Yukawa Potential and Extended Klein-Gordon Equation in Rindler Space-Time

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ABSTRACT

Yukawa potential satisfy Proka equation or Klein-Gordon equation. If we represent Yukawa potential in Rindler space-time, this Yukawa potential satisfy the extended Klein-Gordon equation in Rindler space-time.

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1. Introduction

Atom's nucleus force understand by Yukawa potential. We study Yukawa potential in Rindler Space-time time.

At first, Yukawa potential V describes nucleus's combine force in semi-classical method.

$$V = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar})$$

g is real number, m_π is the meson's mass (1)

Klein-Gordon equation is satisfied by Yukawa potential V.

$$-\partial_i \partial^i V + \frac{m^2 c^2}{\hbar^2} V = -\nabla^2 V + \frac{m_\pi{}^2 c^2}{\hbar^2} V = 0$$

$$V = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar}), i = 1,2,3 \tag{2}$$

2. Yukawa potential from Extended Klein-Gordon Equation in Rindler-Space-Time

Rindler coordinates are

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0}{c}\xi^0) \ , \ x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3 \tag{3}$$

If we write Yukawa potential V in inertial frame,

$$V = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar}) \tag{4}$$

If we rewrite Yukawa potential V_ξ in Rindler space-time,

$$V_\xi = -\frac{g^2}{\sqrt{x^2 + y^2 + z^2}}\exp(-\frac{m_\pi c}{\hbar}\sqrt{x^2 + y^2 + z^2})$$

$$= -\frac{g^2}{\sqrt{\{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}\}^2 + (\xi^2)^2 + (\xi^3)^2}}\exp[-\frac{m_\pi c}{\hbar}\sqrt{\{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}\}^2 + (\xi^2)^2 + (\xi^3)^2}\,] \tag{5}$$

This Yukawa potential satisfy the extended Klein-Gordon equation. At first, energy and momentum are in Rindler space-time,

$$E_\xi = i\hbar\frac{1}{(1 + \frac{a_0}{c^2}\xi^1)}\frac{1}{c}\frac{\partial}{\partial \xi^0}, \vec{p}_\xi = i\hbar\vec{\nabla}_\xi \tag{6}$$

Energy-Momentum equation is in Rindler space-time[1],

$$E_{\xi}{}^{2} = \vec{p}_{\xi} \cdot \vec{p}_{\xi} + m^{2} \qquad (7)$$

Hence, normal Klein-Gordon equation is in Rindler-spacetime,

$$\frac{m^2 c^2}{\hbar^2} V_{\xi} + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial \xi^0)^2} V_{\xi} - \nabla_{\xi}^2 V_{\xi}$$

$$= \frac{m_{\pi}{}^2 c^2}{\hbar^2} V_{\xi} + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial \xi^0)^2} V_{\xi} - \nabla_{\xi}^2 V_{\xi} = 0 \qquad (8)$$

In this time, we focus the gauge Λ equation in Rindler space-time[1],

$$\frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial \xi^0)^2} \Lambda - \nabla_{\xi}{}^2 \Lambda - \frac{\partial \Lambda}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} = 0 \qquad (9)$$

Hence, Eq(8) change extended Klein-Gordon equation in Rindler space-time.

Extended Klein-Gordon Equation is in Rindler space-time,

$$\frac{m^2 c^2}{\hbar^2} V_{\xi} + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial \xi^0)^2} V_{\xi} - \nabla_{\xi}{}^2 V_{\xi} - \frac{\partial V_{\xi}}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)}$$

$$= \frac{m_{\pi}{}^2 c^2}{\hbar^2} V_{\xi} + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial \xi^0)^2} V_{\xi} - \nabla_{\xi}{}^2 V_{\xi} - \frac{\partial V_{\xi}}{\partial \xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} = 0 \qquad (10)$$

Eq(5) ,Yukawa potential V_{ξ} satisfy Eq(10), extended Klein-Gordon equation in Rindler space-time.

3. Conclusion

We found Yukawa potential mechanism in Rindler Space-time..

References

[1]S.Yi, “Electromegnetic Wave Function and Equation, Lorentz Force in Rindler Space-time”, International Journal of Advanced Research in Physical Science,**5**,9(2018)

[1]J.M. Normand, A Lie group: Rotations in Quantum Mechanics(North-Holland Pub. Co., 1980)

[2]J.D. Bjorken & S. D. Drell, Relativistic Quantum Field(McGraw- Hill Co., 1965)

[3]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[4]R.L.Liboff, Quantum Mechanics(Addison-Wesley Publishing Co., Inc.,1990)

[5]A.Beiser, Concept of Modern Physics(McGraw-Hill,Inc.,1991)