On the computability of the cosmic dark matter density from first principles

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Abstract

The tetrahedral tessellation of space is considered for the 3-sphere. It is found that there is a dark matter density $\Omega_{DM} = 0.284$ associated with the curvature of the 3-sphere.

1 Curvature and dark matter density

In this essay, the Universe's large spatial dimensions are modelled as a 3-sphere tessellated into tetrahedra. If there is difficulty envisioning a 3-sphere tessellation made out of tetrahedra, then please see Figure 1 for a 2-sphere tessellation made out of triangles. The 3-sphere tessellations will rely on pseudorandomly placed vertices. We will not be compensating for the variation in tetrahedron extent (e.g. do nothing special even where there are sliver tetrahedra). In effect, the calculation of the curvature will be as simple as possible. The vertex count is N.

For example, where P_{TC} is the ith tetrahedron centre, P_{FC} is the first face's centre, and P_{NTC} is the first neighbouring tetrahedron's centre:

$$\hat{n_1} = \text{normalize}(P_{\text{TC}} - P_{\text{FC}}), \tag{1}$$

$$\hat{o_1} = \text{normalize}(P_{\text{FC}} - P_{\text{NTC}}), \tag{2}$$

and likewise for the other 3 neighbouring tetrahedra, the average dot product d_i is:

$$d_i = \frac{\hat{n_1} \cdot \hat{o_1} + \hat{n_2} \cdot \hat{o_2} + \hat{n_3} \cdot \hat{o_3} + \hat{n_4} \cdot \hat{o_4}}{4}. \tag{3}$$

Because we assume that there are 4 neighbours per tetrahedron, the tetrahedral mesh must be *closed* (precisely 2 tetrahedra per face).

Next, we normalize the curvature:

$$k_i = \frac{1 - d_i}{2}. \tag{4}$$

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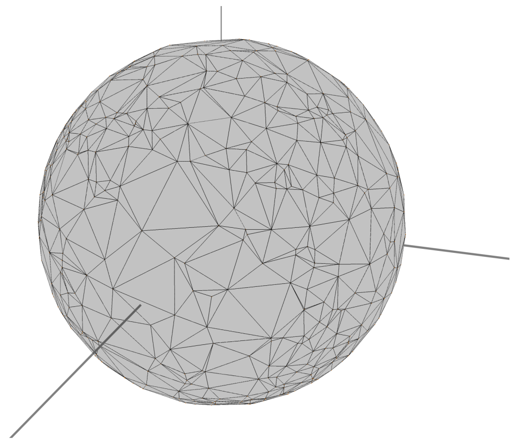

Figure 1: Tessellated 2-sphere, where there are $N = 1,000$ pseudorandomly placed vertices. The 2D triangles exist in 3D space. Accordingly, for a tessellated 3-sphere, the 3D tetrahedra exist in 4D space.

N	K
1,000	0.29473
10,000	0.28821
100,000	0.28413
1,000,000	0.28452

Table 1: Curvature K, where vertex count N is variable. The value of K does not simply increase for $N \geq 1,000,000$, but rather oscillates around 0.284.

Once k_i has been calculated for all tetrahedra, we can then calculate the average normalized measure of curvature K, where T is the number of tetrahedra in the mesh:

$$K = \frac{1}{T}\sum_{i=1}^{T} k_i = \frac{k_1 + k_2 + ... + k_T}{T}. \tag{5}$$

Unlike with a tessellated 2-sphere, it is found that for a tessellated 3-sphere the curvature K does *not* vanish when the tessellation is made up of finer and finer tetrahedra. This curvature exists because the tetrahedral tessellation must consist of irregular tetrahedra. This curvature settles around

$$\lim_{N \to \infty} K(N) = 0.284. \tag{6}$$

This is in line with the dark matter density measure Ω_{DM} used in the various Cold Dark Matter cosmologies. If this is not merely a coincidence, then this is direct evidence of the discrete nature of space, based on a few simple, geometric, first principles.

See Table 1 for curvature K, where the vertex count N is variable.

A C++ code for generating the tessellated 3-sphere can be found at:

`https://github.com/sjhalayka/4d_universe`