

# Derivation of the Muon g-2 Anomaly from Non-Equilibrium Dynamics

Ervin Goldfain

## Abstract

As it is known, the anomalous magnetic moment of muons is a longstanding puzzle of the Standard Model (SM). Commonly referred to as the muon “g-2 problem”, the precise testing of this anomaly provides a sensitive probe for physics beyond SM. Here we show that the leading order contribution to the muon anomaly can be estimated from the onset of non-equilibrium dynamics near the Fermi scale. The derivation is straightforward and evades the postulated existence of new phenomena in the low to the mid TeV range of high-energy physics.

**Key words:** g-2 anomaly, Standard Model, minimal fractal manifold, Fractional Field Theory, non-equilibrium dynamics.

## 1. Introduction

In Quantum Mechanics, a charged particle of mass  $m$  carries a magnetic dipole  $\mu$  linked with the spin vector  $s$  through the following relationship

$$\mu = g \left( \frac{\pm e}{2m} \right) s \quad (1)$$

in which  $g$  is the gyromagnetic ratio and  $e$  the electric charge. Whereas in Quantum Mechanics  $g$  holds a well-defined value ( $g = 2$ ), the cumulative contribution of quantum fluctuations in the SM yields a minute but non-vanishing correction to the gyromagnetic

ratio. This correction is parameterized by the so-called *anomalous magnetic moment*, which is defined as

$$a = \frac{g-2}{2} \quad (2)$$

Another useful parameterization of (2) is the deviation between the experimental and theoretical values, i.e.,

$$\Delta a = \left| a^{\text{exp}} - a^{\text{th}} \right| \quad (3)$$

When applied to the family of SM charged leptons (the electron  $e$ , muon  $\mu$  and the tau-lepton  $\tau$ ), (3) can be measured with various degrees of accuracy and evaluated against all perturbative corrections demanded by the theory. The overall contribution of the SM may be split into three components, namely,

$$a_l^{\text{SM}} = a_l^{\text{QED}} + a_l^{\text{EW}} + a_l^{\text{Had}}, \quad l = \{e, \mu, \tau\} \quad (4)$$

corresponding to the quantum electrodynamics (QED), electroweak (EW) and the hadronic sectors, respectively. The leptonic deviation (3) assumes the form

$$\Delta a_l = \left| a_l^{\text{exp}} - a_l^{\text{SM}} \right| \quad (5)$$

Conventional wisdom holds that (5) can be tracked down to either an experimental error, an incorrect computation of hadronic loop-diagrams or a hint for “*new physics*” phenomena (NP), alleged to surface beyond the energy range of the SM. While the electron anomalous moment  $\Delta a_e$  is relatively insensitive to the EW and strong sectors of

the SM, the muon anomalous moment  $\Delta a_\mu$  contains contributions from all SM sectors. As a result, precise tests of  $\Delta a_\mu$  are believed to provide an excellent opportunity to reveal or constrain the signature of NP, including, for example, supersymmetry, leptoquarks, extended gauge groups, seesaw models, extra dimensions, extended Higgs sectors and other field theories beyond SM (referred below to as BSM scenarios) [1-3].

The most recent experimental values on the magnitude of  $\Delta a_e$  and  $\Delta a_\mu$  are [1, 7]

$$\Delta a_e^{\text{exp}} = (-0.88(0.36)) \times 10^{-12} \quad (6)$$

$$\Delta a_\mu^{\text{exp}} = 2.87(\pm 0.80) \times 10^{-9} \quad (7)$$

The study of lepton magnetic moments represents an active research subject in particle physics. For additional information and in-depth details, the reader is directed to the technical literature devoted to this topic.

## **2. Non-equilibrium dynamics as source of the g-2 anomaly**

The concept of *helicity* in Quantum Field Theory (QFT) refers to the projection of spin onto the direction of the momentum vector. A distinctive feature of the anomalous magnetic moment of leptons is that it can flip helicity in interactions mediated by gauge bosons [2]. These flip transitions are only allowed for massive particles and occur with a probability amplitude proportional to the mass of the particle. Since the probability of such transitions scales with the square of the probability amplitude, the lepton anomalous magnetic moment  $\Delta a_l$  must scale with the square of the lepton mass. In fact, it can be

shown that, in the presence of BSM physics, the leading one-loop contribution to the muon anomalous magnetic  $\Delta a_\mu$  is on the order of [1]

$$\Delta a_\mu \sim \frac{g_{NP}^2}{16\pi^2} \frac{m_\mu^2}{M_{NP}^2} \quad (8)$$

Here,  $g_{NP}$  and  $M_{NP}$  denote the coupling and mass associated with the onset of the NP sector, in particular, *heavy BSM particles* alleged to show up beyond the energy range of the Large Hadron Collider. Many avenues centered around (8) have been explored from various vantage points. For instance, [1] examines the possibility of one or two new fields with different spins and gauge group representations, under the hypothesis of weakly coupling  $|g_{NP}| \leq \sqrt{4\pi}$  and weak-scale masses  $M_{NP} \geq 100 \text{ GeV}$ .

