Excircles of a triangle with two parallel sides

Hiroshi Okumura  
Maebashi Gunma 371-0123, Japan  
e-mail: hokmr@yandex.com

Abstract. For a triangle $ABC$, the radius of the excircle touching $CA$ from the side opposite to $B$ equals 0 if $BC$ and $CA$ are parallel.

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Let us consider the excircle of a triangle $ABC$ touching $CA$ from the side opposite to $B$ (see Figure 1). We show that the radius of this circle equal 0 if $BC$ and $CA$ are parallel.

![Figure 1](image)

We use a rectangular coordinate system such that $A$ and $B$ have coordinates $(a, 0)$ and $(b, 0)$, respectively, where we assume that the point $C$ lies on the region $y > 0$. Let $\theta_a$ (resp. $\theta_b$) be the angles between $\overrightarrow{BA}$ and $\overrightarrow{AC}$ (resp. $\overrightarrow{BC}$). Then the center of the excircle coincides with the point of intersection of the two lines expressed by $y = \tan \frac{\theta_a}{2}(x - a)$ and $y = \tan \frac{\theta_b}{2}(x - b)$. The coordinates of the point are given by:

$$
\left( a \tan \frac{\theta_a}{2} - b \tan \frac{\theta_b}{2}, \frac{(a - b) \sin \frac{\theta_a}{2} \sin \frac{\theta_b}{2}}{\sin \frac{\theta_a - \theta_b}{2}} \right),
$$

where notice that the $y$-coordinate gives the radius of the excircle. Now we fix the points $A$ and $B$ and consider the case $\theta_a = \theta_b$ (see Figure 2). Then by the definition of division by zero, $a/0 = 0$ for any number $a$ [1], we see that the center of the excircle has coordinates $(0, 0)$. Therefore the exradius equals 0.
Remark. The essential part of this note is the fact that the point of intersection of two parallel lines coincides with the origin. The fact is already pointed out in [2].

REFERENCES