Magnetic white dwarfs and gravitomagnetism

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ABSTRACT

The origin of stellar magnetic fields is still uncertain. For decades a fossil field mechanism has been postulated for white dwarfs, based on magnetic flux conservation of Ap or Bp stars. Later on, dynamo action in the common envelope of binary systems has been proposed as an origin of magnetic fields in white dwarfs. Recently, dynamo action in the convective region of isolated white dwarfs has been considered as an explanation.

In this work a gravitational origin for the magnetic fields of rotating massive bodies is reinvestigated. This approach has led to the so-called Wilson-Blackett law that predicts a dipolar magnetic field for all rotating electrically neutral bodies. A short review of the history of this law is given. The validity of the Wilson-Blackett formula for white dwarfs will be examined in this work.

Results are given for ten isolated white dwarfs, eleven AM Herculis systems, one DQ Herculis system and two double-white-dwarf binaries. In most cases only approximate agreement with the predictions of the Wilson-Blackett formula is found. Contributions from electromagnetic origin may be responsible for the deviations. The results for white dwarfs are compared with corresponding classes of pulsars.

1. INTRODUCTION

Attempts to explain the origin of the magnetic fields of celestial bodies has gone through a long and turbulent history. Already in 1891, Schuster [1], considering the magnetic field of the Earth and the Sun, put the question: “Is every large rotating mass a magnet?” He suggested that every moving molecule causes a magnetic field, as if it was electrically charged. Following Schuster, Wilson [2] proposed that electrically neutral matter moving matter bears a residual charge Q^* of magnitude $\beta\, G^{½}\, m$, where β is a dimensionless constant, G is the gravitational constant and m the mass of the moving body (throughout this paper Gaussian units are used). He tried to measure the magnetic field of a swinging bar in the laboratory, but he found no measurable magnetic field. In this experiment he tested his assumption in the case of translational motion, not for rotational motion. Applying the theory of electromagnetism to a massive rotating sphere like the Earth, he implicitly found an approximate form of the relation

$$\mathbf{M} = -\tfrac{1}{2}\beta c^{-1} G^{½}\mathbf{S}, \tag{1.1}$$

where c is the velocity of light, $\mathbf{M}$ the magnetic dipole moment and $\mathbf{S}$ the angular momentum of the rotating sphere.

In 1947 Blackett [3] again considered a gravitational origin of the magnetic field of rotating celestial bodies. He explicitly proposed relation (1.1) and calculated a value of β of 0.3, 1.14 and 1.16 for the Earth, the Sun and the Ap star 78 Virginis, respectively. In addition, Blackett [4] tried to measure the magnetic field of a 10×10 cm gold cylinder, at rest in the laboratory and so rotating with the Earth, but he detected no measurable field.

Since 1977 the validity of (1.1) was reinvestigated by several authors, when magnetic fields of more planets and stars were reported. Ahluwalia and Wu [5] and Sirag [6] extended the series of celestial bodies approximately obeying to (1.1) and proposed to measure the possible magnetic field generated by a rotating metallic sphere in the laboratory, a test already discussed by Blackett [4].

Such an experiment was performed by Surdin [7, 8], who measured an average value of the square of the magnetic field generated by a rotating cylinder of brass and of tungsten, respectively. The magnetic field squared appeared to be in reasonable agreement with the field predicted by (1.1), but the sign did not follow from the experiment. Moreover, the observed field appeared to fluctuate.

Attempts to derive relation (1.1) from a more general theory have been made by several authors [9–15]. Luchak [9], for example, generalized the Maxwell equations by introduction of a gravitational field. Considering rotational motion only, he obtained (1.1). Following Surdin [8] and Luchak [9], this relation will be denoted as the Wilson-Blackett law. A number of authors [10–15] tried to explain equation (1.1) as a consequence of general relativity. For example, it appeared possible to deduce (1.1) from a special version of the gravitomagnetic theory [10–12]. In this approach the so-called "magnetic-type" gravitational field is identified as a *common* magnetic field, resulting into the (gravito)magnetic dipole moment **M**(gm) = **M** of (1.1).

The angular momentum **S** in (1.1) for a spherical star of radius R can be calculated from

$$\mathbf{S} = I\mathbf{\Omega}_\mathrm{s}, \text{ or } S = I\,\Omega_\mathrm{s} = \tfrac{4}{5}\pi\, f\, mR^2 P_\mathrm{s}^{-1}, \tag{1.2}$$

where m is the mass of the star, $\Omega_\mathrm{s} = 2\pi P_\mathrm{s}^{-1}$ is its angular velocity (P_s is the rotational period of the star) and $I = 2/5\ f\ m\ R^2$ is its moment of inertia. The factor f is a dimensionless factor depending on the homogeneity of the mass density in the star (for a homogeneous mass density $f = 1$).

Furthermore, the value of the gravitomagnetic (and electromagnetic) dipole moment **M** is given by the expression

$$\mathbf{M} = \tfrac{1}{2} R^3 \mathbf{B}_\mathrm{p}, \text{ or } M = \tfrac{1}{2} R^3 B_\mathrm{p}. \tag{1.3}$$

Here $\mathbf{B}_\mathrm{p}$ is the magnetic induction field at, say, the north pole of the star at distance R from the centre of the star to the field point where $\mathbf{B}_\mathrm{p}$ is measured.

Combination of (1.1)–(1.3) yields the following gravitomagnetic prediction for $\mathbf{B}_\mathrm{p}$(gm) and B_p(gm), respectively

$$\mathbf{B}_\mathrm{p}(\mathrm{gm}) = -\beta c^{-1} G^{1/2} I R^{-3} \mathbf{\Omega}_\mathrm{s}, \text{ or } B_\mathrm{p}(\mathrm{gm}) = 2\pi c^{-1} G^{1/2} I R^{-3} P_\mathrm{s}^{-1} \text{ for } \beta = -1. \tag{1.4}$$

When β is negative, the directions of $\mathbf{B}_\mathrm{p}$(gm) and $\mathbf{\Omega}_\mathrm{s}$ are parallel. The sign and magnitude of β are unknown, however. See ref. [12] for an ample discussion of this issue.