In what follows we continue to use (8) as a baseline but interpret it in a new light. Specifically, our analysis proceeds with the following assumptions:

- 1) Rather than postulating that (8) arises from the NP sector of particle physics, we conjecture that (8) is rooted in the onset of *non-equilibrium dynamics near* the Fermi scale given by  $M_{NP} \rightarrow M_{EW} \approx O(246 \text{ GeV})$  [4-6]. According to this interpretation, the departure from equilibrium is commensurate with the dimensional deviation from four spacetime dimensions, i.e.  $\varepsilon = 4 - D \approx O(m^2/M_{EW}^2) \ll 1$ .
- 2) we consider the dominant muon decay channel known as the *Michel decay* [2],

$$\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e \quad (9)$$

which is a pure leptonic process mediated by the weak boson  $W^-$ . As such, (9) is a reliable process since it has high-statistics, it enables precise measurements of the Fermi constant ( $G_F$ ) and it accounts for nearly 100% of the branching ratio.

Because we are limiting the discussion to the Michel decay, it makes sense to approximate the scale where non-equilibrium sets in as  $O(M_{EW}) \approx M_W$ . This is to say that, insofar (9) is concerned, the mass of the mediating  $W^-$  boson marks the representative non-equilibrium scale.

The muon coupling in (9) can be extracted from the SM relationship

$$\frac{G_F}{\sqrt{2}} = \frac{g_\mu^2}{8M_W^2} \quad (10)$$

where  $M_W$  is the mass of the  $W$  boson. From (8) and (10) we derive

$$\Delta a_\mu \sim \frac{G_F m_\mu^2}{2\sqrt{2} \pi^2} \quad (11)$$

which yields

$$\boxed{\Delta a_\mu \sim 4.667 \times 10^{-9}} \quad (12)$$

in reasonable agreement with experimental data (7).

A natural follow-up question is the following: Would the same analysis apply to the anomalous magnetic moments of electrons and tau-leptons? Assuming that this line of thinking is correct and that the muon coupling  $g_\mu$  stays the same for electrons and tau-

leptons, we use (8) to estimate the non-equilibrium mass scales that match existing data on  $\Delta a_e$  and  $\Delta a_\mu$  [1-3, 7-8]. Numerical evaluation leads to

$$\Delta a_e \rightarrow \lambda M_W, \lambda = \frac{1}{3} \quad (13)$$

$$\Delta a_\tau \rightarrow \lambda M_W \approx M_{EW}, \lambda = 3 \quad (14)$$

It is also instructive to ponder on the possible corrections to the muon coupling due to the “would-be” contribution of lepton flavor violation (LFV) [2]. This analysis goes beyond the scope of the paper and may be considered elsewhere.

Our findings are summarized in the table below.

$\Delta a_l$	<b>Experiment</b>	<b>Prediction</b>
$\Delta a_e$	$(-0.88(0.36)) \times 10^{-12}$	$(-0.605) \times 10^{-12}$ for $O(M_{EW}) = (\frac{1}{3})M_W$
$\Delta a_\mu$	$2.87(\pm 0.80) \times 10^{-9}$	$4.667 \times 10^{-9}$ for $O(M_{EW}) = M_W$
$\Delta a_\tau$	$-0.0508 < \Delta a_\tau^{\text{exp}} < 0.011823$	$\sim 10^{-7}$ for $O(M_{EW}) = 3M_W$

**Table 1:** Comparison between experimental and predicted values of  $\Delta a_l$

## **References**

- [1] <https://arxiv.org/pdf/1402.7065.pdf>
- [2] <https://arxiv.org/pdf/1610.06587.pdf>
- [3] <https://arxiv.org/pdf/2002.04822.pdf>

[4] Available at the following sites:

<http://www.aracneeditrice.it/aracneweb/index.php/pubblicazione.html?item=9788854889972>

<https://www.researchgate.net/publication/278849474> Introduction to Fractional Field Theory consolidated version

[5] <http://www.ejtp.info/articles/ejtpv7i24p219.pdf>

[6] Available at the following site:

<https://www.researchgate.net/publication/266578650> REFLECTIONS ON THE FUTURE OF PARTICLE THEORY

[7] <https://science.sciencemag.org/content/360/6385/191>

[8] <https://iopscience.iop.org/article/10.1088/1742-6596/912/1/012001/pdf>

