Liquid surface tension.
Forms of hanging and sitting drops.

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The surface tension of a liquid is associated with many effects such as capillary forces, meniscus formation at walls of a vessel containing liquid, shaping of drops of liquid, etc.

In a basis of these physical phenomena are interatomic and intermolecular forces both inside of a liquid and on the surfaces of contact of a liquid with environment and solid substances. It is confirmed by the evident and well-known interrelation of surface tension with vaporation heat of liquids.

Researches of apparently quite clear physical phenomena in liquids with the purpose to get more exact and detailed knowledge are caused by requirements of modern techniques and technologies development. It concerns in full with studying of liquids properties in a drop state.

In given article the solution method of a problem on the form of a drop of liquid in the gravitational field, convenient for the numerical solution on modern computers, specifically is presented. Examples of this problem solution are offered and some characteristic results are shown.

1. Introductory remarks

Evaporation can be represented as fragmentation of a liquid down to the smallest parts (molecules and atoms). The energy, necessary "to disjoint" from each other these small particles making a liquid, actually, is evaporation energy.

It is also clear that the particles, which are being on a surface of a liquid have in part unused potential energy in comparison with those particles, which being in the interior of a liquid interact with a maximum number of surrounding particles. Therefore the liquid, tending to a minimum of potential energy, "tries" to get the form at which its surface is minimal. On the other hand, any increase in a surface of a liquid (having constant volume) causes the expenditure of energy.

Proceeding from these clear reasons, it is possible to draw a wrong conclusion that particles being on the liquid surface are easier to be removed (for example, at evaporation). This conclusion can seem quite natural as expenditure of energy (at evaporation) is connected with break of intermolecular bonds, and in this case bonds on liquid surface are partially absent.

Let's examine more closely this paradoxical situation.

Let's imagine that the small particle was separated from a surface of the liquid having the form of a sphere (for example, in zero-gravity condition).

Let's assume that radius of an initial sphere (for example, of water) was equal to R. Owing to separation of the small particle having radius r (the form of a small particle for simplicity we shall consider also spherical) there was some reduction of initial radius of a water sphere (dashed line, Fig. 1) and accordingly some decrease of area of its surface shell occur.
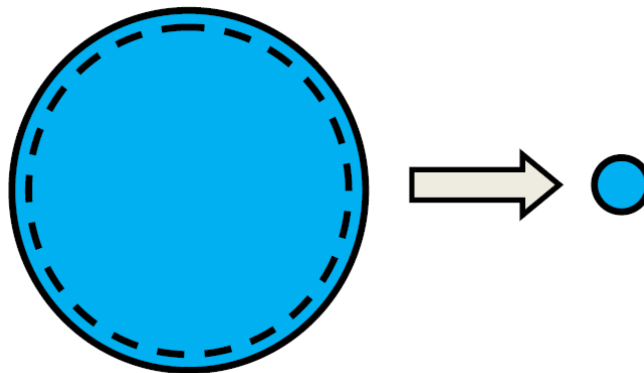

Fig. 1. Separation of a small particle from spherical water mass.

Now we have to compare the total area of water surface before and after separation of a small particle.

An initial water mass in the form of a sphere had had area of a surface equal to $S = 4\pi R^2$.

After separation of a particle the total area of water shares between the separated particle, which area of surface is equal to $4\pi r^2$, and the area of surface of the great bulk of water, which has somewhat decreased.

As the separated particle is very small in comparison with initial volume of water it is possible to get the approximate expression for radius reduction of a water sphere ΔR:

$$\frac{4}{3}\pi r^3 \approx 4\pi R^2 \Delta R, \qquad \Delta R \approx \frac{1}{3}\frac{r^3}{R^2}. \tag{1}$$

In the equation (1) is stated the fact that a small particle volume is equal to the reduction of initial volume of water.

Accordingly reduction of the area of surface of a great bulk of water (in the form of sphere) will be equal to differential ΔS:

$$\Delta S \approx 8\pi R \cdot \Delta R \approx \frac{2}{3}\frac{r}{R} \cdot 4\pi r^2. \tag{2}$$

As we see, surface reduction of great bulk of water is equal to very small part of surface of the separated particle $4\pi r^2$ (as earlier we have put r <<R), and not nearly compensates the increase in general water surface.

If evaporation occurs from water surface in a vessel, then it is a fortiori possible to consider the area of the open surface of a great bulk of water constant.

Hence we can define the specific heat of evaporation L: we shell multiply the area of surface of the separated particle of water by the force of surface tension and next we shall divide the received energy expenditure by the mass of separated small particle:

$$L = \frac{4\pi r^2 \sigma}{\rho(4\pi r^3/3)} = \frac{3\sigma}{\rho r}. \tag{3}$$

Here ρ - density of a liquid,

σ - surface tension force, actually the same as surface potential energy (it is easy to notice, that these physical quantities have the same dimension: N/m = N·m/m^2 = J/m^2).

Expression (3) enables to estimate the size of evaporating small particles on the bases of known values of specific heat of evaporation, surface tension force and density of a liquid:

$$r \approx \frac{3\sigma}{\rho L}. \tag{4}$$

In case of water such estimation gives following result:

$$r \approx \frac{3 \cdot 0,0728}{10^3 \cdot 2,3 \cdot 10^6} \approx 0,095 \cdot 10^{-9} m \approx 10^{-10} m.$$

For mercury we receive the same order of magnitude:

$$r \approx \frac{3 \cdot 0,465}{13,5 \cdot 10^3 \cdot 282 \cdot 10^3} \approx 3,66 \cdot 10^{-10} m.$$

This result indicates that evaporating particles have minimally possible size - at a level of separate molecules and atoms, that is, measured by units of angstrom (1 Å = 10^{-10}m).

