# On Ramanujan's mathematics: on some connections with $\phi$ and various formulas concerning the Particle Physics and in particular the $\mathbf{d}^{*}$-Hexaquark 

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#### Abstract

In this paper we have described some connections between Ramanujan's mathematics, $\phi$ and various formulas concerning the Particle Physics, in particular the $d^{*}$ (2380)-Hexaquark


[^0]
https://www.mathsone.com/blog-details/blog/77-Srinivasa-Ramanujan

## From

## Electromagnetic properties of the $\mathbf{d}^{*}(\mathbf{2 3 8 0})$ hexaquark

M. Bashkanov, D.P. Watts and A. Pastore

Department of Physics, University of York, Heslington, York, Y010 5DD, UK (Dated: July 16, 2019) - arXiv:1905.00713v2 [nucl-th] 15 Jul 2019

We have that:
THE d* (2380) IN A PION CLOUD MODEL

The $C 1$-term ( $D$ - wave $\sigma$ cloud) would not contribute to the quadrupole moment due to the zero net charge of the pion pair. To calculate the $C 2$ term we adopt a relative coordinate system in which $\hat{r}_{\pi}$ is substituted by $\hat{r}_{\pi-\pi}$ - the separation between the two pions. The $r_{\pi}$ is related to $r_{\pi-\pi}$ by $r_{\pi-\pi}=\sqrt{2} r_{\pi}$ - the relative distance between the pions instead of the pion-core distance. Similarly $e_{\pi}$ would be transformed to $2:\left(e_{\pi^{+}}-e_{\pi^{-}}\right)$. To simplify the integration we employ the spherical harmonics addition theorem, decomposing $Y_{0}^{2}\left(\hat{r}_{\pi-\pi}\right)$ into the product $Y_{m}^{2}\left(\hat{r}_{\pi_{1}}\right) Y_{-m}^{2}\left(-\hat{r}_{\pi_{2}}\right)$.

$$
\begin{equation*}
Y_{0}^{2}\left(\hat{r}_{\pi-\pi}\right)=\sqrt{\frac{5}{16 \pi}} \sum_{m=-2}^{2}(-1)^{m} Y_{m}^{2}\left(\hat{r}_{\pi_{1}}\right) Y_{-m}^{2}\left(-\hat{r}_{\pi_{2}}\right) \tag{10}
\end{equation*}
$$

The few non-zero elements in this calculations are shown together with their resulting weights below

$$
\begin{align*}
& \text { a) }<Y_{1}^{1} Y_{-1}^{1}\left|Y_{0}^{2}\right| Y_{1}^{1} Y_{-1}^{1}>>\frac{-1}{16 \sqrt{5 \pi^{3}}} \\
& \text { b) }<Y_{0}^{1} Y_{0}^{1}\left|Y_{0}^{2}\right| Y_{0}^{1} Y_{0}^{1}>>\frac{-1}{4 \sqrt{5 \pi^{3}}} \\
& \text { c) }<Y_{1}^{1} Y_{0}^{1}\left|Y_{0}^{2}\right| Y_{1}^{1} Y_{0}^{1}>: \frac{1}{8 \sqrt{5 \pi^{3}}} \\
& \text { d) }<Y_{1}^{1} Y_{1}^{1}\left|Y_{0}^{2}\right| Y_{1}^{1} Y_{1}^{1}>: \frac{1}{16 \sqrt{5 \pi^{3}}} \tag{11}
\end{align*}
$$

Summing these contributions gives:

$$
\begin{equation*}
<C\left|\hat{Q}_{\pi}\right| C>=-0.52 Q_{\Delta^{+}} \tag{12}
\end{equation*}
$$

The double-pion cloud produces a prolate quadrupole deformation, similar to that observed for the case of the nucleon. Note that the $B$ and $C$ terms have opposite signs. Combining all terms and taking $Q_{\Delta^{+}}=0.043 e \mathrm{fm}^{2}$ from Ref. [24] [30] one gets $Q_{d^{*}} \approx 0.21 Q_{\Delta^{+}}=0.009 \mathrm{efm}^{2}$ with a magnitude mainly arising from the $B-C$ term cancellations. It should

From (10) and (11), we obtain:

## $-2\left(\left(\left(\operatorname{sqrt}(5 /(16 \mathrm{Pi}))+\left[\left(\left((-1) /\left(\left(16 \operatorname{sqrt}\left(5 * \operatorname{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+\left(\left((-1) /\left(\left(4 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)-\right.\right.\right.\right.$ $\left.\left.\left.\left.1 /\left(\left(8 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \mathrm{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]\right)\right)\right)$

## Input:

$-2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)$

## Exact result:

$-2\left(\frac{\sqrt{\frac{5}{\pi}}}{4}-\frac{3}{8 \sqrt{5} \pi^{3 / 2}}\right)$

## Decimal approximation:

-0.57054777856172567291727456375887302514504529480436408594...
-0.57054777856...

## Property:

$-2\left(\frac{\sqrt{\frac{5}{\pi}}}{4}-\frac{3}{8 \sqrt{5} \pi^{3 / 2}}\right)$ is a transcendental number

## Alternate forms:

$$
\begin{aligned}
& \frac{3-10 \pi}{4 \sqrt{5} \pi^{3 / 2}} \\
& -\frac{10 \pi-3}{4 \sqrt{5} \pi^{3 / 2}}
\end{aligned}
$$

$$
\frac{3}{4 \sqrt{5} \pi^{3 / 2}}-\frac{\sqrt{\frac{5}{\pi}}}{2}
$$

## Series representations:

$$
\begin{aligned}
& -2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)= \\
& -\frac{-3+8{\sqrt{z_{0}}}^{2} \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(\frac{5}{\left.16 \pi^{2}-z_{0}\right)^{k_{1}}\left(5 \pi^{3}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}\right.}{k_{1}!k_{2}!}}{4 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{aligned}
$$

for ( $\operatorname{not}\left(z_{0} \in \mathbb{R}\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )

$$
\begin{aligned}
& -2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)= \\
& \quad-\left(\left(-3+8 \exp \left(i \pi\left[\frac{\arg \left(\frac{5}{16 \pi}-x\right)}{2 \pi}\right)\right) \exp \left(i \pi\left[\frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right]\right) \sqrt{x}^{2}\right.\right. \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(\frac{5}{16 \pi}-x\right)^{k_{1}}\left(5 \pi^{3}-x\right)^{k_{2}} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}{k_{1}!k_{2}!}\right) / \\
& \left.\left(4 \exp \left(i \pi\left[\frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(5 \pi^{3}-x\right)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)\right) \text { for }(x \in
\end{aligned}
$$

$$
\begin{aligned}
& -2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)= \\
& -\left(\left(\left(\frac{1}{z_{0}}\right)^{-1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{-1 / 2-1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor}\right.\right. \\
& \left(-3+8\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\left.\arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi) \right\rvert\,+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor\right.}\right. \\
& z_{0}^{1+1 / 2\left\lfloor\left.\arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi) \right\rvert\,+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor\right.} \\
& \quad /\left(4 \sum_{k=0}^{\infty} \frac{\left.\left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)\right)}{k_{1}!k_{2}!}\right)
\end{aligned}
$$

We have also:
$-1 /\left(\left(\left(-2\left(\left(\left(\operatorname{sqrt}(5 /(16 \mathrm{Pi}))+\left[\left(\left((-1) /\left(\left(16 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+\left(\left((-1) /\left(\left(4 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)-\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.1 /\left(\left(8 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]\right)\right)\right)\right)\right)\right)-21 / 10^{\wedge} 3$

## Input:



## Exact result:

$\frac{1}{2\left(\frac{\sqrt{\frac{5}{\pi}}}{4}-\frac{3}{8 \sqrt{5} \pi^{3 / 2}}\right)}-\frac{21}{1000}$

## Decimal approximation:

1.731701592355447782693770201446839213300091395726801137632...
$1.731701592 \ldots \approx \sqrt{3}$ that is the ratio between the gravitating mass $\mathrm{M}_{0}$ and the Wheelerian mass q

$$
M_{0}=\sqrt{3 q^{2}-\Sigma^{2}},
$$

$$
q=\frac{(3 \sqrt{3}) M_{\mathrm{s}}}{2} .
$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

## Property:

$$
-\frac{21}{1000}+\frac{1}{2\left(\frac{\sqrt{\frac{5}{\pi}}}{4}-\frac{3}{8 \sqrt{5} \pi^{3 / 2}}\right)} \text { is a transcendental number }
$$

## Alternate forms:

$\frac{4 \sqrt{5} \pi^{3 / 2}}{10 \pi-3}-\frac{21}{1000}$
$\frac{63-210 \pi+4000 \sqrt{5} \pi^{3 / 2}}{1000(10 \pi-3)}$
$-\frac{-63 \sqrt{5}+210 \sqrt{5} \pi-20000 \pi^{3 / 2}}{1000 \sqrt{5}(10 \pi-3)}$

Series representations:

for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$

$$
\begin{aligned}
& \frac{-1}{-2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)}-\frac{21}{10^{3}}= \\
& -\left(\left(-63-4000 \exp \left(i \pi\left[\frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(5 \pi^{3}-x\right)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}+\right.\right. \\
& 168 \exp \left(i \pi\left\lfloor\frac{\arg \left(\frac{5}{16 \pi}-x\right)}{2 \pi}\right\rfloor\right) \exp \left(i \pi\left\lfloor\frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right\rfloor\right) \sqrt{x}^{2} \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(\frac{5}{16 \pi}-x\right)^{k_{1}}\left(5 \pi^{3}-x\right)^{k_{2}} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}{k_{1}!k_{2}!}\right) / \\
& \left(1 0 0 0 \left(-3+8 \exp \left(i \pi\left[\frac{\arg \left(\frac{5}{16 \pi}-x\right)}{2 \pi}\right]\right) \exp \left(i \pi\left\lfloor\frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right]\right)\right.\right. \\
& \left.\left.\sqrt{x}^{2} \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(\frac{5}{16 \pi}-x\right)^{k_{1}}\left(5 \pi^{3}-x\right)^{k_{2}} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}{k_{1}!k_{2}!}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{-1}{-2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)}-\frac{21}{10^{3}}= \\
& -\left(\left\{-63-4000\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor}\right.\right. \\
& z_{0}^{1 / 2+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right]} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}+ \\
& 168\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right\rfloor+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor} \\
& \left.z_{0}^{1+1 / 2} \arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right]+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right] \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(\frac{5}{16 \pi}-z_{0}\right)^{k_{1}}\left(5 \pi^{3}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}\right) / \\
& \left(1 0 0 0 \left(-3+8\left(\frac{1}{z_{0}}\right)^{1 / 2\left|\arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right|+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor}\right.\right. \\
& \left.z_{0}^{1+1 / 2} \arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right]+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right] \\
& \left.\left.\left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(\frac{5}{16 \pi}-z_{0}\right)^{k_{1}}\left(5 \pi^{3}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}\right)\right)\right)
\end{aligned}
$$

From which:
$-1 /\left(\left(\left(-2\left(\left(\left(\operatorname{sqrt}(5 /(16 \mathrm{Pi}))+\left[\left(\left((-1) /\left(\left(16 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+\left(\left((-1) /\left(\left(4 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)-\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.1 /\left(\left(8 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]\right)\right)\right)\right)\right)\right)-(89+34+11) 1 / 10^{\wedge} 3$

## Input:

$$
\left.-\frac{-1}{-2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right.}\right)-(89+34+11) \times \frac{1}{10^{3}}
$$

## Exact result:

$\frac{1}{2\left(\frac{\sqrt{\frac{5}{\pi}}}{4}-\frac{3}{8 \sqrt{5} \pi^{3 / 2}}\right)}-\frac{67}{500}$

## Decimal approximation:

1.618701592355447782693770201446839213300091395726801137632...
$1.6187015923554 \ldots$. result that is a very good approximation to the value of the golden ratio 1.618033988749...

