Yukawa Potential and Extended Klein-Gordon Equation in Rindler Space-Time

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ABSTRACT
Yukawa potential satisfy Proka equation or Klein-Gordon equation. If we represent Yukawa potential in Rindler space-time, this Yukawa potential satisfy the extended Klein-Gordon equation in Rindler space-time.

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1. Introduction

Atom’s nucleus force understand by Yukawa potential. We study Yukawa potential in Rindler Space-time.

At first, Yukawa potential \( V \) describes nucleus’s combine force in semi-classical method.

\[
V = -\frac{g^2}{r} \exp\left(\frac{-m_r c}{\hbar}\right)
\]

\( g \) is real number, \( m_r \) is the meson’s mass

(1)

Klein-Gordon equation is satisfied by Yukawa potential \( V \).

\[-\partial^2 V + \frac{m^2 c^2}{\hbar^2} V = -\nabla^2 V + \frac{m^2 c^2}{\hbar^2} V = 0
\]

\[
V = -\frac{g^2}{r} \exp\left(\frac{-m_r c}{\hbar}\right), i = 1, 2, 3
\]

(2)

2. Yukawa potential from Extended Klein-Gordon Equation in Rindler-Space-Time

Rindler coordinates are

\[ct = \left(\frac{c^2}{a_0} + \xi^1\right) \sinh\left(\frac{a_0}{c} \xi^0\right), \quad x = \left(\frac{c^2}{a_0} + \xi^1\right) \cosh\left(\frac{a_0}{c} \xi^0\right) - \frac{c^2}{a_0};\]

\[y = \xi^2, \quad z = \xi^3\]

(3)

If we write Yukawa potential \( V \) in inertial frame,

\[
V = \frac{g^2}{r} \exp\left(\frac{-m_r c}{\hbar}\right)
\]

(4)

If we rewrite Yukawa potential \( V_\xi \) in Rindler space-time,

\[
V_\xi = -\frac{g^2}{\sqrt{x^2 + y^2 + z^2}} \exp\left(\frac{m_r c}{\hbar} \sqrt{x^2 + y^2 + z^2}\right)
\]

\[
= -\frac{g^2}{\sqrt{c^4 + 2c^2 \xi^0 \cosh\left(\frac{a_0}{c} \xi^0\right) - \xi^2}} \exp\left[-\frac{m_r c}{\hbar} \sqrt{\left(\frac{c}{a_0} \xi^0\right) \cosh\left(\frac{a_0}{c} \xi^0\right) - \xi^2 + (\xi^2)^2 + (\xi^3)^2}\right]
\]

(5)

This Yukawa potential satisfy the extended Klein-Gordon equation. At first, energy and momentum are in Rindler space-time,

\[
E_\xi = i\hbar \frac{1}{1 + \frac{a_0}{c^2} \xi^1} \cosh\left(\frac{a_0}{c} \xi^0\right) \frac{\partial}{\partial \xi^0}, \quad \vec{p}_\xi = i\hbar \vec{\nabla}_\xi
\]

(6)
Energy-Momentum equation is in Rindler space-time\[1\],

\[
E_\xi^2 = p_\xi \cdot p_\xi + m^2 \ i
\]  

(7)

Hence, normal Klein-Gordon equation is in Rindler-spacetime,

\[
\frac{m^2 c^2 V_\xi}{\hbar^2} + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^2)^2} \left( \frac{\partial^2}{\partial \xi^2} \right) V_\xi - \nabla_\xi^2 V_\xi = \frac{\partial^2}{\partial \xi^2} V_\xi - \nabla_\xi^2 V_\xi = 0
\]  

(8)

In this time, we focus the gauge \( \Lambda \) equation in Rindler space-time\[1\],

\[
\frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^2)^2} \left( \frac{\partial^2}{\partial \xi^2} \right) \Lambda - \nabla_\xi^2 \Lambda - \frac{\partial V_\xi}{\partial \xi^2} \frac{a_0}{c^2} \Lambda \left( 1 + \frac{a_0}{c^2} \xi^2 \right) = 0
\]  

(9)

Hence, Eq(8) change extended Klein-Gordon equation in Rindler space-time.

Extended Klein-Gordon Equation is in Rindler space-time,

\[
\frac{m^2 c^2 V_\xi}{\hbar^2} + \frac{1}{c^2} \frac{1}{(1 + \frac{a_0}{c^2} \xi^2)^2} \left( \frac{\partial^2}{\partial \xi^2} \right) V_\xi - \nabla_\xi^2 V_\xi = \frac{\partial V_\xi}{\partial \xi^2} \frac{a_0}{c^2} \left( 1 + \frac{a_0}{c^2} \xi^2 \right) = 0
\]  

(10)

Eq(5), Yukawa potential \( V_\xi \) satisfy Eq(10), extended Klein-Gordon equation in Rindler space-time.

3. Conclusion
We found Yukawa potential mechanism in Rindler Space-time.

References