Naturally, it is only rank values, as much depends on «a mode of packing» and on the form of molecules of each concrete liquid.

There is one more crucial issue. How much total surface Σs of evaporated particles exceeds surface of liquid in condensed state? We shall make such estimate, for example, for one litre (1 kg) of water:

$$\frac{\Sigma s}{S_1} = \frac{\sigma \cdot \Sigma s}{\sigma \cdot S_1} = \frac{L}{\sigma \cdot S_1} = \frac{2,3 \cdot 10^6}{0,0728 \cdot 6 \cdot 10^{-2}} \approx 5 \cdot 10^8.$$

Here the litre of water is represented in the form of cubic decimeter, and comparison is carried out through comparison of potential surface energy of all evaporated molecules of water (for 1 kg it is equal to specific heat of evaporation) and potential energy of surface of liquid in condensed state (surface of cubic decimeter).

As we see these values differ by many orders of magnitude and hence the conclusion about small influence on evaporation of surface energy of an initial liquid takes one more confirmation.

Let's estimate now binding energy of one molecule of water in condensed state.

As one mole of water (0,018 kg) contains $6,02 \cdot 10^{23}$ molecules (Avogadro number), so at evaporation of water on each molecule it is spent

$$E_1 = \frac{L\mu}{N_A} = \frac{2,3 \cdot 10^6 \cdot 0,018}{6,02 \cdot 10^{23}} \approx 0,69 \cdot 10^{-19} J \approx 0,43 eV.$$

Similar calculation for mercury gives close result:

$$E_1 = \frac{L\mu}{N_A} = \frac{0,282 \cdot 10^6 \cdot 0,2}{6,02 \cdot 10^{23}} \approx 0,94 \cdot 10^{-19} J \approx 0,58 eV.$$

It is high binding energy, for example, in comparison with thermal energy of molecules under normal conditions ($\kappa T \sim 0,03 \ eV$). Therefore in normal conditions evaporation process goes rather slowly.

On the other hand, high-energy processes of evaporation and condensation of water, for example, in atmosphere generate huge flows of energy and determine climate of the Earth.

Knowing binding energy of one molecule it is also possible to make a rough estimate of surface tension force.

Number of molecules in volume unit, for example, of water is:

$$n = \frac{N_A}{\mu / \rho} = \frac{6,02 \cdot 10^{23} \cdot 10^3}{0,018} \approx 3,34 \cdot 10^{28} (1/m^3).$$

Number of molecules on surface unit we shall estimate proceeding from the most simple arrangement of molecules - in the form of a cubic lattice. Then for water this number is about:

$$n_s \approx (33,4 \cdot 10^{27})^{2/3} \approx 10,37 \cdot 10^{18} (1/m^2).$$

Assuming, that molecule being on a surface of liquid has free one sixth part of its potential (binding) energy, we obtain estimated value of surface tension force:

$$\sigma \approx \frac{1}{6} E_1 \cdot n_s \approx \frac{1}{6} \cdot 0,69 \cdot 10^{-19} \cdot 10,37 \cdot 10^{18} \approx 0,12 J/m^2.$$

It is correct order of magnitude though calculation is one and a half times as much than the actual value (0,0728 J/m^2).

Similar calculations for mercury give following result:

$$n = \frac{N_A}{\mu / \rho} = \frac{6,02 \cdot 10^{23} \cdot 13,5 \cdot 10^3}{0,2} \approx 4,06 \cdot 10^{28} (1/m^3).$$

$$n_s \approx (40,6 \cdot 10^{27})^{2/3} \approx 11,8 \cdot 10^{18} (1/m^2).$$

$$\sigma \approx \frac{1}{6} E_1 \cdot n_s \approx \frac{1}{6} \cdot 0,94 \cdot 10^{-19} \cdot 11,8 \cdot 10^{18} \approx 0,185 J/m^2.$$

As we see, the order of magnitude is also correct, though actual value (0,465 J/m^2) in this case exceeds more than twice the calculation.

For rough estimations such results it is possible to recognize quite satisfactory. Similar calculations have been done in paper [1] with use of experimental characteristics of more than 50 liquid substances. Calculations have convincingly shown interrelation of surface tension force, specific heat of evaporation and properties of liquid molecules (structure features).

2. Forms of drops of a liquid under gravitation

The radius of curvature of a liquid surface, as per our point of view, should not influence a surface tension force as it is determined by huge intermolecular bonds. But it is not only the effect of intermolecular interactions (see п.1), but also essential is the fact that any radius of curvature can be considered as infinitely large in comparison with the size of molecules.

Therefore calculation of drops form were carried out assuming that surface tension force is constant and does not depend on direction (along a liquid surface).

Thus the surface tension actually creates effect of some elastic thin film enveloping a liquid while inside of this surface the liquid follows usual laws of hydrostatics (in state of rest).

So the calculation of drops form is based on solution of Laplace equation, or on variational solution. Variational approach gave an opportunity to arrive at the analytical solution and to reduce a problem to numerical integration of some (rather complicated) function.

The detailed review on the given subjects was made in paper [2].

Quit not belittling advantages of already existing methods of calculation which actually were developing from the beginning of the XIX century and up to date, it would be useful to pay attention to one more opportunity of solution of a problem concerning a liquid drop form under gravitation.

In the basis of presented method was put the evident physical model of mechanical equilibrium of a liquid under surface tension force, gravity and internal pressure in a drop of liquid.

First we shall examine simpler problem on internal pressure of a spherical drop of a liquid (in weightlessness).