## Property:

$-\frac{67}{500}+\frac{1}{2\left(\frac{\sqrt{\frac{5}{\pi}}}{4}-\frac{3}{8 \sqrt{5} \pi^{3 / 2}}\right)}$ is a transcendental number

## Alternate forms:

$\frac{4 \sqrt{5} \pi^{3 / 2}}{10 \pi-3}-\frac{67}{500}$
$\frac{201-670 \pi+2000 \sqrt{5} \pi^{3 / 2}}{500(10 \pi-3)}$
$-\frac{-201 \sqrt{5}+670 \sqrt{5} \pi-10000 \pi^{3 / 2}}{500 \sqrt{5}(10 \pi-3)}$

Series representations:

for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$

$$
\begin{aligned}
& \frac{-1}{-2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)}-\frac{89+34+11}{10^{3}}= \\
& -\left(\left(-201-2000 \exp \left(i \pi\left[\frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(5 \pi^{3}-x\right)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}+\right.\right. \\
& 536 \exp \left(i \pi\left[\frac{\arg \left(\frac{5}{16 \pi}-x\right)}{2 \pi}\right]\right) \exp \left(i \pi\left[\frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right]\right) \sqrt{x}^{2} \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(\frac{5}{16 \pi}-x\right)^{k_{1}}\left(5 \pi^{3}-x\right)^{k_{2}} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}{k_{1}!k_{2}!}\right) / \\
& \left(5 0 0 \left(-3+8 \exp \left(i \pi\left[\frac{\arg \left(\frac{5}{16 \pi}-x\right)}{2 \pi}\right]\right) \exp \left(i \pi\left\lfloor\frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right]\right)\right.\right. \\
& \left.\left.\left.\sqrt{x}^{2} \sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(\frac{5}{16 \pi}-x\right)^{k_{1}}\left(5 \pi^{3}-x\right)^{k_{2}} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}{k_{1}!k_{2}!}\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{-1}{-2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)}-\frac{89+34+11}{10^{3}}= \\
& -\left(\left(-201-2000\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor}\right.\right. \\
& z_{0}^{1 / 2+1 / 2}\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}+ \\
& 536\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right\rfloor+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor} \\
& z_{0}^{1+1 / 2}\left\lfloor\arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right]+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(\frac{5}{16 \pi}-z_{0}\right)^{k_{1}}\left(5 \pi^{3}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}\right) / \\
& 500\left(-3+8\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right\rfloor+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor}\right. \\
& \left.z_{0}^{1+1 / 2} \arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right]+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right] \\
& \left.\left.\left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(\frac{5}{16 \pi}-z_{0}\right)^{k_{1}}\left(5 \pi^{3}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}\right)\right)\right)
\end{aligned}
$$

Now, we have for each expression, the following results:
$-1 /\left(\left(16 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)$

## Input:

$-\frac{1}{16 \sqrt{5 \pi^{3}}}$

## Exact result:

$-\frac{1}{16 \sqrt{5} \pi^{3 / 2}}$

## Decimal approximation:

-0.00501961266194286159537606806937899679195414327437159081...
-0.00501961266194...

## Property:

$-\frac{1}{16 \sqrt{5} \pi^{3 / 2}}$ is a transcendental number

## Series representations:

$-\frac{1}{16 \sqrt{5 \pi^{3}}}=-\frac{1}{16 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}}$
$-\frac{1}{16 \sqrt{5 \pi^{3}}}=-\frac{1}{16 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}$
$-\frac{1}{16 \sqrt{5 \pi^{3}}}=-\frac{1}{16 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{k}}{k!}}$ for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$

## $\left(\left(\left(-1 /\left(\left(4 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)\right.$

## Input:

$-\frac{1}{4 \sqrt{5 \pi^{3}}}$

## Exact result:

$$
-\frac{1}{4 \sqrt{5} \pi^{3 / 2}}
$$

## Decimal approximation:

-0.02007845064777144638150427227751598716781657309748636324...
-0.02007845064777...

## Property:

$-\frac{1}{4 \sqrt{5} \pi^{3 / 2}}$ is a transcendental number

## Series representations:

$-\frac{1}{4 \sqrt{5 \pi^{3}}}=-\frac{1}{4 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}}$
$-\frac{1}{4 \sqrt{5 \pi^{3}}}=-\frac{1}{4 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}$
$-\frac{1}{4 \sqrt{5 \pi^{3}}}=-\frac{1}{4 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}$ for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$
$\left(\left(\left(\left(\left(\left(1 /\left(\left(8 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)\right)\right)\right)\right.$

## Input:

$\frac{1}{8 \sqrt{5 \pi^{3}}}$

## Exact result:

$\frac{1}{8 \sqrt{5} \pi^{3 / 2}}$

## Decimal approximation:

0.010039225323885723190752136138757993583908286548743181623...
0.01003922532388...

## Property:

$\frac{1}{8 \sqrt{5} \pi^{3 / 2}}$ is a transcendental number
Series representations:
$\frac{1}{8 \sqrt{5 \pi^{3}}}=\frac{1}{8 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}}$
$\frac{1}{8 \sqrt{5 \pi^{3}}}=\frac{1}{8 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}$
$\frac{1}{8 \sqrt{5 \pi^{3}}}=\frac{1}{8 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}$ for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$
$\left(\left(\left(\left(\left(1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)\right)\right)\right)$

## Input:

$\frac{1}{16 \sqrt{5 \pi^{3}}}$

## Exact result:

$\frac{1}{16 \sqrt{5} \pi^{3 / 2}}$

## Decimal approximation:

$0.005019612661942861595376068069378996791954143274371590811 \ldots$
0.005019612661...

## Property:

$\frac{1}{16 \sqrt{5} \pi^{3 / 2}}$ is a transcendental number

## Series representations:

$\frac{1}{16 \sqrt{5 \pi^{3}}}=\frac{1}{16 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}}$
$\frac{1}{16 \sqrt{5 \pi^{3}}}=\frac{1}{16 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}$
$\frac{1}{16 \sqrt{5 \pi^{3}}}=\frac{1}{16 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}$ for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$

We have thus:
$-0.005019612661942861595$
$-0.02007845064777$
0.01003922532388572319
0.005019612661942861595

From which:
(-0.0050196126619428-0.02007845064777-0.0100392253238852-
$0.0050196126619428)$

## Input interpretation:

-0.0050196126619428-0.02007845064777-
$0.0100392253238852-0.0050196126619428$

## Result:

-0.0401569012955408
-0.0401569012955408
Indeed:
$\left[\left(\left((-1) /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+\left(\left((-1) /\left(\left(4 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)-1 /\left(\left(8 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)-\right.$ $\left.1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]$

## Input:

$$
-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}-\frac{1}{16 \sqrt{5 \pi^{3}}}
$$

## Exact result:

$-\frac{1}{2 \sqrt{5} \pi^{3 / 2}}$

## Decimal approximation:

$-0.04015690129554289276300854455503197433563314619497272649$
-0.040156901295.....

## Property:

$-\frac{1}{2 \sqrt{5} \pi^{3 / 2}}$ is a transcendental number

## Series representations:

$-\frac{\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}-\frac{1}{16 \sqrt{5 \pi^{3}}}}{-\frac{1}{2 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}}}=$

$$
\begin{aligned}
& -\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}-\frac{1}{16 \sqrt{5 \pi^{3}}}= \\
& -\frac{1}{2 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}} \\
& -\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}-\frac{1}{16 \sqrt{5 \pi^{3}}}=-\frac{1}{2 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{aligned}
$$

$$
\text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
$$

The principal result, is:
$\left[\left(\left((-1) /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+(((-\right.$
$\left.\left.\left.1) /\left(\left(4 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+1 /\left(\left(8 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]$

## Input:

$$
-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}
$$

## Exact result:

$-\frac{1}{8 \sqrt{5} \pi^{3 / 2}}$

## Decimal approximation:

$-0.01003922532388572319075213613875799358390828654874318162 \ldots$
-0.01003922532388...

## Property:

$-\frac{1}{8 \sqrt{5} \pi^{3 / 2}}$ is a transcendental number

## Series representations:

$-\frac{\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}{-\frac{1}{8 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}}}=$

$$
\begin{aligned}
& -\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}= \\
& -\frac{1}{8 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}} \\
& -\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}=-\frac{1}{8 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{aligned}
$$

$$
\text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
$$

From which, we obtain:
$-1 /\left[\left(\left((-1) /\left(\left(16 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+(((-\right.$
$\left.\left.\left.1) /\left(\left(4 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+1 /\left(\left(8 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]+29+11$

## Input:

$-\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29+11$

## Exact result:

$40+8 \sqrt{5} \pi^{3 / 2}$

## Decimal approximation:

139.6092793754474586408341868512648763798139401289642745943...
$139.609279375 \ldots$ result practically equal to the rest mass of Pion meson 139.57 MeV

## Property:

$40+8 \sqrt{5} \pi^{3 / 2}$ is a transcendental number

## Alternate form:

$8\left(5+\sqrt{5} \pi^{3 / 2}\right)$

## Series representations:

$$
\begin{aligned}
& -\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5^{3}}}}+29+11= \\
& 40+8 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29+11= \\
& 40+8 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!} \\
& -\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29+11= \\
& 40+8 \sqrt{z_{0} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!} \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)} .
\end{aligned}
$$

$-1 /\left[\left(\left((-1) /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+(((-\right.$
$\left.\left.\left.1) /\left(\left(4 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+1 /\left(\left(8 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \mathrm{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]+29-\mathrm{Pi}$

## Input:

$-\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29-\pi$

## Exact result:

$29-\pi+8 \sqrt{5} \pi^{3 / 2}$

## Decimal approximation:

125.4676867218576654023715434679853734956167707295891687734...
125.4676867218... result very near to the Higgs boson mass 125.18 GeV

## Property:

$29-\pi+8 \sqrt{5} \pi^{3 / 2}$ is a transcendental number

## Alternate form:

$29+\pi(8 \sqrt{5 \pi}-1)$

## Series representations:

$$
\begin{aligned}
& -\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29-\pi= \\
& 29-\pi+8 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29-\pi= \\
& 29-\pi+8 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!} \\
& -\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29-\pi= \\
& 29-\pi+8 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{k}}{k!} \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

$27 * 1 / 2 *\left(\left(\left(-1 /\left[\left(\left((-1) /\left(\left(16 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+(((-\right.\right.\right.\right.$
$\left.\left.\left.\left.\left.\left.1) /\left(\left(4 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+1 /\left(\left(8 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]+29\right)\right)\right)-7$

## Input:

$27 \times \frac{1}{2}\left(-\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29\right)-7$

## Exact result:

$\frac{27}{2}\left(29+8 \sqrt{5} \pi^{3 / 2}\right)-7$

## Decimal approximation:

1729.225271568540691651261522492075831127488191741017707024...
1729.22527156854...

