Beal Conjecture Proved in a Page Margin

"5% of the people think; 10% of the people think that they think; and the other 85%would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

In a page margin, the author proves directly the original Beal conjecture if $A^x + B^y = C^z$, where A,B,C,x,y,z are positive integers and $x,y,z > 2$, then A, B and C have a common prime factor. The principles applied in the proof are based on the properties of the factored Beal equation. The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation.

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Preliminaries

Introduction

The following is from the first page of the author's high school practical physics note book: Science is the systematic observation of what happens in nature and the building up of body of laws and theories to describe the natural world. Scientific knowledge is being extended and applied to everyday life. The basis of this growing knowledge is experimental work. To prove Beal conjecture, one will be guided by the properties of the factored Beal equation.

Observation 1: $\boxed{2^3 + 2^3 = 2^4}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$2^3 + 2^3 = 2^4$$

$$2^3 + 2^3 = 2^3 \bullet 2$$

$$\underbrace{2^3}_{k}(\underbrace{1+1}_{L}) = \underbrace{2^3}_{M} \bullet \underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side..

Note above that the greatest common power of the prime factors on the left of the equation is the same as a power of the prime factor on the right side of the equation.

Note also, the following

The ratio $\frac{K}{M} = \frac{2^3}{2^3} = 1$.

If $\frac{K}{M} = 1$, then $K = M$

Similarly, $\frac{P}{L} = \frac{2}{1+1} = 1$.

If $\frac{P}{L} = 1$, then $P = L$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^y = F^z t^z$$

$$r = s = t = 2$$

$$x = 3 .y = 3, z = 4$$

$$(D = 1, E = 1, F = 1)$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$

$$K = M,\ \ P = L$$

Observation 2: $\boxed{7^6 + 7^7 = 98^3}$
Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$7^6 + 7^7 = 98^3$$
$$7^6 + 7^6 \bullet 7 = (49 \bullet 2)^3$$
$$7^6 + 7^6 \bullet 7 = 7^6 \bullet 2^3$$
$$7^6(1+7) = 7^6 \bullet 2^3$$
$$\underbrace{7^6}_{K}(\underbrace{1+7}_{L}) = \underbrace{7^6}_{M} \bullet \underbrace{2^3}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\dfrac{K}{M} = \dfrac{7^6}{7^6} = 1$.

Similarly, $\dfrac{P}{L} = \dfrac{2^3}{1+7} = 1$.

Corresponding relationship formula
Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 7$$
$$x = 6, y = 7, z = 3$$
$$(D = 1, E = 1, F = 14)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x} F^z}_{P}$$
$$K = M, \; P = L$$

Observation 3: $\boxed{3^3 + 6^3 = 3^5}$
Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$3^3 + 6^3 = 3^5$$
$$3^3 + (3 \bullet 2)^3 = 3^5$$
$$3^3 + 3^3 \bullet 2^3 = 3^5$$
$$3^3(1 + 2^3) = 3^3 \bullet 3^2$$
$$\underbrace{3^3}_{K}(\underbrace{1+8}_{L}) = \underbrace{3^3}_{M} \bullet \underbrace{3^2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\dfrac{K}{M} = \dfrac{3^3}{3^3} = 1$.

Similarly, $\dfrac{P}{L} = \dfrac{3^2}{1+8} = 1$.

Corresponding relationship formula
Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 3$$
$$x = 3, y = 3, z = 5$$
$$(D = 1, E = 2, F = 1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x} F^z}_{P}$$
$$K = M, \; P = L$$

Observation 4: $\boxed{2^9 + 8^3 = 4^5}$

Identify the greatest common factor.of all three terms of the equation and factor it out on the left side.

$$2^9 + 8^3 = 4^5$$
$$2^9 + ([2^3])^3 = ([2^2])^5$$
$$2^9 + 2^9 = 2^{10}$$
$$2^9(1+1) = 2^9 \bullet 2$$
$$\underbrace{2^9}_{k}(\underbrace{1+1}_{L}) = \underbrace{2^9}_{M} \bullet \underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{2^9}{2^9} = 1$.

Similarly, $\frac{P}{L} = \frac{2}{1+1} = 1$.