The first variant of solution. Let's derive the equation describing a balance of forces in horizontal section of sphere, halving it (fig. 2). On the one hand, internal pressure of a liquid drop creates in this section the disruptive force proportional to the area of this section, and on the other hand, the surface tension of a liquid withstands to this internal pressure. So we receive expression for pressure inside of a spherical drop:

$$p \cdot \pi R^2 = \sigma \cdot 2\pi R, \qquad p = \frac{2\sigma}{R}. \tag{5}$$
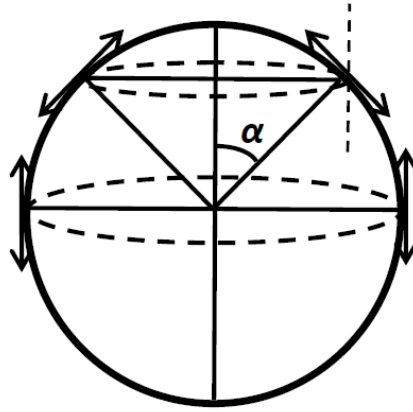
Fig. 2. The surface tension is counterbalanced by internal pressure.

This result corresponds with Laplace's equation.

$$p = \frac{\sigma}{R_1} + \frac{\sigma}{R_2}.$$ (6)

Here R_1 and R_2 radiuses of curvature of a surface in two mutually orthogonal directions. In case of sphere $R_1 = R_2$.

The balance of forces can be got in any section of a sphere (fig. 2). The result of calculation of pressure value, obviously, will turn out the same:

$$p \cdot \pi (R \sin \alpha)^2 = \sigma \sin \alpha \cdot 2\pi R \sin \alpha, \qquad p = \frac{2\sigma}{R}.$$ (7)

The second variant of solution. Let's imagine, that inside a drop the additional small volume of liquid is somehow added (for example, through tubule), and the radius of a sphere increases by a small amount ΔR. Thus energy is spent to overcome the internal pressure. Accordingly there is an increase in a surface area of sphere and hence the potential energy associated with a surface tension also increases. From balance of energy, as expected, we receive the same expression for pressure:

$$p \cdot \Delta V = \sigma \cdot \Delta S, \qquad p \cdot 4\pi R^2 \cdot \Delta R = \sigma \cdot 8\pi R \cdot \Delta R, \qquad p = \frac{2\sigma}{R}.$$ (8)

To solve a problem on the form of a drop of liquid under gravity we also shall use balance of forces, as it was done above (the equations (5), (7)). We shall only consider supplementary forces caused by gravity action (fig. 3).

Calculation is convenient to begin from a drop vertex. The point is that to initialize is expediently at a drop vertex, as well as to do first steps of numerical solution of the problem.
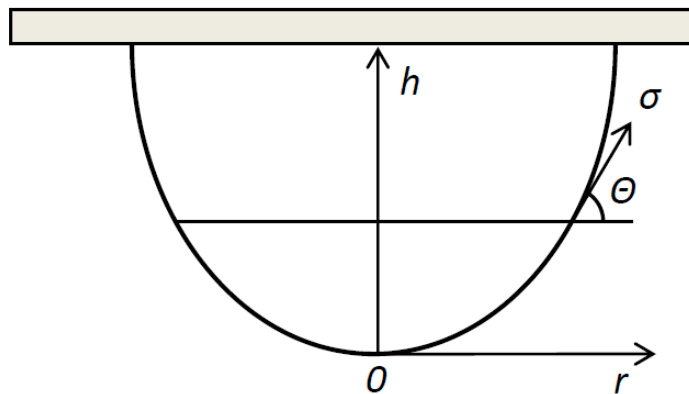
Fig. 3. On calculation of the form of a hanging drop.

Let's set-up balance of forces in any section of a drop (fig. 3):

$$(p - \rho g h) \cdot \pi r^2 + \int_0^h \rho g \pi r^2 dh = 2\pi r \cdot \sigma \sin \Theta. \tag{9}$$

The first member of the equation describes the force arising in given section due to internal pressure (decreases with height).

The second member on the left side of the equation in the form of integral represents weight of the bottom part of a drop, that is, below section (increases with height h).

On the right side of the equation there is surface tension force which counterbalances both forces represented on the left side of the equation.

Unlike the equation for a hanging drop, the equation for a sitting drop (10) differs only in opposite signs. Pressure grows (first member of the equation) when distance from drop vertex grows. The second member has minus sign as the top piece of a drop (above the section) is taken away from the force generated by pressure (fig. 4).
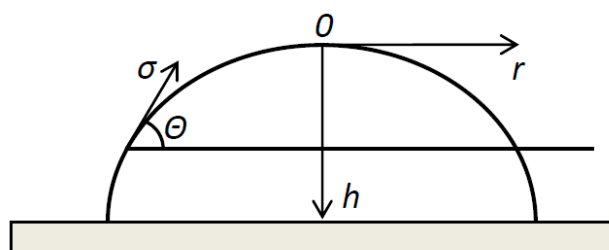

Fig. 4. On calculation of the form of a sitting drop.

Therefore the equation for a sitting drop looks like:

$$(p + \rho g h) \cdot \pi r^2 - \int_0^h \rho g \pi r^2 dh = 2\pi r \cdot \sigma \sin \Theta. \tag{10}$$

Pressure at a drop vertex is determined by radius of curvature, that is, set by initial conditions:

$$p = \frac{2\sigma}{R}. \tag{11}$$

In view of (11) and after division by $\pi\sigma$ equations (9) and (10) it is possible to write down in general view:

$$(\frac{2}{R} \mp \varsigma h) r^2 \pm \int_0^h \varsigma r^2 dh = 2r \sin \Theta. \tag{12}$$

The upper sign regards a hanging drop, and inferior - a sitting drop.