This result is very near to the mass of candidate glueball $\mathbf{f}_{\mathbf{0}}(\mathbf{1 7 1 0})$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the $j$-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the GrossZagier theorem. The number 1728 is one less than the Hardy-Ramanujan number 1729 (taxicab number)

## Property:

$-7+\frac{27}{2}\left(29+8 \sqrt{5} \pi^{3 / 2}\right)$ is a transcendental number

Alternate forms:

$$
\begin{aligned}
& \frac{769}{2}+108 \sqrt{5} \pi^{3 / 2} \\
& \frac{1}{2}\left(769+216 \sqrt{5} \pi^{3 / 2}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{27}{2}\left(-\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29\right)-7= \\
& \frac{769}{2}+108 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k} \\
& \frac{27}{2}\left(-\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29\right)-7= \\
& \frac{769}{2}+108 \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!} \\
& \frac{27}{2}\left(-\frac{1}{-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}}+29\right)-7= \\
& \frac{769}{2}+108 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{k}}{k!} \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

Now, from $2380=1729+728-135-10-9+(135+10+9) / 2$, we obtain:
$-6 *(1729+728-135-10-9+(135+10+9) / 2)\left[-1 /\left(\left(16 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)-1 /\left(\left(4 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)+\right.$ $\left.1 /\left(\left(8 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]-4$

## Input:

$$
\begin{aligned}
& -6\left(1729+728-135-10-9+\frac{1}{2}(135+10+9)\right) \\
& \quad\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4
\end{aligned}
$$

## Exact result:

$$
\frac{357 \sqrt{5}}{\pi^{3 / 2}}-4
$$

## Decimal approximation:

139.3601376250881271639405040614641483782103319160526335850...
$139.360137625 \ldots$ result practically equal to the rest mass of Pion meson 139.57 MeV

## Property:

$-4+\frac{357 \sqrt{5}}{\pi^{3 / 2}}$ is a transcendental number

## Alternate form:

$$
\frac{357 \sqrt{5}-4 \pi^{3 / 2}}{\pi^{3 / 2}}
$$

## Series representations:

$$
\begin{aligned}
& -6\left(1729+728-135-10-9+\frac{1}{2}(135+10+9)\right) \\
& \left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4= \\
& 1785 \\
& \sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k} \\
& -6\left(1729+728-135-10-9+\frac{1}{2}(135+10+9)\right) \\
& \left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4= \\
& -4+\frac{1785}{\sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}} \\
& -6\left(1729+728-135-10-9+\frac{1}{2}(135+10+9)\right) \\
& \left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4= \\
& -4+\frac{1785}{\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}} \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

or:
$-6(2380)\left[-1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)-1 /\left(\left(4 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(8 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)+\right.$ $\left.1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]-4$

## Input:

$-6 \times 2380\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4$

## Exact result:

$$
\frac{357 \sqrt{5}}{\pi^{3 / 2}}-4
$$

## Decimal approximation:

139.3601376250881271639405040614641483782103319160526335850
$139.360137625 \ldots$ result practically equal to the rest mass of Pion meson 139.57 MeV

## Property:

$-4+\frac{357 \sqrt{5}}{\pi^{3 / 2}}$ is a transcendental number

## Alternate form:

$$
\frac{357 \sqrt{5}-4 \pi^{3 / 2}}{\pi^{3 / 2}}
$$

## Series representations:

$$
\begin{aligned}
& -6 \times 2380\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4= \\
& -4+\frac{1785}{\sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}} \\
& -6 \times 2380\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4= \\
& -4+\frac{1785}{\sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}
\end{aligned}
$$

$$
\begin{aligned}
& -6 \times 2380\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4= \\
& -4+\frac{1785}{\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}} \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

$-6 *(1729+728-135-10-9+(135+10+9) / 2)\left[-1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \mathrm{Pi}^{\wedge} 3\right)\right)\right)-1 /\left(\left(4 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)+\right.$ $\left.1 /\left(\left(8 \operatorname{sqrt}\left(5^{*} \operatorname{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \operatorname{sqrt}\left(5^{*} \operatorname{Pi}^{\wedge} 3\right)\right)\right)\right]-4-11-\mathrm{Pi}$

## Input:

$$
\begin{aligned}
& -6\left(1729+728-135-10-9+\frac{1}{2}(135+10+9)\right) \\
& \quad\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4-11-\pi
\end{aligned}
$$

## Exact result:

$-15+\frac{357 \sqrt{5}}{\pi^{3 / 2}}-\pi$

## Decimal approximation:

125.2185449714983339254778606781846454940131625166775277640...
125.2185449714... result very near to the Higgs boson mass 125.18 GeV

## Property:

$-15+\frac{357 \sqrt{5}}{\pi^{3 / 2}}-\pi$ is a transcendental number

## Alternate form:

$\frac{357 \sqrt{5}-15 \pi^{3 / 2}-\pi^{5 / 2}}{\pi^{3 / 2}}$

## Series representations:

$$
\begin{aligned}
& -6\left(1729+728-135-10-9+\frac{1}{2}(135+10+9)\right) \\
& \left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)-4-11-\pi= \\
& -15-\pi+\frac{1785}{\sqrt{-1+5 \pi^{3}} \sum_{k=0}^{\infty}\left(-1+5 \pi^{3}\right)^{-k}\binom{\frac{1}{2}}{k}}
\end{aligned}
$$

$$
\begin{aligned}
& -6\left(1729+728-135-10-9+\frac{1}{2}(135+10+9)\right) \\
& \left.-15-\pi+\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{1785 \sqrt{5 \pi^{3}}}\right)-4-11-\pi= \\
& -6\left(1729+728-135-10-9+\frac{1}{2}(135+10+9)\right) \\
& -\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}+\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+5 \pi^{3}\right)^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}\right. \\
& -17 \sqrt{5 \pi^{3}} \\
& \left.-\frac{1785}{16 \sqrt{5 \pi^{3}}}\right)-4-11-\pi= \\
& -15-\pi+\frac{1}{\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}} \text { for }\left(\text { not }\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

Now, from

$$
-2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)
$$

multiplying by -2380 , i.e. the mass of the $\mathrm{d}^{*}$-Hexaquark and adding 18 and 7 , that are Lucas numbers, we obtain:
$-(2380) *\left(\left(\left(-2\left(\left(\left(\operatorname{sqrt}(5 /(16 \mathrm{Pi}))+\left[\left(\left((-1) /\left(\left(16 \mathrm{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)+\left(\left((-1) /\left(\left(4 \mathrm{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right)\right)-\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.1 /\left(\left(8 \operatorname{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)+1 /\left(\left(16 \mathrm{sqrt}\left(5 * \mathrm{Pi}^{\wedge} 3\right)\right)\right)\right]\right)\right)\right)\right)\right)+18+7$

## Input:

$$
-2380\left(-2\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)\right)+18+7
$$

## Exact result:

$25+4760\left(\frac{\sqrt{\frac{5}{\pi}}}{4}-\frac{3}{8 \sqrt{5} \pi^{3 / 2}}\right)$

## Decimal approximation:

1382.903712976907101543113461746117799845207801634386524549...
1382.903712976... result practically equal to the rest mass of Sigma baryon 1382.8 and very near to the rest mass of $\Sigma^{*^{0}}(1385)=1383.7$ formed by "uds" quarks. The hexaquark might contain either six quarks, resembling two baryons bound together (a dibaryon), or three quarks and three antiquarks.

## Property:

$25+4760\left(\frac{\sqrt{\frac{5}{\pi}}}{4}-\frac{3}{8 \sqrt{5} \pi^{3 / 2}}\right)$ is a transcendental number

## Alternate forms:

$$
\begin{aligned}
& \frac{\sqrt{5}\left(-357+1190 \pi+5 \sqrt{5} \pi^{3 / 2}\right)}{\pi^{3 / 2}} \\
& \frac{-357 \sqrt{5}+1190 \sqrt{5} \pi+25 \pi^{3 / 2}}{\pi^{3 / 2}} \\
& 5\left(5+238 \sqrt{\frac{5}{\pi}}\right)-\frac{357 \sqrt{5}}{\pi^{3 / 2}}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& -2380(-2)\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)+18+7= \\
& \left(5 \left(-357+5 \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}+952{\sqrt{z_{0}}}^{2}\right.\right. \\
& \left.\left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(\frac{5}{16 \pi}-z_{0}\right)^{k_{1}}\left(5 \pi^{3}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}\right)\right) / \\
& \left(\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right) \text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& -2380(-2)\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)+18+7= \\
& \left(5\left(-357+5 \exp \left(i \pi \left\lvert\, \frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right.\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(5 \pi^{3}-x\right)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}+\right. \\
& \left.952 \exp \left(i \pi\left(\frac{\arg \left(\frac{5}{16 \pi}-x\right)}{2 \pi}\right]\right) \exp \left(i \pi \left\lvert\, \frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right.\right]\right) \sqrt{x}^{2} \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(\frac{5}{16 \pi}-x\right)^{k_{1}}\left(5 \pi^{3}-x\right)^{k_{2} x^{-k_{1}-k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}}}{k_{1}!k_{2}!}\right) / \\
& \left.\left(\exp \left(i \pi \left\lvert\, \frac{\arg \left(5 \pi^{3}-x\right)}{2 \pi}\right.\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(5 \pi^{3}-x\right)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)
\end{aligned}
$$

for ( $x \in \mathbb{R}$ and $x<0$ )

$$
\begin{aligned}
& -2380(-2)\left(\sqrt{\frac{5}{16 \pi}}+\left(-\frac{1}{16 \sqrt{5 \pi^{3}}}-\frac{1}{4 \sqrt{5 \pi^{3}}}-\frac{1}{8 \sqrt{5 \pi^{3}}}+\frac{1}{16 \sqrt{5 \pi^{3}}}\right)\right)+18+7= \\
& \left(5 ( \frac { 1 } { z _ { 0 } } ) ^ { - 1 / 2 \lfloor \operatorname { a r g } ( 5 \pi ^ { 3 } - z _ { 0 } ) / ( 2 \pi ) \rfloor } z _ { 0 } ^ { - 1 / 2 - 1 / 2 \lfloor \operatorname { a r g } ( 5 \pi ^ { 3 } - z _ { 0 } ) / ( 2 \pi ) \rfloor } \left(-357+5\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\operatorname{agg}\left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor}\right.\right. \\
& z_{0}^{1 / 2+1 / 2\left\lfloor\operatorname{agg}\left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}+ \\
& \left.952\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right\rfloor+1 / 2\left\lfloor\arg \left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor}\right) \\
& z_{0}^{1+1 / 2\left\lfloor\operatorname{agg}\left(\frac{5}{16 \pi}-z_{0}\right) /(2 \pi)\right]+1 / 2\left\lfloor\operatorname{agg}\left(5 \pi^{3}-z_{0}\right) /(2 \pi)\right\rfloor} \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=0}^{\infty} \frac{(-1)^{k_{1}+k_{2}}\left(-\frac{1}{2}\right)_{k_{1}}\left(-\frac{1}{2}\right)_{k_{2}}\left(\frac{5}{16 \pi}-z_{0}\right)^{k_{1}}\left(5 \pi^{3}-z_{0}\right)^{k_{2}} z_{0}^{-k_{1}-k_{2}}}{k_{1}!k_{2}!}\right) / \\
& \left(\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(5 \pi^{3}-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)
\end{aligned}
$$

