Early Evaluation and Effectiveness of Social Separation Measure for Controlling COVID-19 Outbreaks

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Abstract

Based on real data, we study the effectiveness and we propose an early evaluation method for COVID-19 social separation measures.

Key Words: virology, systems simulation.

1 Introduction

One of the most simple model in Virology is the SIR (Susceptible, Infected, Removed) Model, mostly used for didactics but still very effective. A key parameter of the Model is $R_0$, the basic reproduction number which represent the average number of people an infected transmits the virus to. This Parameter is affected by social separation measure taken by governments to block the infection. However, the model is not designed to take into account a variable $R_0$. In this paper we try to somehow introduce this variable in the model.

2 The SIR Model

The SIR model studies the evolution of the number of people $S(t)$, $I(t)$ and $R(t)$ which are respectively the Susceptible (the ones that can be can be infected), the Infected and the Removed (the ones that have been immunised by the virus or vaccine).

Of course in stationary hypothesis, which is when the period of the outbreak is small enough to consider the total population $N$ constant, we have $S+I+R = N$.

The SIR model is described by the following equations:

$$\begin{cases} x' = -\beta xy \\ y' = \beta xy - \gamma y \end{cases} \quad \text{where} \quad x(t) = \frac{s(t)}{N} \quad y(t) = \frac{i(t)}{N}$$

(1)
where we define also $R_0 = \frac{\beta}{\gamma}$ and $T_i = \frac{1}{\gamma}$ where $T_i$ is the effective period in which an infected person passes the virus to $R_0$ other people (in average). For the COVID-19, first estimations of the parameters from various organizations around the world are $T_i$ between 5 and 7 days and $R_0$ between 2 and 3 people.

With these two parameter the model is fully defined and $R_0$ is basically responsible for the pick and the total infected at the end of the outbreak while $T_i$ is basically responsible for the duration of it.

If the initial conditions are $x(0) = x_0$ and $y(0) \approx 0$, which is the case for COVID-19, then it is possible to expand $y$ to the first order in the equations which became linear in $y$. By using $x = x_0$ in we get:

$$y' = (\beta x_0 - \gamma) y$$

which has the following solution:

$$y = y_0 e^{rt}; \text{ with } r = \beta x_0 - \gamma = \left(\frac{\beta x_0}{\gamma} - 1\right)\gamma = (R_0 x_0 - 1)\gamma$$

(3)

We get easily:

$$R_0 = \frac{1}{x_0} \left[1 + r T_i\right]$$

(4)

If $R_0$ is constant, also $r$ does not change and the initial part of the solution $y(t)$ is a constant slope line in a plot where the vertical axis is in a logarithmic scale. Assuming to have the value for $T_i$, $R_0$ can be estimated using the slope of the linear regression of the above curve.

Finally, assuming that $T_i$ does not depend from social distancing measures in place from the government and $R_0$ changes because of them, we may think to evaluate $R_0$ with the following function $D_0$:

$$D_0 = \frac{1}{x_0} \left[1 + \frac{d}{dt} ln[y(t)] T_i\right]$$

(5)

Where $y(t)$ are real measured data and for the first outbreak we may choose safely $x_0 = 1$. Note that if only a constant fraction of infected are detected, the logarithm transform the multiplicative constant in an additive constant and the derivative take it to zero. For the reason the estimation of $D_0$ is not affected in that case. Note also that when $D_0 = 1$, we see a peak in the active cases of the outbreak.

However, there are some problem with $D_0$, and this will be discussed in the following paragraph.

### 3 $R_0$ Varying with Time

We turn now our attention to a real case which is the outbreak of CoVd-19 of the beginning of 2020. When social separation measure are taken by the government, $R_0$ changes as a step function, from a day to he following, between a value $R_i$, previous measures, to a value $R_f$, post measures. However, analysis of real data shows that the estimation of $D_0$ from Eq. [5] goes down slowly. Fig. 1 shows $D_0$ evaluated on real data from the Chinese outbreak (source [2]).
Figure 1: $D_0$ in China estimated on real data

From the figure $D_0$ looks having an exponential decreasing trend. We make the hypothesis that $D_0$ has an inertia in changing and it is like the output of a first order dynamic systems with transfer function:

$$D_0(s) = \frac{1}{1 + s\tau} R_0(s)$$

which responds to the step function $R_0 = R_i + (R_f - R_i)u(t)$ where $u(t)$ is the Heaviside function.