A capillary constant here also is denoted:

$$\varsigma = \frac{\rho g}{\sigma}. \tag{13}$$

On the right side of equations (12) there is function *sin Θ* expressed through a derivative *dr/dh*:

$$\sin \Theta = \frac{1}{\sqrt{1 + ctg^2 \Theta}} = \frac{1}{\sqrt{1 + \left(\frac{dr}{dh}\right)^2}}. \tag{14}$$

In view of (14), the equations for a hanging drop and for a sitting drop (12) acquire the form:

$$(\frac{2}{R} \mp \varsigma h) r^2 \pm \int_0^h \varsigma r^2 dh = \frac{2r}{\sqrt{1 + \left(\frac{dr}{dh}\right)^2}}. \tag{15}$$

$$\frac{dr}{dh} = \left(\frac{4r^2}{\left[(\frac{2}{R} \mp \varsigma h) r^2 \pm \int_0^h \varsigma r^2 dh \right]^2} - 1 \right)^{1/2}. \tag{16}$$

For the numerical solution of the equations (16), for example, in program *Excel* it is necessary to determine curvature radius R at a drop vertex.

It is also essential to pay attention to an opportunity of broader generalization of equations (12):

$$\left(2 \mp \varsigma R^2 \frac{h}{R}\right) \frac{r^2}{R^2} \pm \int_0^h \varsigma R^2 \frac{r^2}{R^2} d\left(\frac{h}{R}\right) = 2 \frac{r}{R} \sin \Theta, \tag{17}$$

$$\left(2 \mp \Phi \alpha\right) \beta^2 \pm \int_0^\alpha \Phi \beta^2 d\alpha = 2 \beta \sin \Theta. \tag{18}$$

Here the notations for so-called *Bond numbers* and relative coordinates are entered:

$$\Phi = \varsigma R^2 = \frac{\rho g R^2}{\sigma}, \qquad \alpha = \frac{h}{R} \qquad \beta = \frac{r}{R}. \tag{19}$$

At such transformation the angle Θ will not change, as coordinates *h* and *r* are divided by the same value *R*.

Thus, the equations for the form of drops (18) do not contain any physical quantity which has dimension.

Occurrence of dimensionless parameter Φ testifies to the effect that changing the characteristic dimension of a drop it is possible to simulate influence of any other physical characteristic of a liquid entering into this parameter.

For example, the form of a drop of a liquid with the greater surface tension force will coincide completely with the form of a drop of the smaller size, but having smaller surface tension force. The comparison of the form of drops of different liquids, naturally, it is necessary to do in relative coordinates.

As an interesting example it is also possible to mention that in conditions of small free fall acceleration on the Moon (1,62 *m/s*^{2}) drops of a liquid will have the same form as on the Earth, but their size will be 2,46 times as big (root of the ratio 9,8/1,62).

Thus, apparently, abstract mathematical transformation of the equations carries great depth of physical meaning and enables to draw important conclusions. Besides the results obtained for any concrete case, it is possible to use for other liquids and conditions.

The equations of drops after differentiation can be reshaped in the form not containing integration (differentiation of integral on the top limit gives subintegral function).

Differentiation of equations (15) gives following result:

$$\frac{d}{dh}\left(\frac{dr}{dh}\right) = \frac{1}{r}\left(1 + \left(\frac{dr}{dh}\right)^2\right)\left(1 - r\left(\frac{2}{R} \mp \varsigma h\right)\sqrt{1 + \left(\frac{dr}{dh}\right)^2}\right). \tag{20}$$

The equations (18) also can be reshaped convenient for calculations:

$$(2 \mp \Phi\alpha)\beta^2 \pm \int_0^\alpha \Phi\beta^2 d\alpha = 2\beta \frac{1}{\sqrt{1 + \left(\dfrac{d\beta}{d\alpha}\right)^2}}. \qquad (21)$$

$$\frac{d\beta}{d\alpha} = \left(\frac{4\beta^2}{\left[(2 \mp \Phi\alpha)\beta^2 \pm \int_0^\alpha \Phi\beta^2 d\alpha \right]^2} - 1 \right)^{1/2}. \qquad (22)$$

Differentiation of the equation (21) gives a result similar to (20):

$$\frac{d}{d\alpha}\left(\frac{d\beta}{d\alpha}\right) = \frac{1}{\beta}\left(1 + \left(\frac{d\beta}{d\alpha}\right)^2\right)\left(1 - \beta[2 \mp \Phi\alpha]\sqrt{1 + \left(\frac{d\beta}{d\alpha}\right)^2}\right). \qquad (23)$$

Let's go into detail on initial steps by the numerical solution of differential equations in program *Excel*.

Depending on the purposes or preferences with equal success it is possible to solve the equations (16) or (22), containing the first derivative and integral, or the equations (20) and (23) with the first and second derivatives. However in any case initial conditions are required for calculations when making first step near the vertex (zero of coordinate system).

Let's use a circle property consisting in approximate ratio of coordinates (real and relative) valid near the origin of coordinates:

$$r \approx \sqrt{2Rh}, \qquad \beta \approx \sqrt{2\alpha}. \qquad (24)$$

In (24) it is considered, that the radius of curvature *in relative coordinates* at drop vertex is always equal to unit.

Near zero it is possible with high accuracy determine not only coordinates (24), but also the first derivative

$$\frac{dr}{dh} \approx \frac{R}{\sqrt{2Rh}}, \qquad \frac{d\beta}{d\alpha} \approx \frac{1}{\sqrt{2\alpha}}. \qquad (25)$$

Thus, at calculations in program *Excel* the first row (at the origin of coordinates) will contain only zero values of coordinates, while the first and the second derivatives cannot be defined as they are infinite.

