{1.09\times10^{10}} [\sin(\theta)]^{1.09\times10^{10}} \left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos\left[(1.09\times10^{10}+\delta)\left(0-\frac{n}{n-1}1.86\times10^{-7}t\right)\right]\right\} \qquad \text{eq-68cc}$$

where the integer δ can be set as 1E+6.

Table 2. Using eq-56c to calculate a planet's Eigen n' (in both r- and θ-dimension, see SunQM-3s11's Table 1 for details).

NASA's data of planets			{N,n} model			set total n=1 at Sun core		Determine planet r-dimensional n' & w								Determine planet θ-dimensional n' & w				
						calc model n, r_n, v_n										Note: w(ϕ) = w(θ), and $n'_\phi = n'_\theta$				
	Sun's body-r or planets' orbit-r_n	planet's body-r, b=	N	n	q	total n from Sun core	$r_n = r_1 * n^2$	n' = ln(0.01) / [ln(1 + b /r_n)-(b /r_n)]	w= log(n' / n) / log(6)	round up w	n'_r = n*q^w	b(r) =	[(1+b/r_n)*exp(-b r_n)]^n' = 0.01	$r_1 = r_{n'}$ / (n*q^w)^2	r_1 at {N,1//6}	w(θ) =	n'_θ = n*q^w	b(θ) = r*sin(acos[0.01^(1/n')])	b(r) / b(θ)	planet's body-r /b(θ)
unit	m	m					m					m		m				m	m/m	m/m
Sun core	1.74E+08		0	1	6	1	1.74E+08													
SUN	6.96E+08		0	2	6	2	6.96E+08													
Mercury	5.79E+10	2.44E+06	1	3	6	18	5.64E+10	4.91E+09	11.84	12	6.53E+09	2.12E+06	0.010	1.32E-09	{-11,1//6}	12	6.53E+09	2.12E+06	1.00	1.15
Venus	1.08E+11	6.05E+06	1	4	6	24	1.00E+11	2.52E+09	11.31	11	1.45E+09	7.98E+06	0.010	4.76E-08	{-10,1//6}	11	1.45E+09	7.98E+06	1.00	0.76
Earth	1.49E+11	6.38E+06	1	5	6	30	1.57E+11	5.55E+09	11.62	12	1.09E+10	4.55E+06	0.010	1.32E-09	{-11,1//6}	12	1.09E+10	4.55E+06	1.00	1.40
Mars	2.28E+11	3.40E+06	1	6	6	36	2.25E+11	4.06E+10	12.63	13	7.84E+10	2.44E+06	0.010	3.67E-11	{-12,1//6}	13	7.84E+10	2.44E+06	1.00	1.39
Asteroid belt	4.02E+11		1	8	6	48	4.01E+11													
Jupiter	7.78E+11	7.15E+07	2	2	5.33	64.0	7.12E+11	9.12E+08	11.06	11	7.26E+08	8.02E+07	0.010	1.35E-06	{-9,1//6}	11	7.26E+08	8.02E+07	1.00	0.89
Saturn	1.43E+12	6.03E+07	2	3	5.33	95.9	1.60E+12	6.50E+09	11.93	12	6.53E+09	6.01E+07	0.010	3.75E-08	{-10,1//6}	12	6.53E+09	6.01E+07	1.00	1.00
Uranus	2.97E+12	2.56E+07	2	4	5.33	127.9	2.85E+12	1.14E+11	13.37	13	5.22E+10	3.78E+07	0.010	1.04E-09	{-11,1//6}	13	5.22E+10	3.78E+07	1.00	0.68
Neptune	4.51E+12	2.48E+07	2	5	5.33	159.9	4.45E+12	2.97E+11	13.78	14	3.92E+11	2.16E+07	0.010	2.90E-11	{-12,1//6}	14	3.92E+11	2.16E+07	1.00	1.15
Kuiper belt	5.91E+12		2	6	5.33	191.9	6.40E+12													
{3,2}		2.18E+07	3	2	6	383.8	2.56E+13	1.27E+13	16.39	16	5.64E+12	3.27E+07	0.010	8.05E-13	{-13,1//6}	16	5.64E+12	3.27E+07		
{3,3}		1.80E+07	3	3	6	575.6	5.76E+13	9.45E+13	17.28	17	5.08E+13	2.45E+07	0.010	2.24E-14	{-14,1//6}	17	5.08E+13	2.45E+07		
{3,4}		1.59E+07	3	4	6	767.5	1.02E+14	3.83E+14	17.90	18	4.06E+14	1.54E+07	0.010	6.21E-16	{-15,1//6}	18	4.06E+14	1.54E+07		
{3,5}		1.42E+07	3	5	6	959.4	1.60E+14	1.13E+15	18.38	18	5.08E+14	2.16E+07	0.010	6.21E-16	{-15,1//6}	18	5.08E+14	2.16E+07		
Average																			1.00	1.05
StdDev																			0.00	0.27

