Refutation of propositional forgetting theory

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Abstract: We evaluate propositional forgetting theory which is not tautologous. Hence, its existing application to computation tree logic (CTL) as tautologous refutes the claim of inconsistency, to deny CTL as well. Supplementary artifacts in defective proofs are also denied, such as: Representing theorem; Homogeneity proposition; Model checking in forgetting proposition; Entailment on forgetting theorem; and Dual proposition. These results form a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Abstract Computation Tree Logic (CTL) is one of the central formalisms in formal verification. As a specification language, it is used to express a property that the system at hand is expected to satisfy. From both the verification and the system design points of view, some information content of such property might become irrelevant for the system due to various reasons e.g., it might become obsolete by time, or perhaps infeasible due to practical difficulties. Then, the problem arises on how to subtract such piece of information without altering the relevant system behaviour or violating the existing specifications. Moreover, in such a scenario, two crucial notions are informative: the strongest necessary condition (SNC) and the weakest sufficient condition (WSC) of a given property. To address such a scenario in a principled way, we introduce a forgetting-based approach in CTL and show that it can be used to compute SNC and WSC of a property under a given model. We study its theoretical properties and also show that our notion of forgetting satisfies existing essential postulates. Furthermore, we analyse the computational complexity of basic tasks, including various results for the relevant fragment CTL_AF.

1 Introduction ... Although forgetting has been extensively investigated from various aspects of different logical systems, the existing forgetting techniques are not directly applicable in CTL. For instance, in propositional forgetting theory, forgetting atom q from φ is equivalent to a formula

\[ \phi[q/T] \lor \phi[q/\perp], \text{where } \phi[q/X] \text{ is a formula obtained from } \phi \text{ by replacing each } q \text{ with } X \]

\[ (X \in \{T, \perp\}). \]  \hspace{1cm} (1.1.2)

Let \( p, q, r, s: p, q, AG, \phi \text{ or } \psi. \)

\[ (s&(q'(s=s)))+ (s&(q'(s@s))) ; \] 

\[ FFFF FFFF TTTT TTTT \]  \hspace{1cm} (1.1.2)
**Remark 1.1.2:** Eq. 1.1.2 as rendered is *not* tautologous. This refutes the conjectured claim of propositional forgetting theory. The application of forgetting theory to computation tree logic (CTL) is also denied.

This method cannot be extended to a CTL formula. Consider a CTL formula

\[ \psi = AGp \land \neg AGq \land \neg AG\neg q. \]

If we want to forget atom q from \( \psi \) by using the above method, we would have

\[ \psi[q/\top] \lor \psi[q/\bot] = \bot. \]

This is obviously not correct since after forgetting q this specification should not become inconsistent. (1.2.1)

\[
(s=(((r\land p)\land (\neg r \land q)) \land (\neg r \land \neg q))) \land (s((q(s=s)))+(s((q(s=s))))=(s=s)) ;
\begin{array}{cccc}
TTTT & TTTT & TTTT & TTTT
\end{array}
\]

(1.2.2)

**Remark 1.2.2:** Eq. 1.2.2 is *not* contradictory as claimed, that is, can be extended to a CTL formula to produce a theorem. That after forgetting q the specification becomes consistent is due to Eq. 1.1.2 where propositional forgetting theory is denied. This serves also to deny CTL.

From the paper's footnote link for Supplementary material: proof appendix, the following five artifacts are also denied: Representing theorem; Homogeneity proposition; Model checking in forgetting proposition; Entailment on forgetting theorem; and Dual proposition.