From
Neutron star matter equation of state including d*-hexaquark
degrees of freedom A.Mantziris, A. Pastore, I. Vidaña, D. P. Watts, M. Bashkanov, and A. M. Romero - arXiv:2002.06571v1 [nucl-th] 16 Feb 2020

Neutron stars are the remnants of the gravitational collapse of massive stars during a supernova event of Type-II, Ib or Ic. Their masses and radii are typically of the order of $1-2 M_{\odot}\left(M_{\odot} \simeq 2 \times 10^{33} \mathrm{~g}\right.$ is the mass of the Sun) and $10-14 \mathrm{~km}$, respectively. With central densities in the range of $4-8$ times the normal nuclear matter saturation density, $\epsilon_{0} \sim 2.7 \times 10^{14} \mathrm{~g} / \mathrm{cm}^{3}\left(\rho_{0} \sim 0.16\right.$ $\mathrm{fm}^{-3}$ ), neutron stars are most likely among the densest objects in the Universe (Shapiro \& Teukolsky (2008); Glendenning (2000); Haensel et al. (2007); Rezzolla et al. (2018)). These objects are, therefore, excellent laboratories to test our present understanding of the theory of strong interacting matter at extreme conditions, offering an interesting interplay between the physics of dense matter and astrophysical observables.

Relativistic mean-field models are based on effective Lagrangians densities where the interaction between baryons is described in terms of meson exchanges. The couplings of nucleons with mesons are usually fixed by fitting masses and radii of nuclei and the properties of nuclear bulk matter, whereas those of other baryons, like hyperons, are fixed by symmetry relations and hypernuclear observables. Recently, Dutra et al., (Dutra et al. (2014)) have analyzed, as in the case of Skyrme, several parametrizations of 7

Recently, we studied the role of a new degree of freedom $d^{*}(2380)$ (Bashkanov et al. (2019)) on the nuclear EoS (Vidaña et al. (2018)). The $d^{*}(2380)$ is a massive positively charged non-strange particle with integer spin $(\mathrm{J}=3)$ and it represents the first known non-trivial hexaquark evidenced in experiment (Adlarson et al. (2011, 2013, 2014)). The importance of such a new degree-offreedom resides in the fact that it has the same $u, d$ quark composition as neutrons and protons and, therefore, does not involve any strangeness degrees of freedom. Moreover, it is a boson and as such it may condensate within the star. In our previous work we showed that despite its very large mass, the $d^{*}(2380)$ can appear in the neutron star interior at densities similar to those predicted for the appearance of other nucleon resonances, such as the $\Delta$, or hyperons. That work was a first attempt to study the consequences that the presence of the $d^{*}(2380)$ could have on the properties of neutron stars where, however, we assumed the $d^{*}(2380)$ as simple gas of non-interacting bosons. We have, therefore, decided to pursue a more detailed study which accounts for explicit interaction of $d^{*}(2380)$ with the surrounding medium. To this aim, we employ a standard non-linear Walecka model (Dutra et al. (2014)), within the framework of a relativistic mean field theory (RMF). Starting from a well-known nucleonic Lagrangian (Glendenning (2000); Drago et al. (2014b)), we employ the established $d^{*}(2380)$ properties to determine its interaction with other particles. In particular, we aim at providing first constraints on the sign (attractive or repulsive) for the effective interaction of such a particle.

The manuscript is organized in the following way. The Lagrangian density including the $d^{*}(2380)$ is shortly presented in Section 2. Our main results regarding the appearance and effect of $d^{*}(2380)$ on neutron stars are shown and discussed in Section 3. Finally, our concluding remarks and possible directions for future work are given in Section 4.

In a RMF description of infinite nuclear matter, the meson fields are treated as classical fields. Meson field equations in the mean field approximation can be easily derived by applying the Euler-Lagrange equations to the Lagrangian density of Eq. (1) and replacing field operators by their ground-state expectation values $\sigma \rightarrow \bar{\sigma}, \omega_{\mu} \rightarrow \omega_{0}, \boldsymbol{\rho}_{\mu} \rightarrow \bar{\rho}_{0}^{(3)}$. They read simply

$$
\begin{align*}
m_{\sigma}^{2} \bar{\sigma} & =\sum_{B=N, \Delta} g_{\sigma B}\left(\frac{2 J_{B}+1}{2 \pi^{2}} \int_{0}^{k_{F_{B}}} \frac{m_{B}^{*}}{\sqrt{k^{2}+m_{B}^{* 2}}} k^{2} d k\right)-b m_{N} g_{\sigma N}\left(g_{\sigma N} \bar{\sigma}\right)^{2}-c g_{\sigma N}\left(g_{\sigma N} \bar{\sigma}\right)^{3}+g_{\sigma d} \cdot \rho_{d} \cdot  \tag{3}\\
m_{\omega}^{2} \bar{\omega}_{0} & =\sum_{B=N, \Delta} g_{\omega B} \frac{2 J_{B}+1}{6 \pi^{2}} k_{F_{B}}^{3}-g_{\omega d} \cdot \rho_{d} \cdot  \tag{4}\\
m_{\rho}^{2} \bar{\rho}_{0}^{(3)} & =\sum_{B=N, \Delta} g_{\rho B} \frac{2 J_{B}+1}{6 \pi^{2}} k_{F_{B}}^{3} I_{3 B}, \tag{5}
\end{align*}
$$

where $J_{B}$ is the spin of the baryon $\mathrm{B}, k_{F_{B}}$ it Fermi momentum, $m_{B}^{*}=m_{B}-g_{\sigma B}$ its effective mass, $I_{3 B}$ the third component of its isospin, and $\rho_{d^{*}}=2 m_{d^{*}} \phi_{d^{*}}^{*} \phi_{d^{*}}$ is the density of the $d^{*}(2380)$ di-baryon. The energy density and the pressure of the system are obtained from the energy-momentum tensor

$$
\begin{align*}
\varepsilon & =\sum_{B=N, \Delta} \frac{\left(2 J_{B}+1\right)}{2 \pi^{2}} \int_{0}^{k_{F_{B}}} \sqrt{k^{2}+m_{B}^{* 2}} k^{2} d k+\sum_{l=e^{*}, \mu^{-}} \frac{1}{\pi^{2}} \int_{0}^{k_{F}^{l}} \sqrt{k^{2}+m_{l}^{2}} k^{2} d k+\frac{1}{2} m_{\sigma}^{2} \bar{\sigma}^{2}+\frac{1}{3} b m_{N}\left(g_{\sigma} \bar{\sigma}\right)^{3}+\frac{1}{4} c\left(g_{\sigma} \bar{\sigma}\right)^{4} \\
& +\frac{1}{2} m_{\omega}^{2} \bar{\omega}_{0}^{2}+\frac{1}{2} m_{\rho}^{2}\left(\bar{\rho}_{0}^{(3)}\right)^{2}+m_{d^{*}}^{*} \cdot \rho_{d^{*}}  \tag{6}\\
P & =\sum_{B=N, \Delta} \frac{\left(2 J_{B}+1\right)}{6 \pi^{2}} \int_{0}^{k_{F_{B}}} \frac{k^{4} d k}{\sqrt{k^{2}+m_{B}^{* 2}}}+\sum_{l=e^{-}, \mu^{-}} \frac{1}{3 \pi^{2}} \int_{0}^{k_{F}^{\prime}} \frac{k^{4} d k}{\sqrt{k^{2}+m_{l}^{2}}}-\frac{1}{2} m_{\sigma}^{2} \bar{\sigma}^{2}-\frac{1}{3} b m_{N}\left(g_{\sigma} \bar{\sigma}\right)^{3}-\frac{1}{4} c\left(g_{\sigma} \bar{\sigma}\right)^{4} \\
& +\frac{1}{2} m_{\omega}^{2} \bar{\omega}_{0}^{2}+\frac{1}{2} m_{\rho}^{2}\left(\bar{\rho}_{0}^{(3)}\right)^{2} . \tag{7}
\end{align*}
$$

From:
The energy density

$$
\begin{aligned}
\varepsilon & =\sum_{B=N, A} \frac{\left(2 J_{B}+1\right)}{2 \pi^{2}} \int_{0}^{k_{F_{B}}} \sqrt{k^{2}+m_{B}^{* 2}} k^{2} d k+\sum_{l=e^{-}, \mu^{-}} \frac{1}{\pi^{2}} \int_{0}^{k_{F}^{\prime}} \sqrt{k^{2}+m_{l}^{2}} k^{2} d k+\frac{1}{2} m_{\sigma}^{2} \bar{\sigma}^{2}+\frac{1}{3} h m_{N}\left(g_{\sigma} \bar{\sigma}\right)^{3}+\frac{1}{4} c\left(g_{\sigma} \bar{\sigma}\right)^{4} \\
& +\frac{1}{2} m_{\omega}^{2} \bar{\omega}_{0}^{2}+\frac{1}{2} m_{\rho}^{2}\left(\bar{\rho}_{v}^{(3)}\right)^{2}+m_{d}^{*} \cdot \rho_{d}
\end{aligned}
$$

We have that:
$\epsilon_{0} \sim 2.7 \times 10^{14} \mathrm{~g} / \mathrm{cm}^{3}$ the normal nucle ar matter saturation density:
and obtain:
$2.7 \mathrm{e}+14 \mathrm{~g} / \mathrm{cm}^{\wedge} 3$

## Input interpretation:

$2.7 \times 10^{17} \mathrm{~kg} / \mathrm{m}^{3}$ (kilograms per cubic meter)

## Unit conversions:

$2.7 \times 10^{14} \mathrm{~g} / \mathrm{cm}^{3}$ (grams per cubic centimeter)
$0.1515\left(\mathrm{GeV} / \mathrm{c}^{2}\right) / \mathrm{fm}^{3}$ (gigaelectronvolts per speed of light squared per cubic femtometer)