Appendix D. Version-d of NBP formula (for nLL QM state only)

The 4th way (named as version-d) to produce a 3D spherical NBP peak for eq-56a (version-a) is

$$r^2\left|\Psi(r,\theta,\varphi,t)_{planet}\right|^2_{NBP} \propto \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right]^{2n}\left[\sin(\theta)\cos\left(\varphi-\frac{n}{n-1}\omega_n t\right)\right]^{2(n-1)} \qquad \text{eq-56d}$$

with the limitation that $\frac{-\pi}{2} \le \left(\varphi-\frac{n}{n-1}\omega_n t\right) \le \frac{\pi}{2}$, and manually set NBP=0 for both $\left(\varphi-\frac{n}{n-1}\omega_n t\right) > \frac{\pi}{2}$ and $\left(\varphi-\frac{n}{n-1}\omega_n t\right) < \frac{-\pi}{2}$ (to manually eliminate the extra peak of $\cos(\varphi)^m$ at φ=±π). This formula is based on that an orbital moving planet's θ-dimensional movement is in equilibrium (or in a standing wave), so it should use Born probability $|\Theta(\theta)|^2_{Born} \propto \sin(\theta)^{2(n-1)}$ to describe. Then, instead of fitting $|\Theta(\theta)|^2$ to $|\Phi(\varphi)|^2_{NBP}|T(t)|^2_{NBP}$ to obtain eq-56c, we fit $|\Phi(\varphi)|^2_{NBP}|T(t)|^2_{NBP}$ to $|\Theta(\theta)|^2_{Born}$ to obtain eq-56d. In comparison with eq-56a, eq-56d narrows the 3D NBP peak by $\frac{1}{2}$ (see calculation in Table 3). In comparison with eq-56c, eq-56d narrows the 3D NBP peak by $\frac{1}{\sqrt{2}} = 0.707$.

The more accurate but practically less useful form of eq-56d is

$$r^2\left|\Psi(r,\theta,\varphi,t)_{planet}\right|^2_{NBP} \propto \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right]^{2n'}[\sin(\theta)]^{2(n'-1)}\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos\left[(n'-1+\delta)\left(\varphi-\frac{n}{n-1}\omega_n t\right)\right]\right\}^2 \qquad \text{eq-56dd}$$

Table 3. Using eq-56d to calculate a planet's Eigen n' (in both r- and θ-dimension, see SunQM-3s11's Table 1 for details).