As pointed out earlier [11], moving electric charge in the magnetic field from gravitomagnetic origin may cause an additional magnetic field from electromagnetic origin. It is stressed that the magnetic field generated by rotating neutral mass is generally much smaller than the magnetic field generated by moving charge. For a charge e ($e < 0$) and mass m one may compare the following magnetic moment to angular momentum ratios

$$\left(M/S\right)_{\mathrm{electromagnetic}} = \tfrac{1}{2} e\,(mc)^{-1} \text{ and } \left(M/S\right)_{\mathrm{gravitomagnetic}} = -\tfrac{1}{2}\beta c^{-1} G^{1/2}. \tag{1.5}$$

Choosing $\beta = -1$, calculation yields the following dimensionless ratio for an electron

$$(M/S)_{\text{gravitomagnetic}} \,/\, (M/S)_{\text{electromagnetic}} = G^{\frac{1}{2}} m/e = -4.899 \times 10^{-22}. \qquad (1.6)$$

From this relation follows, that magnetic fields from gravitomagnetic origin are usually extremely small and difficult to isolate from fields due to electric charges.

When both a magnetic induction field $\mathbf{B}_p$(gm) from gravitomagnetic origin and a field $\mathbf{B}_p$(em) from electromagnetic origin are present at the north pole of the white dwarf, the total polar magnetic induction field $\mathbf{B}_p$(tot) is given by (see [16])

$$\mathbf{B}_p(\text{tot}) = \mathbf{B}_p(\text{gm}) + \mathbf{B}_p(\text{em}). \qquad (1.7)$$

According to (1.4), the direction of $\mathbf{B}_p$(gm) is parallel to $\mathbf{\Omega}_s$ for $\beta = -1$. It appears helpful to introduce another dimensionless factor β^* following from observations

$$\mathbf{B}_p^{\parallel}(\text{tot}) = \beta^* \mathbf{B}_p(\text{gm}), \qquad (1.8)$$

where $\mathbf{B}_p^{\parallel}$(tot) is the component of parallel to $\mathbf{B}_p$(gm). Usually, the sign of the empirical factor β^* does not follow from observations. For convenience sake, we shall adopt a positive sign for β^* for that reason. When the field $\mathbf{B}_p^{\parallel}$(tot) would only be due to gravitomagnetic origin, $\mathbf{B}_p$(em) = 0, and the factor β^* would reduce to β^*(gm) = 1.

For decades the origin of the magnetic field of white dwarfs has been attributed to an evolutionary scenario. In the fossil field hypothesis, some fraction of the magnetic flux of a magnetically peculiar Ap or Bp star may be conserved producing the compressed magnetic field of a white dwarf. More recently, dynamo action has been used to explain the generation of magnetic fields of white dwarfs. These fields may be formed during the common envelope evolution of *binary* systems containing at least one white dwarf. For recent reviews, see, e.g., Ferrario, Melatos and Zrake [17] and Kawka [18]. Furthermore, Isern *et al.* [19] proposed that a magnetic field is generated by dynamo action in the convective region during the cooling of the *isolated* white dwarf. A related mechanism may be responsible for the magnetic fields of planets.

In section 2 observational data are summarized for ten isolated white dwarfs, eleven white dwarfs that synchronously accrete matter from a low-mass donor star, so-called AM Herculis stars, one white dwarf that asynchronously accretes matter from a donor star, a so-called DQ Herculis star and two double-white-dwarf binaries. Subsequently, values for the quantity β^* in (1.8) are calculated. In addition, in section 3 a discussion of the validity of the Wilson-Blackett formula for white dwarfs is given. Furthermore, in section 4 the calculated values of β^* for different classes of white dwarfs are compared with values for β^* of corresponding classes of pulsars. Conclusions are drawn in section 5.

2. MAGNETIC WHITE DWARFS

In this section data for the investigated white dwarfs are gathered from literature. In particular, values for the effective temperature T_{eff}, mass m, rotation period P_s, radius R and the absolute value of the observed total polar magnetic field B_p(tot) are summarized. See for a review of these data, e.g., Ferrario et al. [20]. In some cases values for R are estimated from the white dwarf mass-radius relation of Provencal *et al.* [21]. Taking $f = 1$, the values of the angular momentum S and the magnetic moment M are calculated from (1.2) and (1.3), respectively. Subsequently, the absolute values of factor β^* have been calculated by combining (1.4), (1.8) and the observed polar magnetic field $\mathbf{B}_p^{\parallel}$(tot), or $\mathbf{B}_p$(tot). The quantity β^* has unity value, when the Wilson-Blackett formula applies.

When data are available, estimates of the angle δ (0° ≤ δ ≤ 180°) between the directions of **M** and **S** are also given. Equation (1.1) predicts parallel directions for **M** and **S** for $\beta = -1$.