## Input interpretation:

$0.1515\left(2.99792 \times 10^{8}\right)^{2}$

## Result:

13616099354496000

## Scientific notation:

$1.3616099354496 \times 10^{16}$
$1.3616099354496 * 10^{16}$
$\left(1.3616099354496 \times 10^{\wedge} 16\right)^{\wedge} 1 / 7-55-8+1 /$ golden ratio
Input interpretation:
$\sqrt[7]{1.3616099354496 \times 10^{16}}-55-8+\frac{1}{\phi}$

## Result:

139.39178751702...
139.39178751702 ... result practically equal to the rest mass of Pion meson 139.57 MeV
$\left(1.3616099354496 \times 10^{\wedge} 16\right)^{\wedge} 1 / 7-76-1 /$ golden ratio

## Input interpretation:

$\sqrt[7]{1.3616099354496 \times 10^{16}}-76-\frac{1}{\phi}$

## Result:

125.15571953952...
$125.15571953952 \ldots$ result very near to the Higgs boson mass 125.18 GeV

Now, we have the following Ramanujan mock theta function:

$$
\begin{aligned}
& \boldsymbol{F}(q)=\mathbf{1}+\frac{q^{2}}{1-q}+\frac{\boldsymbol{q}^{8}}{(1-q)\left(\mathbf{1}-q^{3}\right)}+\ldots, \\
& \boldsymbol{\phi}(-q)+\chi(q)=\mathbf{2} \boldsymbol{F}^{\prime}(q) \\
& 1+\left(0.449329^{\wedge} 2\right) /(1-0.449329)+(0.449329)^{\wedge} 8 /\left((1-0.449329)\left(1-0.449329^{\wedge} 3\right)\right) \\
& 1+\frac{0.449329^{2}}{1-0.449329}+\frac{0.449329^{8}}{(1-0.449329)\left(1-0.449329^{3}\right)} \\
& 1.369955709042580254965844050909072881396600348644448209935 \ldots \\
& \mathbf{F}(\mathbf{q})=\mathbf{1 . 3 6 9 9 5 5 7 0 9} \ldots \\
& \text { We obtain: } \\
& \left(\left(\left(1+\left(0.449329^{\wedge} 2\right) /(1-0.449329)+(0.449329)^{\wedge} 8 /((1-0.449329)(1-\right.\right.\right. \\
& \left.\left.\left.\left.\left.0.449329^{\wedge} 3\right)\right)\right)\right)\right)^{*} 10^{\wedge} 16
\end{aligned}
$$

$$
\left(1+\frac{0.449329^{2}}{1-0.449329}+\frac{0.449329^{8}}{(1-0.449329)\left(1-0.449329^{3}\right)}\right) \times 10^{16}
$$

$1.3699557090425802549658440509090728813966003486444482 \ldots \times 10^{16}$
$1.369955709 \ldots * 10^{16} \quad$ value that is very near to the previous result $1.3616099 * 10^{16}$

We note also that:

|  | $m(\mathrm{MeV})$ | $J$ | $I$ | $I_{3}$ | $b$ | $q$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $n$ | 939 | $1 / 2$ | $1 / 2$ | $-1 / 2$ | 1 | 0 |
| $p$ | 939 | $1 / 2$ | $1 / 2$ | $1 / 2$ | 1 | 1 |
| $\Delta^{-}$ | 1232 | $3 / 2$ | $3 / 2$ | $-3 / 2$ | 1 | -1 |
| $\Delta^{0}$ | 1232 | $3 / 2$ | $3 / 2$ | $-1 / 2$ | 1 | 0 |
| $\Delta^{+}$ | 1232 | $3 / 2$ | $3 / 2$ | $1 / 2$ | 1 | 1 |
| $\Delta^{++}$ | 1232 | $3 / 2$ | $3 / 2$ | $3 / 2$ | 1 | 2 |
| $d^{*}$ | 2380 | 3 | 0 | 0 | 2 | 1 |

Table 1. Mass ( $m$ ), spin $(J)$, isospin $(I)$, isospin third component $\left(I_{3}\right)$, baryon number $(b)$ and electric charge $(q)$ of nucleons, $\Delta$ 's and the di-baryon $d^{*}(2380)$.

We have that:
$(1232+1232+1232+1232+939+939) /(((\operatorname{sqrt}(29 / 35)) * \mathrm{Pi}))$

## Input:

$1232+1232+1232+1232+939+939$

$$
\sqrt{\frac{29}{35}} \pi
$$

## Result:

$6806 \sqrt{\frac{35}{29}}$
$\pi$

## Decimal approximation:

2380.000757547933300770385848365778074047535143357220430745
$2380.0007575479 \ldots$ result very near to the d*-Hexaquark mass

## Property:

$\frac{6806 \sqrt{\frac{35}{29}}}{\pi}$ is a transcendental number

## Alternate form:

$\frac{6806 \sqrt{1015}}{29 \pi}$

## Series representations:

$$
\begin{aligned}
& \frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35}} \pi}=\frac{6806}{\sqrt{\frac{1232+1232+1232+1232+939+939}{\frac{29}{35}} \pi}}=\frac{\pi \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{6}{35}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j}\left(-\frac{6}{35}\right)^{-5} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)} \\
& \frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35}} \pi}=\frac{13612 \sqrt{\pi}}{\pi \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{29}{35}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{aligned}
$$

for $\left(\operatorname{not}\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )

And:
$3((((1232+1232+1232+1232+939+939) /(((\operatorname{sqrt}(29 / 35)) * \mathrm{Pi})))))^{\wedge} 1 / 2-7$

## Input:

$3 \sqrt{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35}} \pi}}-7$

## Exact result:

$3 \sqrt[4]{\frac{35}{29}} \sqrt{\frac{6806}{\pi}}-7$

## Decimal approximation:

139.3557543041318198341464210544885863364980538369793985382...
$139.3557543 \ldots$ result practically equal to the rest mass of Pion meson 139.57 MeV

## Property:

$-7+3 \sqrt[4]{\frac{35}{29}} \sqrt{\frac{6806}{\pi}}$ is a transcendental number

## Alternate form:

$\frac{1}{29}\left(3 \times 29^{3 / 4} \sqrt[4]{35} \sqrt{\frac{6806}{\pi}}-203\right)$

## Series representations:

$3 \sqrt{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35} \pi}}}-7=$
$-7+3 \sqrt{6806} \sqrt{\frac{1}{\pi \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{6}{35}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}}$

$$
\begin{aligned}
& 3 \sqrt{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35} \pi}}-7=} \\
& -7+6 \sqrt{3403} \sqrt{-\frac{\sqrt{\pi}}{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j}\left(-\frac{6}{35}\right)^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}}
\end{aligned}
$$

$$
\begin{aligned}
& 3 \sqrt{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35}} \pi}-7=} \\
& -7+3 \sqrt{6806} \sqrt{\frac{1}{\pi \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{29}{35}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}}
\end{aligned}
$$

for ( $\operatorname{not}\left(z_{0} \in \mathbb{R}\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )

## $3((((1232+1232+1232+1232+939+939) /(((\operatorname{sqrt}(29 / 35)) * \mathrm{Pi})))))^{\wedge} 1 / 2-18-\mathrm{Pi}$

## Input:

$3 \sqrt{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35}} \pi}}-18-\pi$

## Exact result:

$-18+3 \sqrt[4]{\frac{35}{29}} \sqrt{\frac{6806}{\pi}}-\pi$

## Decimal approximation:

125.2141616505420265956837776712090834523008844376042927172...
125.2141616505... result very near to the Higgs boson mass 125.18 GeV

## Alternate forms:

$$
\begin{aligned}
& \frac{1}{29}\left(-522+3 \times 29^{3 / 4} \sqrt[4]{35} \sqrt{\frac{6806}{\pi}}-29 \pi\right) \\
& \frac{3}{29}\left(29^{3 / 4} \sqrt[4]{35} \sqrt{\frac{6806}{\pi}}-174\right)-\pi
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 3 \sqrt{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35} \pi}}}-18-\pi= \\
& -18-\pi+3 \sqrt{6806} \sqrt{\frac{1}{\pi \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{6}{35}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}} \\
& 3 \sqrt{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35} \pi}}}-18-\pi= \\
& -18-\pi+6 \sqrt{3403} \sqrt{-\frac{\sqrt{\pi}}{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j}\left(-\frac{6}{35}\right)^{-5} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}}
\end{aligned}
$$

3

$$
\begin{aligned}
& \frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35}} \pi}-18-\pi= \\
& -18-\pi+3 \sqrt{6806} \sqrt{\frac{1}{\pi \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{\left.(-1)^{k}\left(-\frac{1}{2}\right)\right)_{k}\left(\frac{29}{35}-z_{0}\right)^{k} z_{0}^{k}}{k!}}}
\end{aligned}
$$

for ( $\operatorname{not}\left(z_{0} \in \mathbb{R}\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )
$27 * 1 / 2\left(\left(\left(3((((1232+1232+1232+1232+939+939) /(((\operatorname{sqrt}(29 / 35)) * P i)))))^{\wedge} 1 / 2-18\right)\right)\right)-$ $4+1 / 5$

## Input:

$27 \times \frac{1}{2}\left(3 \sqrt{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35}} \pi}}-18\right)-4+\frac{1}{5}$

## Exact result:

$\frac{27}{2}\left(3 \sqrt[4]{\frac{35}{29}} \sqrt{\frac{6806}{\pi}}-18\right)-\frac{19}{5}$

## Decimal approximation:

1729.002683105779567760976684235595915542723726799221880266
1729.002683105...

This result is very near to the mass of candidate glueball $\mathbf{f}_{\mathbf{0}}(\mathbf{1 7 1 0 )}$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the $j$-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the GrossZagier theorem. The number 1728 is one less than the Hardy-Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):
"The fundamental group of the complex form, compact real form, or any algebraic version of $E_{6}$ is the cyclic group $\mathbf{Z} / 3 \mathbf{Z}$, and its outer automorphism group is the cyclic group $\boldsymbol{Z} / 2 \boldsymbol{Z}$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, $E_{6}$ plays a role in some grand unified theories".