Formulas (24), (25) are used while filling in the second row (first step). The stride parameter of coordinate axis *h* (or *α*) must be small enough to achieve high precision of calculation.

The second row should also contain the second derivative (formulas (20) or (23)). After that in the same row, using value of the second derivative, the increment of the first derivative, and, using value of the first derivative, the increment of radius (the product of derivatives and stride parameter) are to be located.

The third and subsequent rows are filled in similarly: coordinate h (or α) has the increment at the rate of selected step, then radius (r or β) and the first derivative get calculated in previous row increments. The second derivative is again determined according to formulas (20) or (23) and then, as well as it was done in previous row, increments of the first derivative and radius are filled in.

Automatic calculation and completion of consequent rows presents no difficulties. Opportunity of graphic representation of results, which is also provided by the program *Excel,* enables to carry out visually an estimation and comparison of various variants of calculation.

Computations in accordance with the formulas containing integrals actually differ little from described above sequence of operations.

It is necessary also to note, that calculations in real coordinates enable to receive the additional information (for example, drop weight and pressure inside of the drop).

Below some calculation results describing forms of hanging and sitting drops of water in normal conditions are presented.

When drop size is rather small the form of the drop little differs from sphere (in fig. 5 drops of water with radius of curvature at vertex *R=1mm* are shown, that corresponds to relatively small Bond number $\Phi \approx 0{,}135$).

To show in full measure drop forms with different Bond numbers, we conditionally remove the restriction imposed by wetting angle depending on interphase factors (σ_{SG} - solid body - gas, σ_{SL} - solid body - liquid, σ_{LG} - liquid - gas):

$$\cos \Theta = \frac{\sigma_{SG} - \sigma_{SL}}{\sigma_{LG}}. \qquad (26)$$

Young equation (26) is a condition of mechanical balance of a drop and usually is considered as a basis for calculations of the drop form.

However, assuming universality of the drop form which follows from above mentioned equations, it is logical to impose constraints (26) on the available complete solution.

For example, in fig. 5 is shown that, depending on angle of wetting, it is possible to make imaginary horizontal section so that tangent line in point of the intersection with generating line of drop surface will subtend angle (26) with these horizontal section.
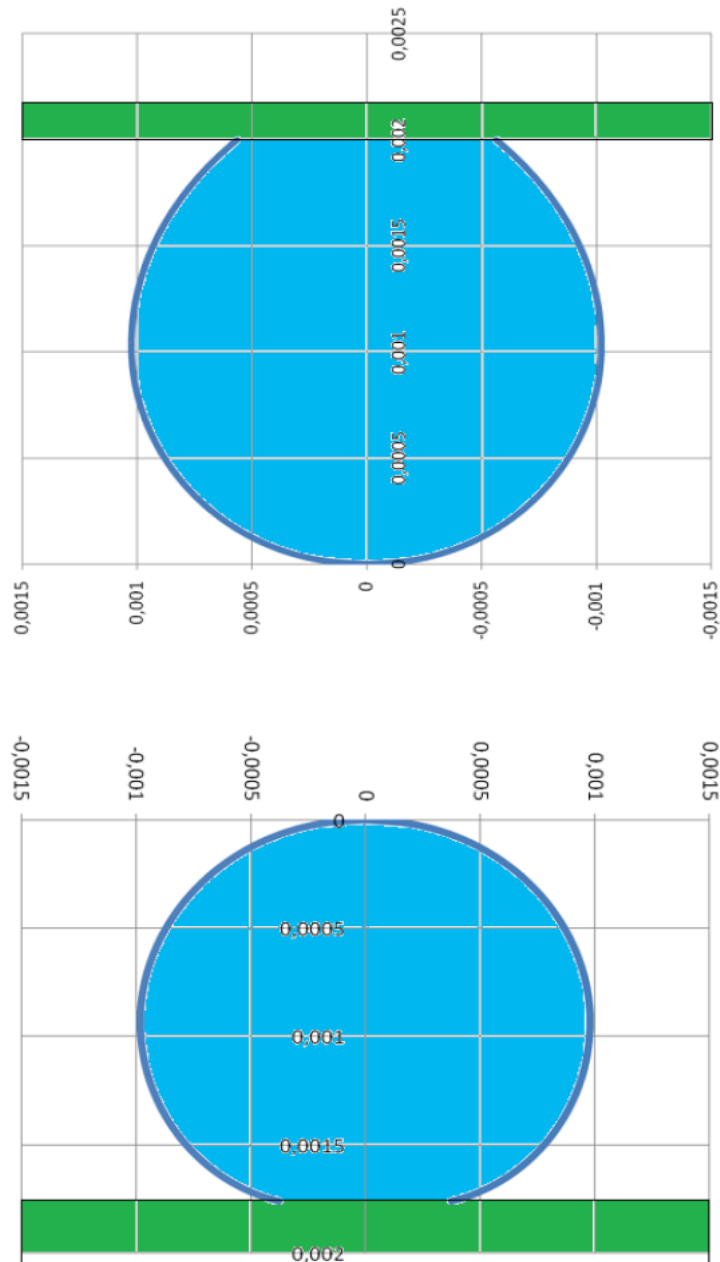

Fig. 5. Hanging drop (above) and sitting drop (below) with radius of curvature at vertex *R=1mm=0,001m* (Φ≈0,135).

Such method, hence, enables an opportunity to use single problem solution (one drop form) for various boundary conditions (26).

Calculations show, that depending on Bond number hanging drop can have two form versions. With small numbers Φ the hanging drop has form close to spherical, as in fig. 5, or a little bit elongated, as in fig. 6.

