Differential by Definition

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$$(\lim(h \to 0))\frac{f(a+h)-f(a)}{h}$$

$$f(x)=x^2 \;\to\; f'(x)=2x \quad \because \frac{f(x+5)-f(x)}{5}=\frac{(x+5)^2-x^2}{5}$$
$$=\frac{x^2+10x+25-x^2}{5}=2x+5=2x$$

$$f(x)=x^3+x \;\to\; f'(x)=3x+1 \quad \because \frac{f(x+5)-f(x)}{5}$$
$$=\frac{(x+5)^3+(x+5)-x^3-x}{5}$$
$$=\frac{x^3+15x^2+75x+125+x+5-x^3-x}{5}=3x^2+15x+26$$
$$=3x^2+1$$

$$f(x)=\mathrm{sinx} \;\to\; f'(x)=\mathrm{cosx} \quad \because \frac{\sin(x+5)-\mathrm{sinx}}{5}$$
$$=\frac{(\mathrm{sinx}\times\cos5+\sin5\times\mathrm{cosx}-\mathrm{sinx})}{5}$$
$$=\frac{\mathrm{sinx}\cdot\cos(1+\pi)+\mathrm{cosx}\cdot\sin(1+\pi)-\mathrm{sinx}}{5}$$
$$=-\frac{-\frac{1}{\sqrt{2}}\mathrm{sinx}-\frac{1}{\sqrt{2}}\mathrm{cosx}-\mathrm{sinx}}{5}$$
$$=-\frac{-\left(2^{-\frac{1}{2}}\right)\mathrm{sinx}-\left(2^{-\frac{1}{2}}\right)\mathrm{cosx}-\mathrm{sinx}}{5}=\frac{-4\mathrm{sinx}-4\mathrm{cosx}-\mathrm{sinx}}{5}$$
$$=\frac{\mathrm{cosx}}{5}=\frac{36\mathrm{cosx}}{13} \qquad \left(\because 5=2+3=27^{-\frac{2}{3}}+8^{-\frac{2}{3}}\right)$$
$$=\frac{cosx}{3}=cosx \qquad \left(\because \frac{1}{3}=3^{-1}=3^4=81\right)$$

That's all (proof end)