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 2$$
$$x = 9, y = 3, z = 5$$
$$(D = 1, E = 4, F = 2)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \ P = L$$

Observation 5: $\boxed{34^5 + 51^4 = 85^4}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$34^5 + 51^4 = 85^4$$
$$(17 \bullet 2)^5 + (17 \bullet 3)^4 = (17 \bullet 5)^4$$
$$17^5 \bullet 2^5 + 17^4 \bullet 3^4 = 17^4 \bullet 5^4$$
$$17^4(17 \bullet 2^5 + 3^4) = 17^4 \bullet 5^4$$
$$\underbrace{17^4}_{k}(\underbrace{17 \bullet 2^5 + 3^4}_{L}) = \underbrace{17^4}_{M} \bullet \underbrace{5^4}_{P}$$

(Note: $17 \bullet 2^5 + 3^4 = 17 \bullet 32 + 81 = 625$; $5^4 = 625$)

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{17^4}{17^4} = 1$.

Similarly, $\frac{P}{L} = \frac{5^4}{17 \bullet 2^5 + 3^4} = 1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 17$$
$$x = 5, y = 4, z = 4$$
$$(D = 2, E = 3, F = 5)$$
$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$$
$$K = M, \ P = L$$

Note above that one factored out s^y .

One will apply the switch from r^x to s^y in the conjecture proof.

Observation 6: $\boxed{3^9+54^3=3^{11}}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$3^9+54^3=3^{11}$$
$$3^9+(3^3\bullet 2)^3=3^{11}$$
$$3^9+3^9\bullet 2^3=3^{11}$$
$$3^9(1+2^3)=3^9\bullet 3^2$$
$$\underbrace{3^9}_{k}(\underbrace{1+2^3}_{L})=\underbrace{3^9}_{M}\bullet\underbrace{3^2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side

Note the following

The ratio $\dfrac{K}{M}=\dfrac{3^9}{3^9}=1$.

Similarly, $\dfrac{P}{L}=\dfrac{3^2}{1+2^3}=1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A=Dr$ **,** $B=Es$ **,** $C=Ft$ **.**

$$(Dr)^x+(Es)^y=(Ft)^z$$
$$D^xr^x+E^ys^y=F^zt^z$$
$$r=s=t=3$$
$$x=9, y=3, z=11$$
$$(D=1, E=18, F=1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K=M,\ P=L$$

Observation 7: $\boxed{33^5+66^5=33^6}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$33^5+66^5=33^6$$
$$(11\bullet 3)^5+(11\bullet 2\bullet 3)^5=(11\bullet 3)^6$$
$$11^5\bullet 3^5+11^5\bullet 2^5\bullet 3^5=11^6\bullet 3^6$$
$$11^5(3^5+2^5\bullet 3^5)=11^5\bullet 11\bullet 3^6$$
$$\underbrace{11^5}_{k}(\underbrace{3^5+2^5\bullet 3^5}_{L})=\underbrace{11^5}_{M}\bullet\underbrace{11\bullet 3^6}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\dfrac{K}{M}=\dfrac{11^5}{11^5}=1$.

Similarly, $\dfrac{P}{L}=\dfrac{11\bullet 3^6}{3^5+2^5\bullet 3^5}=1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A=Dr$ **,** $B=Es$ **,** $C=Ft$ **.**

$$(Dr)^x+(Es)^y=(Ft)^z$$
$$D^xr^x+E^ys^y=F^zt^z$$
$$r=s=t=11$$
$$x=5, y=5, z=6$$
$$D=3, E=6, F=3$$
$$\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K=M,\ P=L$$

Surprise: The above properties will apply to $\boxed{6^2+8^2=10^2}$

$$6^2+8^2=10^2$$
$$(2\bullet 3)^2+(2^3)^2=(2\bullet 5)^2$$
$$2^2\bullet 3^2+2^6=2^2\bullet 5^2$$
$$\underbrace{2^2}_{K}(\underbrace{3^2+2^4}_{L})=\underbrace{2^2}_{M}\bullet\underbrace{5^2}_{P}\ ;K=M,\ L=P$$

Summary of Observations 1-7

The most important and useful observation in the above examples is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation. This observation will be useful in proving Beal conjecture.