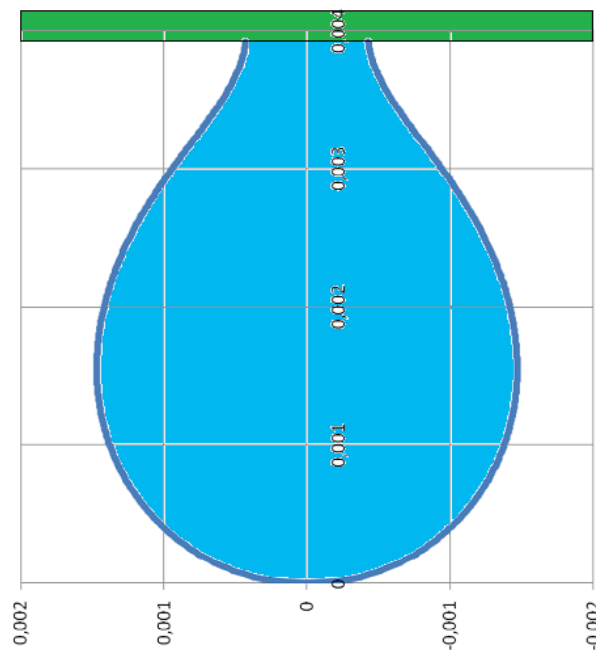At Φ increase the intermediate form version of the drop is observed, shown in fig. 7, and further the drop gets rather complex form having expansion in the top part (fig. 8).

Fig. 6. Elongated form of a hanging drop (*R=1,4mm*, Φ≈0,264).

Interestingly enough, that the hanging drop with expansion in the top part to some extent reminds suction cap as its internal pressure in attaching place on a flat horizontal surface turns out to be negative (in relation to external air pressure).

In fig. 9 the envelope curve of drop in the form of suction cap and decrease of internal pressure with increasing height (on the right) are shown. Pressure is measured in Pa, coordinates - in real sizes (m).

Negative pressure inside of a drop comes nearer to - 20 Pa and actually makes possible its attaching to a horizontal surface. It is effect of suction cap which sometimes is taken as increased adhesion of liquids in a drop state.
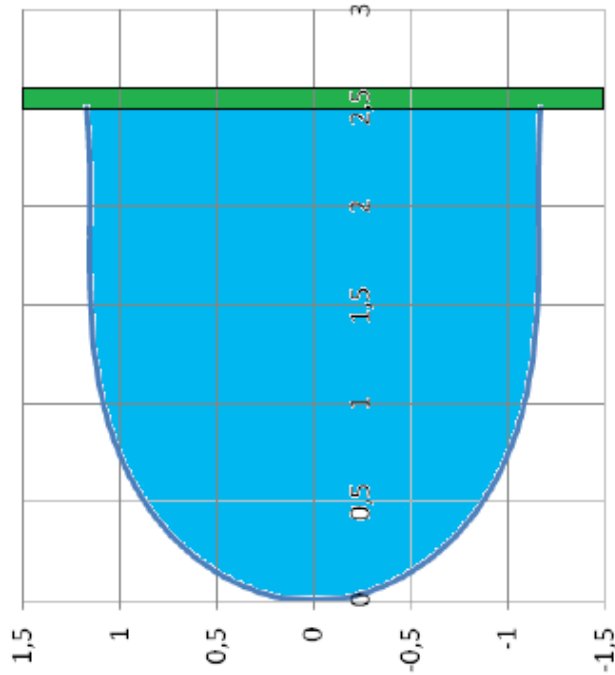
Fig. 7. Intermediate form of a hanging drop (*R=2,1mm*, Φ≈0,6).
Calculation was carried out in relative coordinates.
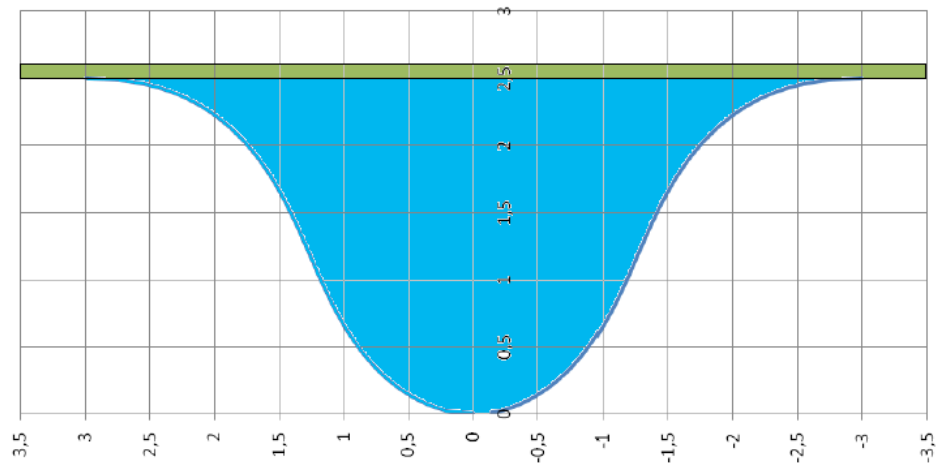

Fig. 8. Hanging drop with expansion in the top part (*R=2,85mm*, Φ≈1,093).
Calculation was carried out in relative coordinates.

As of sitting drops, the increase in radius of curvature *R* at drop vertex (increase in parameter Φ) results in increasing of deviation of drop form from the spherical form (fig. 10).