NASA's data of planets			{N,n} model			set total n=1 at Sun core		Determine planet r-dimensional n' & w								Determine planet θ-dimensional n' & w				
						calc model n, r_n, v_n										Note: w(ϕ) = w(θ), and $n'_\phi = n'_\theta$				
	Sun's body-r or planets' orbit-r_n	planet's body-r, b=	N	n	q	total n from Sun core	$r_n = r_1$*n^2	n' = ln(0.1) / [ln(1 + b /r_n) - (b /r_n)]	w= log(n' / n) / log(6)	round up w	n'_r = n*q^w	b(r) =	[(1+b/r_n)*exp(-b r_n)]^(2n') = 0.01	$r_1 = r_{n'}$ / (n*q^w)^2	r_1 at {N,1//6}	w(θ) =	n'_θ = n*q^w	b(θ) = r*sin(acos[0.01^(1/(2n'))])	b(r) / b(θ)	planet's body-r /b(θ)
unit	m	m					m					m		m				m	m/m	m/m
Sun core	1.74E+08		0	1	6	1	1.74E+08													
SUN	6.96E+08		0	2	6	2	6.96E+08													
Mercury	5.79E+10	2.44E+06	1	3	6	18	5.64E+10	2.46E+09	11.45	11	1.09E+09	3.67E+06	0.010	4.76E-08	{-10,1//6}	11	1.09E+09	3.67E+06	1.00	0.67
Venus	1.08E+11	6.05E+06	1	4	6	24	1.00E+11	1.26E+09	10.92	11	1.45E+09	5.64E+06	0.010	4.76E-08	{-10,1//6}	11	1.45E+09	5.64E+06	1.00	1.07
Earth	1.49E+11	6.38E+06	1	5	6	30	1.57E+11	2.77E+09	11.24	11	1.81E+09	7.89E+06	0.010	4.76E-08	{-10,1//6}	11	1.81E+09	7.89E+06	1.00	0.81
Mars	2.28E+11	3.40E+06	1	6	6	36	2.25E+11	2.03E+10	12.25	12	1.31E+10	4.23E+06	0.010	1.32E-09	{-11,1//6}	12	1.31E+10	4.23E+06	1.00	0.80
Asteroid belt	4.02E+11		1	8	6	48	4.01E+11													
Jupiter	7.78E+11	7.15E+07	2	2	5.33	64.0	7.12E+11	4.56E+08	10.67	11	7.26E+08	5.67E+07	0.010	1.35E-06	{-9,1//6}	11	7.26E+08	5.67E+07	1.00	1.26
Saturn	1.43E+12	6.03E+07	2	3	5.33	95.9	1.60E+12	3.25E+09	11.54	12	6.53E+09	4.25E+07	0.010	3.75E-08	{-10,1//6}	12	6.53E+09	4.25E+07	1.00	1.42
Uranus	2.97E+12	2.56E+07	2	4	5.33	127.9	2.85E+12	5.71E+10	12.98	13	5.22E+10	2.67E+07	0.010	1.04E-09	{-11,1//6}	13	5.22E+10	2.67E+07	1.00	0.96
Neptune	4.51E+12	2.48E+07	2	5	5.33	159.9	4.45E+12	1.48E+11	13.39	13	6.53E+10	3.73E+07	0.010	1.04E-09	{-11,1//6}	13	6.53E+10	3.73E+07	1.00	0.66
Kuiper belt	5.91E+12		2	6	5.33	191.9	6.40E+12													
{3,2}		2.18E+07	3	2	6	383.8	2.56E+13	6.35E+12	16.00	16	5.64E+12	2.31E+07	0.010	8.05E-13	{-13,1//6}	16	5.64E+12	2.31E+07		
{3,3}		1.80E+07	3	3	6	575.6	5.76E+13	4.73E+13	16.89	17	5.08E+13	1.73E+07	0.010	2.24E-14	{-14,1//6}	17	5.08E+13	1.73E+07		
{3,4}		1.59E+07	3	4	6	767.5	1.02E+14	1.92E+14	17.51	18	4.06E+14	1.09E+07	0.010	6.21E-16	{-15,1//6}	18	4.06E+14	1.09E+07		
{3,5}		1.42E+07	3	5	6	959.4	1.60E+14	5.67E+14	18.00	18	5.08E+14	1.53E+07	0.010	6.21E-16	{-15,1//6}	18	5.08E+14	1.53E+07		
Average																			1.00	0.96
StdDev																			0.00	0.28