## Property:

$-\frac{19}{5}+\frac{27}{2}\left(-18+3 \sqrt[4]{\frac{35}{29}} \sqrt{\frac{6806}{\pi}}\right)$ is a transcendental number

## Alternate forms:

$81 \sqrt[4]{\frac{35}{29}} \sqrt{\frac{3403}{2 \pi}}-\frac{1234}{5}$
$\frac{1}{10}\left(405 \sqrt[4]{\frac{35}{29}} \sqrt{\frac{6806}{\pi}}-2468\right)$
$\frac{1}{290}\left(405 \times 29^{3 / 4} \sqrt[4]{35} \sqrt{\frac{6806}{\pi}}-71572\right)$

Series representations:

$$
\begin{aligned}
& \frac{27}{2}\left(\sqrt[3]{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35} \pi}}}-18\right)-4+\frac{1}{5}= \\
& -\frac{1234}{5}+81 \sqrt{\frac{3403}{2}} \sqrt{\frac{1}{\pi \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{6}{35}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{27}{2}\left(\sqrt[3]{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35}} \pi}}-18\right)-4+\frac{1}{5}
\end{aligned}=
$$

$$
\frac{27}{2}\left(\sqrt[3]{\frac{1232+1232+1232+1232+939+939}{\sqrt{\frac{29}{35}} \pi}}-18\right)-4+\frac{1}{5}=
$$

$$
-\frac{1234}{5}+81 \sqrt{\frac{3403}{2}} \sqrt{\frac{1}{\pi \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{29}{3!}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}}
$$

for ( $\operatorname{not}\left(z_{0} \in \mathbb{R}\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )

From:
A new possibility for light-quark dark matter
M Bashkanov and D P Watts
Department of Physics, University of York, Heslington, York, Y010 5DD, United Kingdom E-mail: mikhail.bashkanov@york.ac.uk and daniel.watts@york.ac.uk Received 1 July 2019, revised 11 December 2019 - Accepted for publication 6 January 2020 - Published 12 February 2020
the drop model then gives the binding energy per
$d^{*}(2380)(B / D)$ in a linear (lin) and spherical (sph) BEC configurations as:

$$
\begin{array}{ll}
B / D=a_{v}^{\prime} \cdot(D-1)-a_{c}^{\prime} \cdot \frac{(D-1)}{D} ; & a_{c}^{\prime}(\mathrm{MeV})=0.064 \cdot \frac{\rho}{\rho_{0}} ; \\
B / D=a_{v}^{\prime} \cdot(D-1)-a_{c}^{\prime} \cdot \frac{(D-1)}{D^{1 / 3}} ; & a_{c}^{\prime}(\mathrm{MeV})=0.73 \cdot \sqrt[3]{\frac{\rho}{\rho_{0}}} ; \quad(\mathrm{sph}) \tag{2}
\end{array}
$$

$\mathrm{BEC}=$ Bose - Einstein condensates
We have that:

$$
a_{c}^{\prime}(\mathrm{MeV})=0.73 \cdot \sqrt[3]{\frac{\rho}{\rho_{0}}} ; \quad(\mathrm{sph})
$$

$\left(\left(\left(0.73 *(10)^{\wedge} 1 / 3\right)\right)\right)$

## Input:

$0.73 \sqrt[3]{10}$

## Result:

1.572737323723275116884284303559125861539321807800239265217...
1.57273732372...
$1+1 /\left(\left(\left(0.73 *(10)^{\wedge} 1 / 3\right)\right)\right)-18 / 10^{\wedge} 3$

## Input:

$1+\frac{1}{0.73 \sqrt[3]{10}}-\frac{18}{10^{3}}$

## Result:

1.617834086796271081152065253550609120075527277398800498308 .
$1.61783408679 \ldots$ result that is a very good approximation to the value of the golden ratio 1.618033988749...

From:

$$
\begin{equation*}
a_{c}^{\prime}(\mathrm{MeV})=0.064 \cdot \frac{\rho}{\rho_{0}} \tag{lin}
\end{equation*}
$$

We obtain:
$(0.064 * 10)$

## Input:

$0.064 \times 10$

## Result:

0.64
0.64
$1+(0.064 * 10)$

## Input:

$1+0.064 \times 10$

## Result:

1.64
$1.64 \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

From

## Dark Matter in the Standard Model?

Christian Gross, Antonello Polosa, Alessandro Strumia, Alfredo Urbano, Wei Xue arXiv:1803.10242v2 [hep-ph] 3 Sep 2018

We have that:

At $N_{\text {in }} \approx 20$ e-folds before the end of inflation, the Higgs background value $h$ is brought by quantum fluctuation to some $h_{\text {in }} \neq 0$. This configuration must be spatially homogeneous on an inflating local patch large enough to encompass our observable Universe today.

From:

$$
N_{\text {in }}>\frac{1}{2} \ln \left[7.2 \times 10^{21} \frac{H_{0}}{\bar{M}_{\mathrm{Pl}}}\right]=18.3 \text { for } H_{0}=10^{-6} \bar{M}_{\mathrm{Pl}} .
$$

we obtain:
$1 / 2 \ln (7.2 \mathrm{e}+15)$

## Input interpretation:

$\frac{1}{2} \log \left(7.2 \times 10^{15}\right)$
$\log (x)$ is the natural logarithm

## Result:

18.2564287104663474...
18.2564287104663474... (very near to the Lucas number 18)

From which:
$30 /(((1 / 2 \ln (7.2 \mathrm{e}+15))))-(18+7) / 10^{\wedge} 3$
Input interpretation:
$\frac{30}{\frac{1}{2} \log \left(7.2 \times 10^{15}\right)}-\frac{18+7}{10^{3}}$

## Result:

1.61825676591945410 ..
$1.61825676591945410 \ldots .$. result that is a very good approximation to the value of the golden ratio 1.618033988749...

## Now, we have that:



Figure 8: Left: Sample evolution of the classical Higgs background ( $h_{\mathrm{cl}}$, red solid line) and of a perturbation with $k / a_{\text {in }} H_{0}=10^{2}$ (dashed lines). Right: Higgs curvature perturbation $\zeta_{h}^{(k)}$ during inflation. We compare the full numerical result with the analytical approximation (last term in eq. (52), solid horizontal black line). The vertical dashed gray line marks the instant of horizon exit. We use the analytical approximation in eq. (32) with $h_{\mathrm{cr}}=410^{12} \mathrm{GeV}$, $b=0.09 /(4 \pi)^{2}$ (which corresponds to $M_{t}=172 \mathrm{GeV}$ ) and $H_{0}=10^{12} \mathrm{GeV}$.

We will show precise numerical results for the SM case. However the discussion is clarified by introducing a simple approximation that encodes the main features of the SM effective potential in eq. (31):

$$
\begin{equation*}
V_{\mathrm{eff}}(h) \approx-b \log \left(\frac{h^{2}}{h_{\mathrm{cr}}^{2} \sqrt{e}}\right) \frac{h^{4}}{4}-6 \xi_{H} H_{0}^{2} h^{2}, \tag{32}
\end{equation*}
$$

where $h_{\text {cr }}$ is the position of the maximum of the potential with no extra mass term, $\xi_{H}=0$. The parameters $b$ and $h_{\text {cr }}$ depend on the low-energy SM parameters such as the top mass: they can be computed by matching the numerical value of the Higgs effective potential at the gauge-invariant position of the maximum, $V_{\text {eff }}\left(h_{\text {cr }}\right)=b h_{\text {cr }}^{4} / 8$. The result is shown in the right panel of fig. 7 .

From (32):

## For

$\xi_{H}=-10^{-3}$
$\mathrm{e} \approx 20$
$h \gg 174 \mathrm{GeV}=2112=$ rest mass of strange D meson
we obtain:
$-0.09 /\left(4^{*} \mathrm{Pi}\right)^{\wedge} 2 \ln \left(\left(\left(\left(2112^{\wedge} 2\right) /\left(\left(\left(4^{*} 10^{\wedge} 12\right)^{\wedge} 2^{*} \operatorname{sqrt} 20\right)\right)\right)\right)\right)^{*}\left(2112^{\wedge} 4\right) / 4-6(-10)^{\wedge}-$ $3 *\left(10^{\wedge} 12\right)^{\wedge} 2^{*}\left(2112^{\wedge} 2\right)$

## Input interpretation:

$-\frac{0.09}{(4 \pi)^{2}} \log \left(\frac{2112^{2}}{\left(4 \times 10^{12}\right)^{2} \sqrt{20}}\right) \times \frac{2112^{4}}{4}-\frac{6\left(10^{12}\right)^{2} \times 2112^{2}}{(-10)^{3}}$

## Result:

$2.6763264000000000125364 \ldots \times 10^{28}$
$2.6763264 \ldots * 10^{28}$

From:
$V_{\mathrm{eff}}\left(h_{\mathrm{cr}}\right)=b h_{\mathrm{cr}}^{4} / 8$.
we obtain:
$1 / 8\left(\left(\left(0.09 /(4 * \mathrm{Pi})^{\wedge} 2^{*}\left(4^{*} 10^{\wedge} 12\right)^{\wedge} 4\right)\right)\right)$

Input interpretation:
$\frac{1}{8}\left(\frac{0.09}{(4 \pi)^{2}}\left(4 \times 10^{12}\right)^{4}\right)$

## Result:

$1.8237813055620798859898303377750975002784579441004378 \ldots \times 10^{46}$
$1.8237813055 \ldots * 10^{46}$

From:

The position of the potential barrier - defined by the field value where the effective potential has its maximum - strongly depends on the value of the top mass, on the non-minimal coupling to gravity, and, after inflation, on the temperature of the thermal bath which provides and extra mass term. For $\xi_{H} \neq 0$, the maximum of the Higgs potential gets shifted from $h_{\mathrm{cr}}$ to

$$
\begin{equation*}
h_{\max }=H_{0}\left[-\frac{b}{12 \xi_{H}} \mathcal{W}\left(\frac{-12 \xi_{H} H_{0}^{2}}{b h_{\mathrm{cr}}^{2}}\right)\right]^{-1 / 2}, \tag{33}
\end{equation*}
$$

where $\mathcal{W}(z)$ is the product-log function defined by $z=\mathcal{W} e^{\mathcal{W}}$. The condition

$$
\begin{equation*}
-12 \xi_{H} H_{0}^{2}>-\frac{b h_{\mathrm{cr}}^{2}}{e}, \tag{34}
\end{equation*}
$$

must be satisfied otherwise the effective mass is too negative and it erases the potential barrier, thus leading to a classical instability.
for

$$
\begin{aligned}
& \xi_{H}=-10^{-3} \\
& \mathrm{e} \approx 20
\end{aligned}
$$

we obtain:
$10^{\wedge} 12\left(\left(\left(\left(\left(-0.09 /(4 \mathrm{Pi})^{\wedge} 2 * 1 /\left(12 *(-10)^{\wedge}-3\right) * \mathrm{e}\left(\left(\left(-12(-10)^{\wedge}-3 *\left(10^{\wedge} 12\right)^{\wedge} 2\right)\right)\right) /\right.\right.\right.\right.\right.$ $\left.\left.\left.\left(\left(\left(0.09 /(4 \mathrm{Pi})^{\wedge} 2 *\left(4 * 10^{\wedge} 12\right)^{\wedge} 2\right)\right)\right)\right)\right)\right)^{\wedge}(-1 / 2)$

## Input interpretation:

$$
10^{12}\left(-\frac{0.09}{(4 \pi)^{2}} \times \frac{1}{\frac{12}{(-10)^{3}}} e \times \frac{-\frac{12\left(10^{12}\right)^{2}}{(-10)^{3}}}{\frac{0.09}{(4 \pi)^{2}}\left(4 \times 10^{12}\right)^{2}}\right)^{-1 / 2}
$$