2.1 Isolated magnetic white dwarfs

In table 1 necessary data and calculated absolute values for angular momentum *S*, magnetic dipole moment *M* and factor β^* are summarized for ten isolated white dwarfs. The sequence of the stars is given in decreasing order of the rotation period P_s. Additional details of the observations and analyses of these white dwarfs are added:

1. WD 1953−011 (= G92–40).
Spectroscopic observations presented by Maxted *et al*. [22] showed Zeeman-split components and variations in time of the equivalent width of the Balmer Hα absorption lines. They constructed a model consisting of two components: a high field region with a nearly uniform field of strength 490 kG covering about 10 per cent of the surface area of the star, and a weak centred dipolar field with $B_p \approx 100$ kG. From more recent spectropolarimetric observations Valyavin *et al*. [23] also deduced a related best fit for a magnetic field consisting of a high- and a low-field component in the photosphere of the star. The strong-field component had a localized geometry (magnetic spot) of 515 kG that could not be understood as a high-field term in the multipolar expansion of the star's magnetic dipole and quadrupole. From the combined dipole and quadrupole they deduced a value of 178 kG for the polar magnetic field. Moreover, an angle 8º (or 172º) for the angle δ between the directions of the rotational axis and the direction of the dipolar magnetic field was calculated. Furthermore, they calculated a rotation period of 1.448 days for the star from the rotationally modulated low-field component.

2. G195–19 (= WD 0912+536).
Angel [24, 25] reported a rotation period of 1.331 days for this white dwarf from circular polarization measurements. In addition, a magnetic field of about 100 MG was calculated from polarimetry.

3. WD 2047+372 (= G210–36).
Landstreet *et al.* [27] found that the shape of the Zeeman split of the Hα line core is almost constant, but polarimetry revealed that the line of sight component of the magnetic field varied strongly. A simple dipolar model with a polar field B_p of 91.8 kG gave a good fit with an angle δ of 86.5º (or 93.5º), but a value of 27.0º (or 163º) for δ is also possible.

4. PG 1031+234 (= WD 1031–234).
Schmidt *et al*. [28] investigated this white dwarf by time-resolved Zeeman spectroscopy and polarimetry. From the Balmer Hα and Hβ lines and the Lyman Hα line they deduced a model with a centred magnetic dipole moment with a polar magnetic field of about 200 MG. An angle δ = 35º (or 145º) between the rotational and magnetic axes was obtained. Furthermore, a strong magnetic dipole moment was added to the model, displaced by $\Delta R = 0.4\ R$ from the stellar centre. The strongest polar field of the second dipole amounted to 500 MG. The added dipole resembles a strong "magnetic spot" containing primarily radial field lines. An angle δ = 55º between the directions of the rotational axis and the direction of the strong polar field was calculated. In addition, Brinkworth *et al*. [30] deduced a rotation period of 3.53 hr for this white dwarf from photometric variability.

5. PG 2329+267 (= WD 2329+267).
Moran *et al*. [31] deduced a centred dipolar magnetic field of strength 2.3 MG from linear Zeeman splitting and quadratic Zeeman shifts of the Hα Balmer lines. Furthermore, Brinkworth *et al*. [30] deduced a rotation period of 2.767 hr from photometric variabilities.

Table 1. Isolated white dwarfs.

	Name star main comp. [references]	$T_{\rm eff}$ (K)	m (g) ($m_\odot$)	R (cm)	P_s (s) (min)	S (g.cm^2.s^{-1})	B_p(tot) (G)	M (G.cm^3)	β*	δ (º)
1	WD 1953–011 H [22, 23]	7,920 [20]	1.68×10^{33} 0.844 [22]	6.6×10^{8} [23]	1.251×10^{5} 2085 [23]	1.5×10^{46}	1.78×10^{5} [23]	2.6×10^{31}	0.4	8 or 172 [23]
2	G195–19 He [24, 25, 26]	7,160 [26]	1.5×10^{33} 0.75 [26]	7.2×10^{8} [26]	1.150×10^{5} 1917 [24, 25]	1.7×10^{46}	~10^{8} [25]	1.9×10^{34}	~ 260	
3	WD 2047+372 H [27]	14,712 [27]	1.6×10^{33} 0.81 [27]	7.3×10^{8} [27]	2.101×10^{4} 350.2 [27]	1.0×10^{47}	9.18×10^{4} [27]	1.8×10^{31}	0.04	86.5 or 93.5
4	PG 1031+234 H [28, 29, 30]	15,000 [28]	1.85×10^{33} 0.93 [29]	6×10^{8} [21]	1.27×10^{4} 212 [30]	1.3×10^{47}	~ 2×10^{8} [28]	2×10^{34}	~ 35	35/145 and 55 [28]
5	PG 2329+267 H [30, 31]	11,730 [30]	2.35×10^{33} 1.18 [30]	4×10^{8} [21]	9.961×10^{3} 166.0 [30]	9×10^{46}	2.3×10^{6} [31]	7×10^{31}	0.2	
6	Feige 7 He [32, 33, 34]	21,000 [32]	1.2×10^{33} 0.6 [34]	9×10^{8} [21]	7.896×10^{3} 131.6 [32]	3.1×10^{47}	3.5×10^{7} [32, 33]	1.3×10^{34}	10	90 or 30/150 [33]
7	GD 356 He [26, 35, 36]	7,500 [35]	1.3×10^{33} 0.67 [26]	8.0×10^{8} [26]	6.94×10^{3} 116 [36]	3.0×10^{47}	1.3×10^{7} [35]	3.3×10^{33}	2.5	~ 1 or ~ 60 [36]
8	PG 1533–057 H [29, 30, 37]	20,000 [29]	1.9×10^{33} 0.94 [29]	6×10^{8} [21]	6.804×10^{3} 113.4 [30]	2.5×10^{47}	3.1×10^{7} [37]	3.3×10^{33}	3	
9	PG 1015+014 H [29, 30, 38, 39]	14,000 [29]	2.29×10^{33} 1.15 [29]	5×10^{8} [21]	6.30×10^{3} 105 [30]	2.3×10^{47}	9×10^{7} [29, 38, 39]	5.6×10^{33}	6	
10	G99–47 H [26, 30, 40]	5,790 [26]	1.4×10^{32} 0.71 [26]	7.7×10^{8} [26]	1.608×10^{3} 26.8 [30]	1.3×10^{48}	2.0×10^{7} [40]	4.6×10^{33}	0.8	10 or 170 [40]

6. Feige 7 (= WD 0041–102).
Achilleos *et al*. [32] analysed the He and H absorption lines of this white dwarf obtained by time-resolved Zeeman spectroscopy and by optical polarimetry. To fit the spectra they found a magnetic dipole for the star with a polar field strength of 35 MG, displaced by ΔR = 0.15 R from the stellar centre. Martin *et al*. [33] deduced an angle δ = 90º or, alternatively, δ ~ 30º (or ~ 150º) between the rotational and magnetic axes.