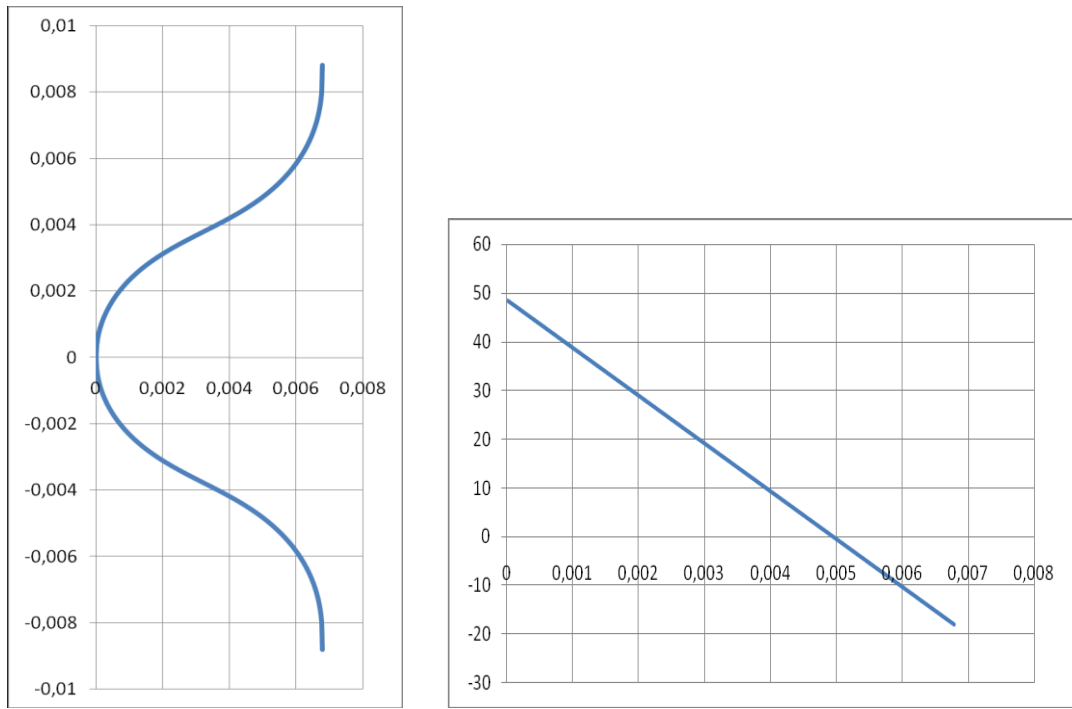At parameter Φ exceeding 10, sitting drop even greater reminds the form of a disk (fig. 11).

Fig. 9. Envelope curve of a drop in real coordinates (on the left) and pressure inside of a drop as a function of height ($R=3mm$, $\Phi\approx1,21$).

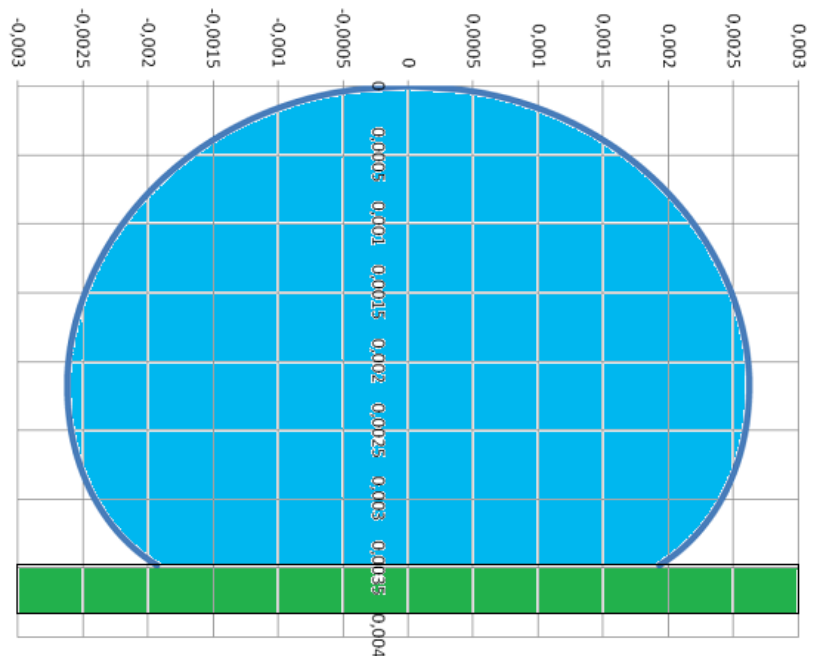

Fig. 10. Sitting drop of water ($R=3mm$, $\Phi\approx1,21$).

Calculation was carried out in real coordinates.

As well as in case of hanging drops, we give general solution of a problem of a sitting drop form.
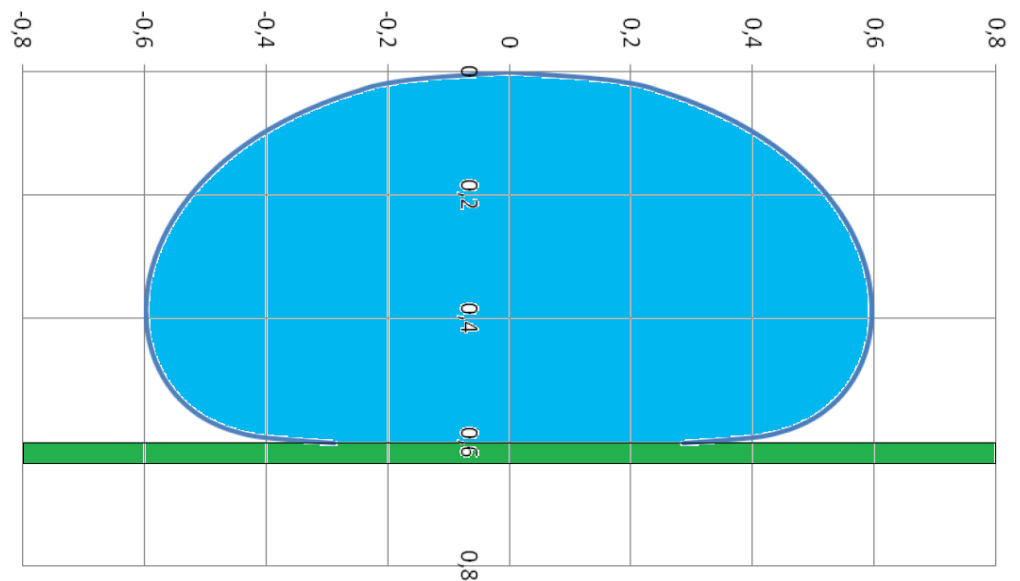