## Result:

$2.4261226388505336944151981399647218137676725419487478 \ldots \times 10^{12}$
$2.4261226388 \ldots * 10^{12}$

From which:
$\left(\left(\left(10^{\wedge} 12\left(\left(\left(\left(\left(-0.09 /(4 \mathrm{Pi})^{\wedge} 2^{*} 1 /\left(12^{*}(-10)^{\wedge}-3\right) *\right.\right.\right.\right.\right.\right.\right.\right.$ e $\left(\left(\left(-12(-10)^{\wedge}-3 *\left(10^{\wedge} 12\right)^{\wedge} 2\right)\right)\right) /$ $\left.\left.\left.\left.\left.\left.\left.\left.\left(\left(\left(0.09 /(4 \mathrm{Pi})^{\wedge} 2 *\left(4^{*} 10^{\wedge} 12\right)^{\wedge} 2\right)\right)\right)\right)\right)\right)^{\wedge}(-1 / 2)\right)\right)\right)\right)\right)^{\wedge} 2$

## Input interpretation:

$$
\left(10^{12}\left(-\frac{0.09}{(4 \pi)^{2}} \times \frac{1}{\frac{12}{(-10)^{3}}} e \times \frac{-\frac{12\left(10^{12}\right)^{2}}{(-10)^{3}}}{\frac{0.09}{(4 \pi)^{2}}\left(4 \times 10^{12}\right)^{2}}\right)^{-1 / 2}\right)^{2}
$$

## Result:

$5.8860710587430771455283803225833738791329780965082853 \ldots \times 10^{24}$
$5.88607105874 \ldots * 10^{24}=h_{\max }^{2}$

Dividing the (32) by this result, we obtain:
$\left(\left(\left(-0.09 /\left(4^{*} \mathrm{Pi}\right)^{\wedge} 2 \ln \left(\left(\left(\left(2112^{\wedge} 2\right) /\left(\left(\left(4^{*} 10^{\wedge} 12\right)^{\wedge} 2^{*} \mathrm{sqrt20}\right)\right)\right)\right)\right)^{*}\left(2112^{\wedge} 4\right) / 4-6(-10)^{\wedge}-\right.\right.\right.$ $\left.\left.\left.3 *\left(10^{\wedge} 12\right)^{\wedge} 2^{*}\left(2112^{\wedge} 2\right)\right)\right)\right) / 5.886071058743077 \times 10^{\wedge} 24$

## Input interpretation:

$$
-\frac{0.09}{(4 \pi)^{2}} \log \left(\frac{2112^{2}}{\left(4 \times 10^{12}\right)^{2} \sqrt{20}}\right) \times \frac{2112^{4}}{4}-\frac{6\left(10^{12}\right)^{2} \times 2112^{2}}{(-10)^{3}}
$$

$5.886071058743077 \times 10^{24}$

## Result:

4546.880887590759...
4546.880887590759...

From which:

1/golden ratio $+11+4+1 / 3\left(\left(\left(-0.09 /(4 * \mathrm{Pi})^{\wedge} 2 \ln \right.\right.\right.$ $\left.\left.\left(\left(\left(\left(2112^{\wedge} 2\right) /\left(\left(\left(4^{*} 10^{\wedge} 12\right)^{\wedge} 2^{*} \operatorname{sqrt20)}\right)\right)\right)\right)^{*}\left(2112^{\wedge} 4\right) / 4-6(-10)^{\wedge}-3^{*}\left(10^{\wedge} 12\right)^{\wedge} 2^{*}\left(2112^{\wedge} 2\right)\right)\right)\right) /$ $5.886071058743077 \times 10^{\wedge} 24$

## Input interpretation:

$\frac{1}{\phi}+11+4+\frac{1}{3} \times \frac{-\frac{0.09}{(4 \pi)^{2}} \log \left(\frac{2112^{2}}{\left(4 \times 10^{12}\right)^{2} \sqrt{20}}\right) \times \frac{2112^{4}}{4}-\frac{6\left(10^{12}\right)^{2} \times 2112^{2}}{(-10)^{3}}}{5.886071058743077 \times 10^{24}}$

## Result:

1531.244996519003...
1531.244996519003... result practically equal to the rest mass of Xi baryon 1531.80

Now, we have that:

Finally, we can now compute the mass and amount of PBH generated by Higgs fluctuations. The radius of hubble horizon or the wavelengh of the modes determines the typical mass of the PBHs [36]:

$$
\begin{equation*}
M_{\mathrm{PBH}} \approx \frac{\gamma}{2} \frac{M_{\mathrm{Pl}}^{2}}{H_{0}} e^{2 N}, \tag{60}
\end{equation*}
$$

where $N$ is the number of $e$-folds when the $k$-mode leave the horizon; $\gamma \approx 0.2$ is a correction factor [37]. For example $H_{0}=10^{12} \mathrm{GeV}$ and $N=20$ gives $M_{\mathrm{PBH}} \approx 10^{-15} M_{\odot}$.
$\left(\left(0.2 * \mathrm{e}^{\wedge}(40)^{*}(2.435 \mathrm{e}+18)^{\wedge} 2\right)\right) /\left(\left(2 * 10^{\wedge} 12\right)\right)$

## Input interpretation:

$\frac{0.2 e^{40}\left(2.435 \times 10^{18}\right)^{2}}{2 \times 10^{12}}$

## Result:

$1.39565 \ldots \times 10^{41}$
$1.39565 \ldots * 10^{41}$

$$
\begin{gathered}
F^{\prime}(q)=1+\frac{q^{2}}{1-q}+\frac{q^{8}}{(1-q)\left(1-q^{3}\right)}+\ldots \\
\phi(-q)+\chi(q)=2 F^{\prime}(q)
\end{gathered}
$$

$1+\left(0.449329^{\wedge} 2\right) /(1-0.449329)+(0.449329)^{\wedge} 8 /\left((1-0.449329)\left(1-0.449329^{\wedge} 3\right)\right)$

## Input interpretation:

$1+\frac{0.449329^{2}}{1-0.449329}+\frac{0.449329^{8}}{(1-0.449329)\left(1-0.449329^{3}\right)}$

## Result:

1.369955709042580254965844050909072881396600348644448209935...

Open code
$F(q)=1.369955709 . .$.
$\left(\left(1+(0.449329 \wedge 2) /(1-0.449329)+(0.449329)^{\wedge} 8 /((1-0.449329)(1-\right.\right.$
$\left.\left.\left.\left.0.449329^{\wedge} 3\right)\right)\right)\right)^{*} 10^{\wedge} 41$

Input interpretation:
$\left(1+\frac{0.449329^{2}}{1-0.449329}+\frac{0.449329^{8}}{(1-0.449329)\left(1-0.449329^{3}\right)}\right) \times 10^{41}$

## Result:

$1.3699557090425802549658440509090728813966003486444482 \ldots \times 10^{41}$
$1.369955709 \ldots * 10^{41}$

From
$\left(\left(0.2 * \mathrm{e}^{\wedge}(40)^{*}(2.435 \mathrm{e}+18)^{\wedge} 2\right)\right) /\left(\left(2 * 10^{\wedge} 12\right)\right)$
we obtain also:
$\left[\ln \left(\left(\left(\left(0.2^{*} \mathrm{e}^{\wedge}(40) *(2.435 \mathrm{e}+18)^{\wedge} 2\right)\right) /\left(\left(2 * 10^{\wedge} 12\right)\right)\right)\right)\right]^{\wedge} 1 / 9$

## Input interpretation:

$\sqrt[9]{\log \left(\frac{0.2 e^{40}\left(2.435 \times 10^{18}\right)^{2}}{2 \times 10^{12}}\right)}$
$\log (x)$ is the natural logarithm

## Result:

1.65811439...
$1.65811439 \ldots$. result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505} / G_{101 / 5}\right)^{3}=1164,2696$ i.e. $1,65578 \ldots$
$\left[\ln \left(\left(\left(\left(0.2^{*} \mathrm{e}^{\wedge}(40)^{*}(2.435 \mathrm{e}+18)^{\wedge} 2\right)\right) /\left(\left(2^{*} 10^{\wedge} 12\right)\right)\right)\right)\right]^{\wedge} 1 / 9-(29+11)^{*} 1 / 10^{\wedge} 3$

## Input interpretation:

$\sqrt[9]{\log \left(\frac{0.2 e^{40}\left(2.435 \times 10^{18}\right)^{2}}{2 \times 10^{12}}\right)}-(29+11) \times \frac{1}{10^{3}}$
$\log (x)$ is the natural logarithm

## Result:

1.61811439...
$1.61811439 \ldots$ result that is a very good approximation to the value of the golden ratio 1.618033988749...

## From: Manuscript Book 2 of Srinivasa Ramanujan

Page 131

$(((1+(1 * 3) /(16(2+1))+(1 * 3 * 5 * 7) /(16 * 32(2+1)(2+3)))))$

## Input:

$1+\frac{1 \times 3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}$

## Exact result:

551
$\overline{512}$
Decimal form:
1.076171875
1.076171875

And:
$1 /(89+13+5)(((1+(1 * 3) /(16(2+1))+(1 * 3 * 5 * 7) /(16 * 32(2+1)(2+3)))))$

## Input:

$\frac{1}{89+13+5}\left(1+\frac{1 \times 3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right)$

## Exact result:

## $\frac{551}{54784}$

## Decimal approximation:

0.010057681074766355140186915887850467289719626168224299065 .
0.010057681074.....
$\left(((7 \mathrm{e}) / 55)^{\wedge}(1 / 4) \operatorname{sqrt}(3 /(2 \pi))\right)^{*}$
$(((1+(1 * 3) /(16(2+1))+(1 * 3 * 5 * 7) /(16 * 32(2+1)(2+3)))))$

## Input:

$\left(\sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}\right)\left(1+\frac{1 \times 3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right)$

## Exact result:

$\frac{551}{512} \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}$

## Decimal approximation:

0.570308427395712852444384563045926563627105731731972819798 .
$0.5703084273957 \ldots$.