7. GD 356 (= WD 1639–537).
Ferrario *et al*. [35] and Brinkworth *et al*. [36] investigated the Hα and Hβ emission lines of this white dwarf by Zeeman and circular polarimetry spectroscopy. The first authors found a centred magnetic dipole with a polar field strength of 13 MG as best fit for the observed data. The latter authors deduced a rotation period of 0.0803 day from the near-sinusoidal photometric (V-band) variability in GD 356. The variability was modelled by a dark spot covering 10 per cent of the stellar surface. An angle δ ~ 1º between the rotational and magnetic axes was deduced. Alternatively, a value of δ ~ 60º could be chosen.

8. PG 1533–057 (= WD 1533–057).
From linear Zeeman splitting of Hα and Hβ lines and polarimetry Achilleos *et al*. [36] deduced a centred magnetic dipole for this star with a polar field B_p of 31 MG. An

alternative model with a magnetic dipole moment, displaced by less than $\Delta R = 0.1\ R$ from the stellar centre, yielded a slightly better description of the observational data.

9. PG 1015+014 (= WD 1015+014).
Euchner *et al*. [38] applied Zeeman tomography and polarimetry to this white dwarf, covering a whole rotation period. They found a magnetic field geometry more complex than a centred magnetic dipole, or a single moderately offset dipole. Zeeman features between 50 and 90 MG were found. Therefore, as an order of magnitude, we choose a polar magnetic field B_p of 90 MG in our calculation (compare with the value $B_p \sim 120$ MG, discussed by Schmidt and Norsworthy [37]).

10. G99-47 (= WD 0553+053).
Putney and Jordan [39] applied Zeeman spectroscopy and polarimetry to this white dwarf. An off-centred dipole with a polar magnetic field B_p of 20 MG, or a dipole plus quadrupole configuration best fit observations.

2.2 AM Herculis white dwarfs

AM Herculis systems or polars are close binary systems, in which a primary compact white dwarf accretes matter from a Roche-lobe filling late type secondary star. It is usually assumed that the magnetic field is strong enough to channel the accreted matter along its magnetic field lines to the white dwarf. In addition, the rotation period of the white dwarf P_s and the binary orbital period P_{orb} are synchronized, so that $P_s = P_{orb}$. In table 2 values are given for the effective temperature T_{eff}, mass m, rotation period P_s, radius R, calculated absolute values for angular momentum S, magnetic dipole moment M and factor β^* of eleven AM Her white dwarfs. Additional details of the observations and analyses of these stars are added below:

1. V1309 Ori (RX J0515.6+0105).
A number of system parameters of this long-period, eclipsing polar was presented by Staude *et al.* [40]. In addition, from polarimetric observations Katajainen *et al.* [41] calculated values of about 50 MG for the magnetic field at two cyclotron emission regions. Moreover, they located these two accreting regions at almost diametrically opposite positions, and centred at colatitudes 35° and 145° on the surface of the white dwarf (145° for the positive magnetic pole).

2. RX J1007−20.
Thomas *et al*. [42] reported optical and X-ray observations of this high-field polar. They applied spectrophotometry and circular spectropolarimetry to this system and found an orbital period $P_{orb} = P_s$ of 208.6 min. From cyclotron spectra a magnetic pole with a field strength of 94 MG was deduced.

3. AM Her (3U 1809+50).
Gänsicke *et al*. [43] discussed various parameters like T_{eff}, mass m and radius R data for this polar. Campbell *et al*. [44] adopted a single cyclotron emission region near the main pole and extracted an angle $\delta = 85°$ between the rotational axis and the direction of the magnetic field at this accretion site.

4. HS 1023+3900.
From the cyclotron emission line spectrum of the eclipsing white dwarf Reimers *et al.* [45] deduced a main accretion pole with a field strength of 60 MG and a secondary pole with a field strength of 68 MG. Schwarz *et al*. [47] found that the magnetic dipole axis is perpendicular to the line connecting both stars and inclined into the orbital plane. In

addition, they deduced the values δ ~ 100º and ~ 85º for the angle between the rotational axis and the magnetic primary and secondary pole, respectively.

Table 2. AM Herculis white dwarfs or polars.

