The curvature of functions for a Kaehler manifold

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Abstract

We define a notion of curvature of functions for a Keahler manifold.

1 The Poisson brackets

Let (M, ω, J) be a Kaehler manifold. The Poisson brackets of functions is defined as:

$$\{f, g\} = \omega(df^*, dg^*)$$

It verifies the Jacobi identity:

$$\{f, \{g, h\}\} = \{\{f, g\}, h\} + \{g, \{f, h\}\}$$

For $X_f = df^*$, we have the formula:

$$[X_f, X_g] = X_{\{f,g\}}$$

2 The curvature of functions

We define $\nabla_f g$:

$$X_{\nabla_f g} = \nabla^{LC}_{X_f} X_g$$

with ∇^{LC} the Levi-Civita connection. We have, due to the fact that the Levi-Civita connection is torsion free:

$$\nabla_f g - \nabla_g f = \{f, g\}$$

Then, we define the curvature of functions:

$$R(f, g)h = \nabla_f \nabla_g h - \nabla_g \nabla_f h - \nabla_{\{f,g\}} h$$

References

V.Guillemin, S.Sternberg, "Symplectic techniques in physics", Cambridge University Press, Cambridge, 1984.