1. $2^3+2^3=2^4$ $\underbrace{2^3}_{K}(\underbrace{1+1}_{L})=\underbrace{2^3}_{M}\bullet\underbrace{2}_{P}$	**2.** $3^3+6^3=3^5$ $\underbrace{3^3}_{K}(\underbrace{1+8}_{L})=\underbrace{3^3}_{M}\bullet\underbrace{3^2}_{P}$	**3.** $7^6+7^7=98^3$ $\underbrace{7^6}_{K}(\underbrace{1+7}_{L})=\underbrace{7^6}_{M}\bullet\underbrace{2^3}_{P}$
4. $2^9+8^3=4^5$ $\underbrace{2^9}_{K}(\underbrace{1+1}_{L})=\underbrace{2^9}_{M}\bullet\underbrace{2}_{P}$	**5.** $34^5+51^4=85^4$ $\underbrace{17^4}_{K}(\underbrace{17\bullet 2^5+3^4}_{L})=\underbrace{17^4}_{M}\bullet\underbrace{5^4}_{P}$	**6.** $3^9+54^3=3^{11}$ $\underbrace{3^9}_{K}(\underbrace{1+2^3}_{L})=\underbrace{3^9}_{M}\bullet\underbrace{3^2}_{P}$
7. $33^5+66^5=33^6$ $\underbrace{11^5}_{K}(\underbrace{3^5+2^5\bullet 3^5}_{L})=\underbrace{11^5}_{M}\bullet\underbrace{11\bullet 3^6}_{P}$	**Corresponding relationship formulas** $\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^{z}}_{P}$ $K=M,\ P=L$ **or** $\underbrace{s^y}_{K}[\underbrace{E^y+D^xr^x\bullet s^{-y}}_{L}]=\underbrace{t^y}_{M}\underbrace{t^{z-y}F^{z}}_{P}; K=M,\ P=L$	

Properties of the Factored Beal Equation

Let r, s and t be prime factors of A, B and C .respectively, such that $A=Dr$ **,** $B=Es$ **,** $C=Ft$ **.** where D, E and F are positive integers; and the equation becomes $(Dr)^x+(Es)^y=(Ft)^z$.

Step 1: Factor out r^x on the left side of the equation and on the right side of the equation, replace t^z by $t^x\bullet t^{z-x}$ (Note $t^x\bullet t^{z-x}=t^z$)

$$(Dr)^x+(Es)^y=(Ft)^z$$

$$D^xr^x+E^ys^y=F^zt^z$$

$$\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P};\ K=M,\ P=L$$

For the factorization $\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$ with respect to r^x, $r^x=t^x$ $(K=M)$

Step 2: Factor out s^y on the left side of the equation and on the right side of the equation, replace t^z by $t^y\bullet t^{z-y}$ (Note $t^y\bullet t^{z-y}=t^z$)

$$(Es)^y+(Dr)^x+=(Ft)^z$$

$$E^ys^y+D^xr^x=F^zt^z$$

$$\underbrace{s^y}_{K}[\underbrace{E^y+D^xr^x\bullet s^{-y}}_{L}]=\underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}\ ;K=M,\ P=L$$

For the factorization $\underbrace{s^y}_{K}[\underbrace{E^y+D^xr^x\bullet s^{-y}}_{L}]=\underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$ with respect to s^y, $s^y=t^y$ $(K=M)$

Beal Conjecture Proved in a Page Margin

Given: $A^x + B^y = C^z$, A, B, C, x, y, z are positive integers and $x, y, z > 2$.

Required: To prove that A, B and C have a common prime factor.

Plan: Let r , s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.** The proof would be complete after showing that $r = s = t$

Proof

Step 1: Factor out r^x

$(Dr)^x + (Es)^y = (Ft)^z$

$D^x r^x + E^y s^y = F^z t^z$

$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] =$

$\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P} K = M,\ P =\ L$

(Properties of factored Beal equation)

From above, $r^x = t^x$;

If $r^x = t^x$, then $r = t$.

$(\log r^x = \log t^x; x\log r = x\log t; \log r = \log t; r = t)$

Step 2: Factor out s^y

$(Es)^y + (Dr)^x + = (Ft)^z$

$E^y s^y + D^x r^x = F^z t^z$

$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] =$

$\underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P};\ K = M,\ \ P =\ L$

(Properties of factored Beal equation)

From above, $s^y = t^y$;

If $s^y = t^y$,then $s = t$.

$(\log s^y = \log t^y; y\log s = y\log t;\ \log s = \log t;\ s = t)$

It has been shown in Step 1 that $r = t$,and in Step 2 that, $s = t$; therefore,$r = s = t$.

Since $A = Dr$ **,** $B = Es$ **,** $C = Ft$ and $r = s = t$, A, B and C have a common prime factor, QED.

Conclusion

The original Beal conjecture has been proved in a page margin. The principles applied in the proof are based on the analytic observations of the factorization of numerical Beal equations. Since the main concern of this conjecture is a common prime factor, it was appropriate that factorization was the main tool in observing the structure of the factorization of the equations. The factorization of the equation revealed the relationships between the prime factors involved.

PS: Other proofs of Beal Conjecture by the author are at viXra:2001.0694; ;viXra:1702.0331; viXra:1609.0383; viXra:1609.0157;

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