	Name star (used spectr.) [references]	T_{eff} (K)	m (g) ($m_\odot$)	R (cm)	P_s (s) (min)	S (g.cm^2.s^{-1})	B_p(tot) (G)	M (G.cm^3)	β*	δ (º)
1	V1309 Ori (cyclotron) [41, 42]	20,000 or less [41]	1.4×10^{33} 0.70 [41]	7.5×10^{8} est. [41]	2.874×10^{4} 479.0 [41]	6.9×10^{46}	5.0×10^{7} [42]	1.1×10^{34}	37	145 [42]
2	RX J1007–20 (cyclotron) [43]	22,000 [43]	1.6×10^{33} 0.8 [43]	7.4×10^{8} [43]	1.252×10^{4} 208.6 [43]	1.8×10^{47}	9.4×10^{7} [43]	1.9×10^{34}	24	accr. spot 87.7 [43]
3	AM Her (cyclotron) [44, 45]	19,800 [44]	1.6×10^{33} 0.78 [44]	7.6×10^{8} [44]	1.114×10^{4} 185.6 [45]	2.0×10^{47}	1.36×10^{7} [45]	3.0×10^{33}	3.5	85 [45]
4	HS 1023+3900 (cyclotron) [46, 47]	13,000 [46]	1.8×10^{33} 0.9 [46]	6.3×10^{8} [46]	1.002×10^{4} 166.9 [47]	1.8×10^{47}	6.4×10^{7} [46]	8.0×10^{33}	10	~ 100 (prim. pole) [47]
5	UZ For (cyclotron) [48]	7,590 [48]	1.4×10^{33} 0.71 [48]	7.5×10^{8} [48]	7.592×10^{3} 126.5 [48]	2.6×10^{47}	5.3×10^{7} [48]	1.1×10^{34}	10	12 [48]
6	AR Uma (cycl., Zeeman) [49, 50]	3,200 [50]	2.1×10^{33} 1.07 [50]	6×10^{8} [21]	6.955×10^{3} 115.9 [49]	2.7×10^{47}	2.0×10^{8} [49]	2.2×10^{34}	19	5-10 [49]
7	V834 Cen (Zeeman, cycl.) [20, 51]	14,300 [20]	1.3×10^{33} 0.66 [20]	8.1×10^{8} [51]	6.090×10^{3} 101.5 [20]	3.5×10^{47}	3.1×10^{7} [20]	8.2×10^{33}	5.4	10 [51]
8	LSQ1725-64 (Zeeman) [52]	12,650 [52]	1.9×10^{33} 0.97 [52]	5.9×10^{8} [52]	5.64×10^{3} 94 [52]	2.9×10^{47}	1.25×10^{7} [52]	1.3×10^{33}	1.0	10-59 [52]
9	DP Leo (cyclotron) [53]	13,500 [53]	1.2×10^{33} 0.6 [53]	8×10^{8} [53]	5.388×10^{3} 89.80 [53]	3.6×10^{47}	4.5×10^{7} [53]	1.2×10^{34}	8	100 [53]
10	EF Eri (cyclotron) [54, 55]	9,500 [54]	1.2×10^{33} 0.6 [54]	8.7×10^{8} [54]	4.861×10^{3} 81.02 [54]	4.7×10^{47}	4.4×10^{7} [55]	1.4×10^{34}	7	75 [55]
11	EV Uma (cyclotron) [56]	56,000 [56]	2.2×10^{33} 1.1 [56]	6×10^{8} [21]	4.781×10^{3} 79.69 [56]	4×10^{47}	3.5×10^{7} [56]	4×10^{33}	2	

5. UZ For (EXO 0333−25).
Data for this eclipsing polar are taken from light curve fitting of Kube *et al*. [48]. For the angle between the rotational and magnetic axis a value of δ = 12º was calculated.

6. AR UMa (1ES 1113+432).
Gänsicke *et al*. [49] discussed data for this high-field polar (P_{orb} = 1.932 hr) and confirmed that the Lyman Hα splittings could be explained by a magnetic field of ~ 200 MG. Low-level accretion on both poles may cause cyclotron emission. The deduced field strengths for the northern and the southern poles were ~ 240 MG and ≥ 160 MG, respectively, broadly consistent with the field derived from the Zeeman lines. A mass range for the white dwarf in AR Uma of 0.91-1.24 $m_\odot$ was inferred by Bai *et al.* [50].

7. V834 Cen (1E 1405−451)

Data for this eclipsing polar are taken from Ferrario *et al*. [20]. For the polar field strength B_p a value of ~ 31 MG is chosen from Ferrario *et al*. (1992) in ref. [20], offset by ~ – 0.1 R from the centre of the white dwarf along the magnetic axis. In addition, Mauche [51] proposed a simple model of accretion from a ballistic stream along the field lines of a tilted magnetic dipole centred on the white dwarf. From this model an angle δ = 10º between the rotational and magnetic axis was calculated.

8. LSQ1725−64

Fuchs *et al.* [52] presented new photometric and spectroscopic data for this eclipsing binary. They reported a surface-averaged magnetic field of 12.5 MG and an estimate for the angle δ between 10º and 59º.

9. DP Leo (1E1114+182)

Schwope *et al*. [53] discussed old and new data of this eclipsing polar (P_{orb} = 0.06236 d). They refer to the two-pole accretor proposed by Cropper and Wickramasinghe (1993), who deduced field strengths of 30.5 MG and 59 MG for the two poles from cyclotron emission lines.

10. EF Eri

Hoard *et al*. [54] discussed photometric new data for this polar. From a best-fit offset dipole model Beuermann *et al*. [55] obtained a value for the polar magnetic field B_p of 44 MG, inclined to the rotational axis by 75º.

11. EV UMa (RE J1307+535)

Ramsey and Cropper [56] discussed various properties of this eclipsing polar. This star displays a short orbital period P_{orb} of 79.69 min and a magnetic field of 30−40 MG.

2.3 Asynchronously rotating white dwarfs in binaries

In general, the rotation period of the white dwarf P_s and the binary orbital period P_{orb} need not to be synchronized ($P_s \neq P_{orb}$). For example, in the subclass of DQ Herculis stars, or intermediate polars, the magnetic field of the white dwarf may not be not strong enough to channel the accreted matter along the magnetic field lines to the star. An accretion disk may then be formed. Furthermore, a related class of binaries consists of two white dwarfs, one magnetic and one not. In table 3 values for the effective temperature T_{eff}, mass m, rotation period P_s, radius R and absolute value of the observed total polar magnetic field B_p(tot) are given for two double-white-dwarf binaries and one DQ Herculis white dwarf. Results of the calculated absolute values for angular momentum S and magnetic dipole moment M and factor β^* are also shown. Additional details of the observations and analyses of the white dwarfs are given below:

1. NLTT 12758 (= 0410–114)

For this binary system consisting of two white dwarfs Kawka *et al*. [57] found a rotation period P_s of 23 min for the magnetic white dwarf and an orbital period P_{orb} of 1.154 days. In addition, they deduced a polar magnetic field B_p of 3.1 MG, offset by + 0.1 R from the stellar centre of the magnetic white dwarf.

