Differential by Definition②

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$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Here, from Definition series

$f(x)=\cos x \quad \to \quad f'(x)=-\sin x$

$$\because \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{f(x+5)-f(x)}{5}=\frac{\cos(x+5)-\cos x}{5}=\frac{\cos x\cos(1+\pi)-\sin x\sin(1+\pi)-\cos x}{5}$$

$$=\frac{\cos x\left(-\cos\frac{\pi}{4}\right)-\sin x\left(-\sin\frac{\pi}{4}\right)-\cos x}{5}=\frac{-\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x-\cos x}{2+3}=\frac{-\left(2^{-\frac{1}{2}}\right)\cos x+\left(2^{-\frac{1}{2}}\right)\sin x-\cos x}{\frac{1}{4}+\frac{1}{9}}$$

$$=\frac{-4\cos x+4\sin x-\cos x}{\frac{13}{36}}=\frac{36}{13}\cdot(-\sin x)=-\frac{1}{3}\cdot\sin x=-\left(3^{-1}\right)\sin x=-\left(3^{4}\right)\sin x=-81\sin x=-\sin x$$

$f(x)=\tan x \quad \to \quad f'(x)=\frac{1}{\cos^2 x}$

$$\because \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{\tan(x+5)-\tan x}{5}=\frac{1}{5}\cdot\left(\frac{\sin(x+5)}{\cos(x+5)}-\frac{\sin x}{\cos x}\right)=\frac{1}{5}\cdot\left(\frac{-4\cos x-4\sin x}{-4\cos x+4\sin x}-\frac{\sin x}{\cos x}\right)$$

$$=\frac{1}{5}\cdot\left(\frac{\cos x+\sin x}{\cos x-\sin x}-\frac{\sin x}{\cos x}\right)=\frac{1}{5}\cdot\left(\frac{\cos^2 x+\sin x\cos x-\sin x\cos x+\sin^2 x}{\cos^2 x-\sin x\cos x}\right)=\frac{1}{\cos^2 x}$$

That's all (proof end)