## Series representations:

$$
\begin{aligned}
& \left(1+\frac{3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right) \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}= \\
& \frac{551}{512} \sqrt[4]{\frac{7}{55}} \sqrt[4]{e} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+\frac{3}{2 \pi}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}
\end{aligned}
$$

$$
\begin{array}{r}
\left(1+\frac{3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right) \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}= \\
\quad \frac{551}{512} \sqrt[4]{\frac{7}{55}} \sqrt[4]{e} \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{3}{2 \pi}-z_{0}\right)^{k} z_{0}^{-k}}{k!}
\end{array}
$$

for ( $\operatorname{not}\left(z_{0} \in \mathbb{R}\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )

$$
\begin{aligned}
& \left(1+\frac{3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right) \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}=\frac{551}{512} \sqrt[4]{\frac{7}{55}} \sqrt[4]{e} \\
& \left.\quad \exp \left(i \pi \left\lvert\, \frac{\arg \left(\frac{3}{2 \pi}-x\right)}{2 \pi}\right.\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(\frac{3}{2 \pi}-x\right)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!} \text { for }(x \in \mathbb{R} \text { and } x<0)
\end{aligned}
$$

$1 /\left(\left(\left(\left(((7 \mathrm{e}) / 55)^{\wedge}(1 / 4) \operatorname{sqrt}(3 /(2 \pi))\right)^{*}\right.\right.\right.$
$(((1+(1 * 3) /(16(2+1))+(1 * 3 * 5 * 7) /(16 * 32(2+1)(2+3))))))))-21 / 10^{\wedge} 3$

## Input:

$$
\frac{1}{\left(\sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}\right)\left(1+\frac{1 \times 3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right)}-\frac{21}{10^{3}}
$$

## Exact result:

$$
\frac{512}{551} \sqrt[4]{\frac{55}{7 e}} \sqrt{\frac{2 \pi}{3}}-\frac{21}{1000}
$$

## Decimal approximation:

1.732437178837517637417220623589237697969043359275249906221...
$1.7324371788375 \ldots \approx \sqrt{3}$ that is the ratio between the gravitating mass $\mathrm{M}_{0}$ and the Wheelerian mass q

$$
\begin{aligned}
M_{0} & =\sqrt{3 q^{2}-\Sigma^{2}}, \\
q & =\frac{(3 \sqrt{3}) M_{\mathrm{s}}}{2} .
\end{aligned}
$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? arXiv: 1909.13052 v 1 [gr-qc] 28 Sep 2019)

## Alternate forms:

$\frac{512000 \sqrt[4]{\frac{55}{7 e}} \sqrt{\frac{2 \pi}{3}}-11571}{551000}$
$\frac{512000 \times 7^{3 / 4} \sqrt[4]{\frac{55}{e}} \sqrt{6 \pi}-242991}{11571000}$

## Series representations:

$$
\begin{aligned}
& \frac{1}{\left(1+\frac{3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right) \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}}-\frac{21}{10^{3}}= \\
& -\frac{21}{1000}+\frac{512 \sqrt[4]{\frac{55}{7}}}{551 \sqrt[4]{e} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-1+\frac{3}{2 \pi}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}} \\
& \frac{\left(1+\frac{3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right) \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}}{1}-\frac{21}{10^{3}}= \\
& 512 \\
& -\frac{21}{1000}+\frac{\sqrt[4]{\frac{55}{7}}}{551 \sqrt[4]{e} \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{3}{2 \pi}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{aligned}
$$

$$
\text { for }\left(\operatorname{not}\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
$$

$$
\begin{aligned}
& \frac{1}{\left(1+\frac{3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right) \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}}-\frac{21}{10^{3}}= \\
& -\frac{21}{1000}-\frac{1024 \sqrt[4]{\frac{55}{7}} \sqrt{\pi}}{551 \sqrt[4]{e} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j}\left(-1+\frac{3}{2 \pi}\right)^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}
\end{aligned}
$$

$1 /\left(\left(((() \mathrm{e}) / 55)^{\wedge}(1 / 4) \operatorname{sqrt}(3 /(2 \pi))\right)\right)^{*}$
$(((1+(1 * 3) /(16(2+1))+(1 * 3 * 5 * 7) /(16 * 32(2+1)(2+3))))))))-(89+34+13) * 1 / 10^{\wedge} 3$
Input:
$\frac{1}{\left(\sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}\right)\left(1+\frac{1 \times 3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right)}-(89+34+13) \times \frac{1}{10^{3}}$

## Exact result:

$\frac{512}{551} \sqrt[4]{\frac{55}{7 e}} \sqrt{\frac{2 \pi}{3}}-\frac{17}{125}$

## Decimal approximation:

1.617437178837517637417220623589237697969043359275249906221...
1.6174371788375. result that is a very good approximation to the value of the golden ratio 1.618033988749...

## Alternate forms:

$\frac{64000 \sqrt[4]{\frac{55}{7 e}} \sqrt{\frac{2 \pi}{3}}-9367}{68875}$
$\frac{64000 \times 7^{3 / 4} \sqrt[4]{\frac{55}{e}} \sqrt{6 \pi}-196707}{1446375}$

## Series representations:

$$
\frac{1}{\left(1+\frac{3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right) \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}}-\frac{59+34+13}{10^{3}}=
$$

$$
\begin{gathered}
\frac{1}{\left(1+\frac{3}{16(2+1)}+\frac{35 \times 7}{16 \times 32(2+1)(2+3)}\right) \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}}-\frac{89+34+13}{10^{3}} \\
-\frac{17}{125}+\frac{512 \sqrt[4]{\frac{55}{7}}}{551 \sqrt[4]{e} \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(\frac{3}{2 \pi}-z_{0}\right)^{k} z_{0}^{-k}}{k!}}
\end{gathered}
$$

for ( $\operatorname{not}\left(z_{0} \in \mathbb{R}\right.$ and $\left.-\infty<z_{0} \leq 0\right)$ )

$$
\begin{gathered}
\frac{1}{\left(1+\frac{3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right) \sqrt[4]{\frac{7 e}{55}} \sqrt{\frac{3}{2 \pi}}}-\frac{89+34+13}{10^{3}}= \\
-\frac{17}{125}-\frac{1024 \sqrt[4]{\frac{55}{7}} \sqrt{\pi}}{551 \sqrt[4]{e} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j}\left(-1+\frac{3}{2 \pi}\right)^{-5} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}
\end{gathered}
$$

or:
$(((1+(1 * 3) /(16(2+1))+(1 * 3 * 5 * 7) /(16 * 32(2+1)(2+3)))))^{\wedge} 7-(47+7) 1 / 10^{\wedge} 3$
Input:
$\left(1+\frac{1 \times 3}{16(2+1)}+\frac{3 \times 5 \times 7}{16 \times 32(2+1)(2+3)}\right)^{7}-(47+7) \times \frac{1}{10^{3}}$

## Exact result:

1865139355483821682171
$\overline{1152921504606846976000}$

## Decimal approximation:

1.617750512963018404639752023665977276323246769607067108154...
$1.617750512963 \ldots$ result that is a very good approximation to the value of the golden ratio 1.618033988749...

## Alternate form:

1865139355483821682171
1152921504606846976000

## Observations

Figs.


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, $\phi$. Red arrows show the classical motion of $\phi$. When $\phi$ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_{f}$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of $\phi$ : Even near regions (b) and (d), $\phi$ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," $\rho$ remains nearly constant. Only after $\phi$ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.


The ratio between $\mathrm{M}_{0}$ and q

$$
\begin{aligned}
M_{0} & =\sqrt{3 q^{2}-\Sigma^{2}}, \\
q & =\frac{(3 \sqrt{3}) M_{\mathrm{s}}}{2} .
\end{aligned}
$$

i.e. the gravitating mass $\mathrm{M}_{0}$ and the Wheelerian mass q of the wormhole, is equal to:
$\frac{\sqrt{3\left(2.17049 \times 10^{37}\right)^{2}-0.001^{2}}}{\frac{1}{2}\left((3 \sqrt{3})\left(4.2 \times 10^{6} \times 1.9891 \times 10^{30}\right)\right)}$
1.732050787905194420703947625671018160083566548802082460520...
1.7320507879
$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass $\mathrm{M}_{0}$ and the Wheelerian mass $q$ of the wormhole

We note that:
$\left(-\frac{1}{2}+\frac{i}{2} \sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2} \sqrt{3}\right)$
$i \sqrt{3}$
$1.732050807568877293527446341505872366942805253810380628055 \ldots i$
$r \approx 1.73205$ (radius), $\theta=90^{\circ}$ (angle)
1.73205

This result is very near to the ratio between $\mathrm{M}_{0}$ and q , that is equal to 1.7320507879 $\approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result
$\left(-\frac{1}{2}+\frac{i}{2} \sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2} \sqrt{3}\right)=i \sqrt{3}=$
$=1.732050807568877293527446341505872366942805253810380628055$ $i$
$r \approx 1.73205$ (radius), $\quad \theta=90^{\circ}$ (angle)
can be related with:
$u^{2}(-u)\left(\frac{1}{2} \pm \frac{i \sqrt{3}}{2}\right)+v^{2}(-v)\left(\frac{1}{2} \pm \frac{i \sqrt{3}}{2}\right)=q$

Considering:
$(-1)\left(\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)=q$
$=i \sqrt{3}=1.732050807568877293527446341505872366942805253810380628055 \ldots i$
$r \approx 1.73205$ (radius), $\quad \theta=90^{\circ}$ (angle)
Thence:
$\left(-\frac{1}{2}+\frac{i}{2} \sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2} \sqrt{3}\right) \Rightarrow$
$\Rightarrow(-1)\left(\frac{1}{2}-\frac{i \sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i \sqrt{3}}{2}\right)=q=1.73205 \approx \sqrt{3}$


#### Abstract

We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.


From:
https://www.scientificamerican.com/article/mathematics-
ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA
Ramanujan's statement concerned the deceptively simple concept of partitions-the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9)=30, p(9+5)=135, p(9+10)=490, p(9+15)=1,575$ and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11 -there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by $5^{\wedge} 3=125$ units, saying that the corresponding $p(n)$ 's should all be divisible by 125 . In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

## From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field $\phi$ and a Dirac field $\psi$. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between
the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T=0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$
g_{22}=\sqrt{(1+\sqrt{2})}
$$

Hence

$$
\begin{array}{rlr}
64 g_{22}^{24} & = & e^{\pi \sqrt{22}}-24+276 e^{-\pi \sqrt{22}}-\cdots \\
64 g_{22}^{-24} & = & 4096 e^{-\pi \sqrt{22}}+\cdots
\end{array}
$$

so that

$$
64\left(g_{22}^{24}+g_{22}^{-24}\right)=e^{\pi \sqrt{22}}-24+4372 e^{-\pi \sqrt{22}}+\cdots=64\left\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\right\}
$$

Hence

$$
e^{\pi \sqrt{22}}=2508951.9982 \ldots
$$

Thence:

$$
64 g_{22}^{-24}=\quad 4096 e^{-\pi \sqrt{22}}+\cdots
$$

And

$$
64\left(g_{22}^{24}+g_{22}^{-24}\right)=e^{\pi \sqrt{22}}-24+4372 e^{-\pi \sqrt{22}}+\cdots=64\left\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\right\}
$$

That are connected with $64,128,256,512,1024$ and $4096=64^{2}$
(Modular equations and approximations to $\pi-S$. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350-372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants $\pi, \phi, 1 / \phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted $F_{n}$, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of $n$ and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

```
0,1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946,
17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309,
3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155\ldots
```

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842-91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio. ${ }^{[1]}$ The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.
The sequence of Lucas numbers is:

```
2,1,3,4,7,11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127,
24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349,
4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803......
```

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiralwhose growth factor is $\varphi$, the golden ratio. ${ }^{[1]}$ That is, a golden spiral gets wider (or further from its origin) by a factor of $\varphi$ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies ${ }^{[3]}$ - golden spirals are one special case of these logarithmic spirals

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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Manuscript Book 2 of Srinivasa Ramanujan


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