2. RE J0317–853 (= EUVE J0317−85.5 or WD 0316–849).

Burleigh *et al.* [58] investigated the magnetic white dwarf by time-resolved Zeeman spectroscopy of the Lyman lines. Using a multipolar expansion they found a dipolar contribution of 206 MG and an angle δ ~ 29º between the rotational and magnetic axes. The close proximity of the non-magnetic white dwarf, LB 09802, allowed to estimate the mass of RE J0317–853 at 1.35 $m_\odot$. Vennes *et al*. [59] extended the observations and concluded that both offset-dipole and multipolar field models are inadequate to explain the data. They suggested that the surface of RE J0317–853 may possess an additional high-field magnetic spot.

3. V405 Aur (= RX J0558.0+5353)

Brunschweiger *et al*. [61] reported a mass m = 0.89 $m_\odot$ for the white dwarf of the intermediate polar V405 Aur. Piirola *et al*. [62] reported a magnetic field of 31.5 MG for the white dwarf with spin period P_s = 9.09 min and P_{orb} = 4.15 hr.

Table 3. Asynchronously rotating white dwarfs in binaries.

	Name (used spectrum) [references]	T_{eff} (K)	m (g) ($m_\odot$)	R (cm)	P_s (s) (min)	S (g.cm^2.s^{-1})	B_p(tot) (G)	M (G.cm^3)	β*	δ (º)
1	NLTT 12758 (Zeeman) [57]	7,220 [57]	1.4×10^{33} 0.69 [57]	8.0×10^{8} [57]	1,380 23 [57]	1.6×10^{48}	3.1×10^{6} [57]	7.9×10^{32}	0.11	
2	RE J0317–853 (Zeeman) [58, 59]	33,000 [59]	2.69×10^{33} 1.35 [58]	2.9×10^{8} [59]	725.7 12.095 [59]	7.8×10^{47}	2.06×10^{8} [58]	2.5×10^{33}	0.75	29/151 [58, 59]
3	V405 Aur (cyclotron) [60, 61, 62]	~4,100 [60]	1.8×10^{33} 0.89 [61]	6.5×10^{8} [21]	545 9.09 [62]	3.5×10^{48}	3.15×10^{7} [62]	4.3×10^{33}	0.29	82 [62]

3. DISCUSSION OF THE RESULTS

Until now, the origin of magnetic fields in stars remains a major unresolved problem in astrophysics. To my knowledge, no generally accepted, complete theory is available for any planet, star or galaxy. White dwarfs are no exception. For a long time the origin of the magnetic fields of white dwarfs has been attributed to the fossil hypothesis. More recently, dynamo action was proposed as an origin of the magnetic field in white dwarfs. For example, a magnetic field may be generated during the common envelope evolution of *binary* systems containing at least one white dwarf. For reviews, see, e.g., Ferrario, Melatos and Zrake [17] and Kawka [18]. More recently, another dynamo mechanism was studied by Isern *et al.* [19]. They assumed that a magnetic field is generated in the convective region of the *isolated* white dwarf during its cooling. The latter approach also predicts a magnetic field in celestial bodies with a liquid convective mantle on top of the solid core like planets Earth and Jupiter.

In this work a gravitomagnetic origin of the basic magnetic field of white dwarfs is considered (see refs. [1−16]). This approach may lead to the so-called Wilson-Blackett formula (1.1), with parallel magnetic moment **M** and angular momentum **S** for the choice β = –1. When β* = +1 in (1.8), $\mathbf{B}_p^{\parallel}$(tot) is equal to the gravitomagnetic field $\mathbf{B}_p$(gm) of (1.4). The latter expression follows from the Wilson-Blackett formula (1.1). Observed values of β* different from the value β* = +1 are thus an indication of the invalidity of the Wilson-Blackett formula (1.1). In tables 1, 2 and 3 data and calculated results, among them the values for the factor β*, are summarized for 10 isolated white dwarfs, 11 AM Herculis white dwarfs and 3 other binary white dwarfs, respectively.

If available, values for the angle δ between the directions of **M** and **S** are also added to the tables 1 through 3. Some preference for values like δ ~ 0º, δ ~ 90º and δ ~ 180 is found. Apart from centred magnetic dipoles, magnetic dipoles with an offset and combinations of magnetic quadrupoles and octupoles are found. Moreover, sometimes a localized geometry of the magnetic field ("magnetic spot") has to be added to the multipolar expansion of the star's general field (see, e.g., comment to WD 1953−011).

So, in many cases the predicted magnetic dipole following from (1.1) for $\beta = -1$ only yields a poor description of the observed magnetic field. Moreover, the complexity of the observed magnetic fields suggests that different mechanisms are at work.

In table 4 mean values of mass m, radius R, angular momentum S, total magnetic field B_p(tot), magnetic moment M and factor β^* from the white dwarfs in tables 1 through 3 are summarized. In addition, corresponding standard deviations s of the various parameters are added.

Table 4. Mean values of m, R, S, B_p(tot), M and β^* for white dwarfs, and their corresponding standard deviations s.