We propose to modify the model, in order to take into account the delay of the system to respond to a change in $R_0$, as shown in Fig. 2.

Figure 2: Simulation with variable $R_0$

4 Evaluation of the Time Constant

A first approximation of the value of the time constant $\tau$ can be done directly from the plot of $D_0$. For example, from real data of the Chinese outbreak in Wuhan of Jan. 2020 (source [2]), after the lockdown $D_0$ has gone from a value of about $R_i = 2.53$ to a value of about $R_f = 0.5$ (we will see later that this value is much lower). Since the time constant (assuming a first order system) is the time required for $D_0$ to decrease by 63.2% of the interval $(R_i - R_f)$, the time constant can be evaluated as the number of days required to $D_0$ to get to a value of about 1.26.

From the data and based on the above consideration we get a time constant of exactly:

$$\tau = 14 \text{ days}$$

5 Evaluation of $R_0$ in the Early Days

We want to evaluate $R_0$ from the early days after social separating measures have been applied to see if it has been effective. Given the above assumption of a first order system behaviour, the value of $D_0(t)$ is function of the three parameter, $R_i$, $R_f$ and $\tau$ as follows:

$$f_{D_0}(t, R_i, R_f, \tau) = R_i + (R_f - R_i) \left(1 - e^{-\frac{t}{\tau}}\right)$$

Given the real data $D_0(t)$, the above three parameters can be evaluated minimizing the functional:

$$J(R_i, R_f, \tau) = \sum_{n=\text{days of data}} |D_0(t_n) - f_{D_0}(t_n, R_i, R_f, \tau)|^2 dt$$

6 Characterization of Outbreaks

We have evaluated the above three parameters using real data from the Chinese outbreak in Wuhan (source [2]) and for the Italian outbreak (source [1]) of the beginning of 2020 and we have found as follows:

**Chinese outbreak:**

- $R_i = 2.533$ [people]
- $R_f = 0.290$ [people]
- $\tau = 15.805$ [days]
- $T_p = 20$ [days] time of peak from lockdown
- $T_i = 6$ [days] assumed value

The Fig. 3 shows the comparison between real data and relevant characterizing curve:

![Figure 3: Chinese COVID-19 Outbreak](image-url)
Italian outbreak:

\[
\begin{align*}
R_i &= 2.367 \ [\text{people}] \\
R_f &= 0.822 \ [\text{people}] \\
\tau &= 17.271 \ [\text{days}] \\
T_p &= 39 \ [\text{days}] \ \text{time of peak from lockdown} \\
T_i &= 6 \ [\text{days}] \ \text{assumed value}
\end{align*}
\] (11)

The Fig. 4 shows the comparison between real data and the relevant characterizing curve:

Figure 4: Italian COVID-19 Outbreak

In the above figure, real data (blue line) are reported till the data of writing of this paper (i.e. 26/03/20). The red line shows the theoretical evolution of the outbreak if the trend of the data continues to follow the same pattern. It has to be noted that the real data contains a lot of noise which has a major effect of the final parameters.

Comparing the two examples above we note that according the the data, Chinese measure have been more effective leading to an final \( R_0 \) much lower. This because \( R_0 \) is affected by \( \beta \) which in turn is affected by the probability for people to meet. Throughout the paper we have assumed that \( T_i \) and therefore \( \gamma \) are not affected by this measures.

Finally, in Fig. 5, we propose a plot of the theoretical evolution of the active cases \( I(t) \) for the outbreak in Italy (red line) versus real data when available (blue line).

We note that, according to the data, Italy is quite marginal in having an effective value for \( R_f \), so marginal that, further fluctuation of the measured \( D_0 \) value, could even put in danger the possibility to reach an early peak.
Figure 5: Theoretical Evolution of Italian Outbreak

References