Fig. 11. Sitting drop of water reminding form of a disk (*R=8,8mm*, Φ≈10,42). Calculation was carried out in relative coordinates.
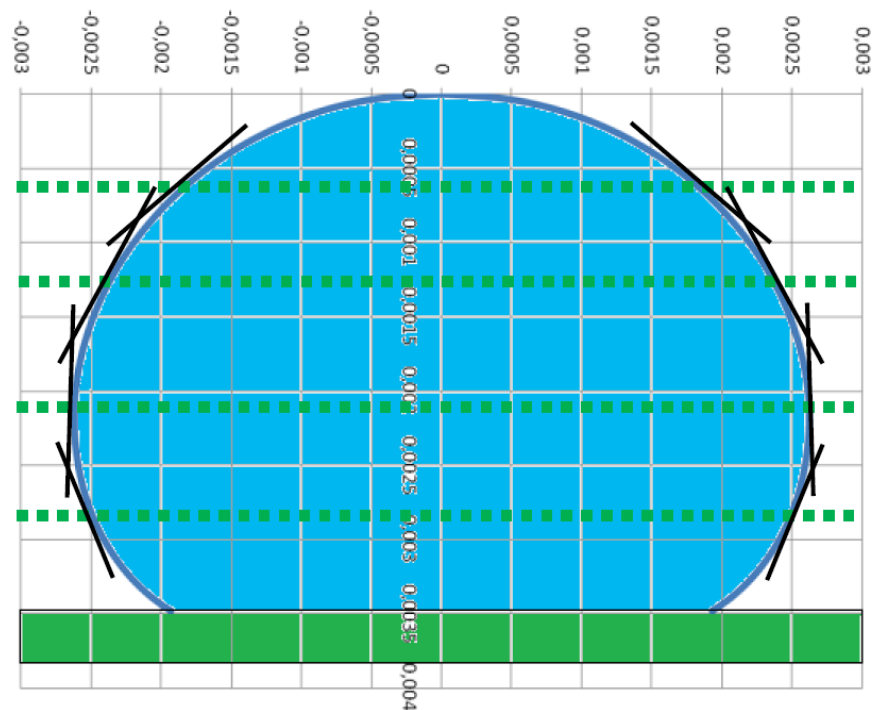

Fig. 12. General solution can be used to get variants with various wetting angles.

Depending on concrete conditions (wetting angle (26)) it is necessary to determine a horizontal secant plane to be imposed on the general solution to meet these conditions. In this section the necessary condition of mechanical equilibrium of a drop (fig. 12) will be routinely provided.

Let's notice, that for simplification of writing we used in the text and in formulas the contracted notation of a surface tension force of a liquid σ though, strictly speaking, it would be necessary to use more exact notation - σ_{LG} as it was done in the formula (26).

In calculations, as it was already mentioned, everywhere water was used as a liquid, but it does not reduce a generality of results, because the form of a drop depends on unique parameter - Bond number.

3. Conclusion

The problem of liquid drop form in gravitational field, undoubtedly, is very interesting both from the point of view of physics of liquids, and from the viewpoint of used methods of analysis.

Traditionally these methods are based on Laplace equation (capillary pressure law), or on the solution of a variational problem of search of total energy minimum of a drop (surface energy plus potential energy of liquid in gravitational field).

In the given paper the drop form equation was deduced on the basis of balance of forces acting in horizontal section, conditionally dissecting a liquid drop at some height. Advantage of such approach is not only simplicity and clearness of solution of the problem, but also the universality of results which do not depend on particular boundary conditions.

The opportunity to abstract from concrete boundary conditions enables to get adequate representation of the phenomenon and to discover on this basis regular dependences.

The form of a drop in relative coordinates is determined by only one (and too dimensionless) parameter - Bond number.

Thus, envelope lines of drops (solutions of equation of drop form) actually are *special mathematical functions* β_Φ *(α)*, having at least three versions. One version meets the form of sitting drop and two more versions of functions describe different forms of hanging drop.

As we saw, when Bond number exceeds Φ≈0,6 hanging drops essentially change their form. The usual, close to spherical or extended drop form turns into other version of a drop which has expansion in the top part and in shape reminds suction cap.

However this is not only similarity of appearance. It turned out, that internal pressure in the top part of such drop is negative in relation to external air pressure. Therefore this kind of hanging drop attaches to a horizontal surface as suction cap - due to a difference of external and internal pressure.

Calculation and construction of graphs of hanging and sitting drops of a liquid was accomplished on the basis of the introduced method of analysis of equilibrium conditions of a liquid drop with use of the most widespread and accessible computer program *Excel*.

Naturally, stated approach is not universal or replacing other methods of analysis, but, undoubtedly, it can appear very convenient and useful addition to already known methods of studying of liquids properties in a drop state.

References

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