		Number	$\overline{m}$ s ($m_\odot$)	$\overline{R}$ s (cm)	P_s (min)	$\overline{S}$ s (g cm^2 s^{-1})	$\overline{B_p\text{(tot)}}$ s (G)	$\overline{M}$ s (G.cm^3)	$\overline{\beta^*}$ s
1	Isolated white dwarfs from table 1	10	0.86 0.19	6.7×10^8 1.5×10^8	26.8 – 2085	2.7×10^{47} 3.8×10^{47}	4.9×10^7 6.4×10^7	0.69×10^{34} 0.77×10^{34}	32 81
2	AM Herculis white dwarfs from table 2	11	0.81 0.18	7.2×10^8 1.0×10^8	79.69 – 479	2.8×10^{47} 1.2×10^{47}	5.8×10^7 5.2×10^7	1.0×10^{34} 0.64×10^{34}	12 11
3	Binary white dwarfs from table 3	3	0.98 0.34	5.8×10^8 2.6×10^8	9.09 – 23	20×10^{47} 14×10^{47}	8.0×10^7 11×10^7	2.5×10^{33} 1.8×10^{33}	0.38 0.33

It is found that observed values of β^* in tables 1 through 3 differ from unity value up to two orders of magnitude, whereas the mean values $\overline{\beta^*}$ in table 4 differ by about an order of magnitude. Note that the three white dwarfs with the shortest rotation period P_s in table 4 possess the smallest value $\overline{\beta^*} = 0.38$. The abundance of this class of white dwarfs is relatively low. Furthermore, it is noticed that the standard deviations s given in table 4 are large in general.

The mean values $\overline{m}$ and $\overline{R}$ from the isolated white dwarfs in table 4 can be used to calculate an approximate value of gravitomagnetic field B_p(gm) from (1.4). One obtains

$$B_p(\text{gm}) = 2\pi c^{-1} G^{1/2} I R^{-3} P_s^{-1} = 5.5\times10^{10} P_s^{-1} \text{ for } \beta = -1. \tag{3.1}$$

In calculating (3.1), a value $f = 1$ has been inserted into the moment of inertia $I = 2/5\, f\, m\, R^2$ of (3.1). In that case I equals to $I = 3.07\times10^{50}$ g.cm^2. Since the individual values of m and R in table 4 differ not too much from $\overline{m}$ and $\overline{R}$, respectively, substitution of the rotation period of the white dwarf P_s into (3.1) yields the correct order of magnitude for B_p(gm).

4. COMPARISON WITH PULSARS

White dwarfs and pulsars are both compact stars and often display strong magnetic fields. Therefore, comparison of β^* values for pulsars with the present analysis of white dwarfs may be useful. Previously, absolute values of β^* have been calculated for a large sample of pulsars [16]. For 14 *binary*, accretion-powered X-ray pulsars these values have been calculated from electron cyclotron resonance spectral features (CRSFs) (see table 2 of ref. [16], where β^* is put equal to $\beta^* = |B_p(\text{tot})|/|B_p(\text{gm})|$). The gravitomagnetic field B_p(gm) follows from (1.4) and can be written as

$$B_p(\text{gm}) = 2\pi c^{-1} G^{1/2} I R^{-3} P_s^{-1} = 5.414\times10^{13} P_s^{-1} \text{ for } \beta = -1. \tag{4.1}$$

In order to calculate (4.1) a value of $I = 10^{45}$ g.cm^2 for pulsars has been inserted. The choice for I implies a value $f = 0.898$, when the convential values $m = 1.4\, m_\odot$ and $R = 10^6$ cm are substituted into $I = 2/5\, f\, m\, R^2$. Note that the values of B_p(gm) for white dwarfs in (3.1) differ by about three orders of magnitude from that for pulsars in (4.1). The obtained mean values for $\overline{\beta^*}$ are summarized in table 5.

In addition, from data of a sample of 96 isolated pulsars and 3 binary millisecond pulsars in table 4 of ref. [16] mean values $\overline{\beta^*}$ and corresponding standard deviations s for both classes of pulsars have also been calculated. For all these pulsars, the field B_p(tot) has been obtained from the formula for the magnetic dipole radiation

$$B_p(\mathrm{sd}) = \left(\frac{3c^3 I}{8\pi^2 R^6}\right)^{1/2} (P_s.\dot{P}_s)^{1/2} = 3.200\times 10^{19}(P_s.\dot{P}_s)^{1/2}, \tag{4.2}$$

where $\dot{P}_s$ is the time derivative (spin down) of the rotation period P_s. In this case the parameter β^* is approximated by the relation $\beta^* = |B_p(\mathrm{sd})|/|B_p(\mathrm{gm})|$. The results from table 4 in ref. [16] are also summarized in table 5.

Table 5. Mean value $\overline{\beta^*}$ and standard deviation *s* for pulsars.

		Number	m ($m_\odot$)	R (km)	P_s (s)	$\overline{\beta^*}$	s
1	Isolated pulsars [16, table 4]	96	1.4	10	0.06496 – 4.962	0.061	0.134
2	Slowly rotating pulsars in binaries [16, table 2]	14	1.4	10	1.238 – 837.7	14.2	17.8
3	Fast rotating pulsars in binaries [16, table 4]	3	1.4	10	0.009348 – 0.08828	6.4×10^{-6}	7.0×10^{-6}

Comparison of the *isolated* white dwarfs of table 1 and the *isolated* pulsars in table 5 of this work shows that the absolute values of β^* of both classes display a large spread. For example, the isolated white dwarf WD 1953–011 has a value $\beta^* = 0.4$, whereas a value of $\beta^* \sim 260$ was obtained for the isolated white dwarf G195–19 (The rotation periods P_s of the white dwarfs do not differ much: $P_s = 2085$ min and $P_s = 1917$ min, respectively). An explanation for this big difference could be, that the white dwarfs with a low magnetic field strength are born as *isolated* stars, corresponding to $\beta^* \approx 1$ and in agreement with the gravitomagnetic hypothesis. On the other side, the isolated high-field magnetic white dwarfs may be the result of core merging in a common envelope episode.

Comparison of the AM Herculis white dwarfs in table 2 of this work and the *binary*, accretion-powered X-ray pulsars in table 2 of ref. [16] shows that the absolute values of β^* increase for increasing rotation periods P_s in both classes. For pulsars this increase is more pronounced. If this trend may be extrapolated to AM Herculis white dwarfs with a very long rotation period P_s, then the latter stars would possess strong magnetic fields. In this case, the high magnetic fields may be due to a magnetic dynamo operating during common envelope evolution.

Another example of an analogy between both kind of stars may be the observed low value of $\overline{\beta^*}$ for short-period *binary* white dwarfs ($\overline{\beta^*} = 0.38$ in table 4) and the extreme low mean value for short-period pulsars in binaries ($\overline{\beta^*} = 6.4\times10^{-6}$ for millisecond pulsars in table 5). It is noticed that the abundance of millisecond pulsars is also relatively low. As has been discussed in ref. [16], this effect may be caused by a toroidal current in these pulsars leading to a contribution $\beta^*_{\mathrm{current}} \approx -1$. In that case the observed value of β^* is given by $\beta^* = \beta^*(\mathrm{gm}) + \beta^*_{\mathrm{current}} = +1 + \beta^*_{\mathrm{current}} \approx 0$. It is noticed that values for $\beta^*_{\mathrm{current}}$ are calculated for a number of pulsars displaying high frequency quasi periodic oscillations (QPOs) [63].

5. CONCLUSIONS

The origin of magnetic fields in stars remains an unanswered question in astrophysics. For decades it has been assumed that magnetic fields in white dwarfs are fossil fields from Ap and Bp stars. More recently, dynamo action in various forms has been proposed as an explanation (see e.g., refs. [17–19]). In this work a gravitomagnetic origin of the basic magnetic field of white dwarfs has been investigated (see refs. [1–16]). Usually, the so-called Wilson-Blackett formula (1.1) is the starting point of the gravitomagnetic hypothesis. The validity of the latter approach may be tested by the dimensionless factor β^*. Deviations of the observed value β^* from the value $\beta^* = 1$ are a measure of the validity of the Wilson-Blackett formula. In section 3 values of β^* are summarized for 10 *isolated* white dwarfs, 11 AM Herculis white dwarfs and 3 *short-period* white dwarfs in *binary* systems, respectively. In table 4 the corresponding mean values of $\overline{\beta^*}$ are given.

It is found that observed values of β^* in section 3 differ from $\beta^* = 1$ up to two orders of magnitude, whereas the mean values $\overline{\beta^*}$ in table 4 differ by about an order of magnitude from unity value. In table 5 mean values $\overline{\beta^*}$ for 96 *isolated* pulsars, 14 *binary*, accretion-powered X-ray pulsars and 3 *binary millisecond* pulsars are given. These classes of pulsars are related to the white dwarfs considered in table 4. At first sight, the found discrepancies may be a good reason to abandon the gravitomagnetic hypothesis.

However, earlier calculations carried out by several authors, e.g., Blackett [4], Ahluwalia and Wu [5], Sirag [6], Surdin [7, 8] and Biemond [11] showed an almost linear relationship between the observed magnetic moment $|\mathbf{M}|$ and angular momentum $|\mathbf{S}|$ of many massive bodies, in agreement with the Wilson-Blackett formula (1.1). For example, for a series of about 14 rotating bodies ranging from metallic cylinders in the laboratory, moons, planets, stars and the Galaxy (M and S varied over an interval of *sixty decades*) such a relationship was found from a linear regression analysis. This linear relationship for so many, *very different*, rotating bodies is amazing! For this sample an average value of $\overline{\beta^*} = 0.076$ was obtained from a weighted least-squares fit to the data, also distinctly different from the gravitomagnetic prediction $\beta^* = 1$. In view of large intervals of the values of M and S, however, the bandwidth of the mean values $\overline{\beta^*}$ is relatively small.

From the theoretical point of view, the discrepancies between observed value B_p(tot) and proposed gravitomagnetic field B_p(gm) may be attributed to various magnetic field contributions from electromagnetic origin. Such contributions could explain the large deviations of the dimensionless factor β^* from unity value. As has been pointed out in the introduction (see (1.6)), however, magnetic fields from gravitomagnetic origin are usually extremely small and difficult to isolate from fields due to moving electric charges.

It is noticed that the Wilson-Blackett formula (1.1) may be extrapolated to neutrinos, resulting into $M \approx c^{-1} G^{½} S$, where $S = ½\, \hbar = 5.27\times10^{-28}$ g cm^2 s^{-1}. An alternative expression for the magnetic moment M, proportional to the mass m_i of the neutrino i, has been deduced for massive Dirac neutrinos in the context of electroweak interactions. Combination of the latter expression for M for the lightest neutrino of mass m_1 and $M = c^{-1} G^{½} S$ yields a value of 1.53 meV/c^2 for mass m_1 [64, 65]. Conformation of this value would provide another indication for the validity of the gravitomagnetic hypothesis.

Summing up, a gravitomagnetic origin to the basic magnetic field of white dwarfs may be an essential ingredient in the explanation of the total magnetic field of rotating massive spheres. Moving electric charges in the magnetic field from gravitomagnetic origin may cause dominating magnetic fields from electromagnetic origin. According to the fossil hypothesis [17, 18], these fields may be retained when white dwarfs are formed from an ancestor like an Ap star. Dynamo-generated magnetic fields may also contribute to the explanation for the magnetic fields of white dwarfs [17–19].

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