Optimal Binary Number System When Numbers Are Energy?

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May 17, 2020

Abstract

In this short note, we will quickly look at optimal binary number systems used in communication (or transactions) under the assumption that one must use energy to give away (send) numbers. We show that the current binary system is not the optimal binary number system as it can be arbitraged. We also show that there exist other optimal binary number systems in such a scenario. Naturally, one has to ask, "Optimal for whom? – For the one sending the number out, or for the one receiving the number?" Alternatively, we can have a binary number system that, on average, is neutral for both sender and receiver. Numbers are typically only considered to have symbolic value, but if the money units were so small that they came in the smallest possibly energy units, then we could be forced to switch to a number system where the physical value of each number was equal to its symbolic value. That is to say, the physical value of three must be higher than the physical value of two, for example. Numbers are always physical because storing or sending a number from a computer requires bits, and bits of information require energy.

Key Words: binary number systems, energy, time, value of numbers.

1 Binary Number Systems and the Value of Numbers

Binary systems go back a long time and can, for example, be found as the basis for I-Ching [1], a book that supposedly was written by Fuxi, the philosopher "King" who was also the first emperor of China, according to legends. Leibniz clearly had inspiration from I-Ching and created the foundation for the binary system used in the modern world today as for example the foundation of computer science, see for example Shannon [2] and Lande [3]. Leibniz [4] published his binary system in an article in 1703, where he states

> *What is amazing in this reckoning is that this arithmetic by 0 and 1 is found to contain the mystery of the lines of an ancient King and philosopher named Fuxi, who is believed to have lived more than 4000 years ago, and whom the Chinese regard as the founder of their empire and their sciences. It was scarcely more than two years ago that I sent to Reverend Father Bouvet, the celebrated French Jesuit who lives in Peking, my method of counting by 0 and 1, and nothing more was required to make him recognize that this was the key to the figures of Fuxi.* – Leibniz, 1703

Let's play a little game. In a four-bit binary system, we have 16 combinations that describe 16 numbers from zero to 15. If I asked you whether you would prefer the number three or the number eight in the form of binary numbers, which one would you choose? Or would you be neutral? If we assume 1 is an energy unit and zero is a no-energy unit, then actually the number three (0011) has twice the amount of energy as eight (1000), so the number three is naturally preferable.

Extending this idea, imagine a case where numbers are exchanged between two parties. Assume the numbers represent real value. The simplest model is to think of numbers as money, and that the higher the number the greater the value; therefore, 2 must be more valuable than 1 and 7 more valuable than 6, for example. Most numbers in computers and information systems are binary. Assume we are working with a four-bit binary system that can handle numbers from 0 to 15. In a standard binary system, zero is represented by 0000, and one is represented by 0001. Now assume these numbers are sent as discrete energy units. Let us choose to represent 1 with a unit of energy, and 0 as a unit of no energy (no energy basically means nothing is sent out), as posited previously. We can also also say that the sender and receiver both have clocks ticking at a uniform rate. In each time unit, one observes either nothing, which means 0, (this means zero sent or received energy units), or one observes 1, which means one energy unit has been sent or received. Clearly, energy is valuable, as it can be used for many other purposes besides sending and receiving information.

In a standard binary system, when we assume 1 represents energy and 0 represents no energy, then the number four, which has a binary representation of 0100, contains less energy than the number 3. This mean if the energy in the numbers were more valuable than other symbolic information they represented, then one would prefer receiving three instead of four; after all, three is now twice as valuable as four. Further, eight, which is represented by 1000, is much less valuable than seven, because seven contains three energy units, as it is represented by 0111. Even the number three contains twice as much energy as eight does. This simply means that the standard binary system is not ordered by energy value; it is not a fair binary system in this way.

However, we can easily construct other types of binary number systems that can be more fair in the sense that higher numbers are more valuable. This is based again on the concept that the energy content in each number should be in the same order sequence as the number it is representing. A higher number should always contain more energy than a lower number. We all also know that time is money. It is better to receive one dollar today than to receive one dollar tomorrow, as you can put it to productive work today and will get return on the capital by tomorrow. In the same way, it is better to give away one energy unit tomorrow than today, and better to receive one energy unit today than tomorrow. Let us take this analogy into our binary numbers. Assume we will represent the number one with 1000 and the number two with 0100. Then if the bits are sent with the bits from the right hand side first, then two is indeed more valuable than one for a receiver of the number, not because it contains more energy, but because we get the one energy unit in number two exactly one time unit earlier than for number one. Column three in Table 1 shows a binary number system, where higher numbers always have higher value for the receiver, based on taking energy and time into account. However, for the sender it is not so, as he would prefer to send the number two rather than one, as he would then keep the energy unit one time-unit longer. Column four is what we can call a "sender-friendly" binary number system. That is, when the the value increases for higher numbers (for the sender), based on our assumption that 1 represents energy and the goal for the sender it to retain the energy for himself as long as possible. In the last column, we have tried to make a number system that is neutral. Here, the number one, represented by 0001, is more valuable than the number two, which is 1000 for a receiver, but the number two is more valuable than number three for a sender. So, on average, if all numbers are used in equal proportions, then both receiver and sender will be just as well off.

If we do not take the value of time into account, i.e., to get a bit of energy earlier rather than later, then we cannot compress the numbers in the way it is done in the systems described so far. However, we can use a number system with a variable number of bits, as described in the second column from the right in the table. The reason we start with a zero here is for the receiver (or sender to know there is a new number coming). The sender or receiver of numbers must still have a clock that checks, at each uniform time interval, if there is energy or no energy in the number "detector." We can also use the 16-bit system, as described in the right-most column; this would require that one already knows the highest number one plans to use. If the highest number is 255, then one needs to use a 256-bit system for all numbers. In other words, if we want to have a number system where each number has a higher value in the physical world as well, we cannot compress the information in the way done in most of today's number systems, where the numbers only have symbolical value. This is due to the fact that the symbolic value is much higher than the physical value.

Number	Standard binary system	Receiver friendly	Sender friendly	"Neutral" on average	Energy ranked number system	Energy ranked 16 bits system
0	0000	0000	0000	0000	00	000000000000000
1	0001	0001	1000	0001	01	000000000000001
2	0010	0010	0100	1000	011	000000000000011
3	0011	0100	0010	0010	0111	000000000000111
4	0100	1000	0001	0100	01111	000000000001111
5	0101	0011	1100	0110	011111	000000000011111
6	0110	0101	1010	1001	0111111	000000000111111
7	0111	1001	1001	0011	01111111	000000001111111
8	1000	0110	0110	1100	011111111	000000011111111
9	1001	1010	0101	1010	0111111111	000000111111111
10	1010	1100	0011	0101	01111111111	000001111111111
11	1011	0111	1110	1011	011111111111	000011111111111
12	1100	1011	1101	1101	0111111111111	000111111111111
13	1101	1101	1011	0111	01111111111111	001111111111111
14	1110	1110	0111	1110	01111111111111	011111111111111
15	1111	1111	1111	1111	011111111111111	111111111111111
16					0111111111111111	
17					01111111111111111	
18					⋮	

Table 1: The table shows standard numbers with six corresponding binary number systems. The table assumes that the numbers are sent as single bit streams from left to right.

2 Similarity with Money

This has some similarities in concepts to physical money and even electronic money. Let's start with physical money. A 5-cent coin is worth 5 cents only if the notional value is considerably higher than the value of its mass. The mass of a 5-cent coin is mostly nickel. If the metal value is higher than the notional value, then some would be tempted to melt the coin to get the value of its mass in form of the metal value, see also Haug and Stevenson [5] that discusses this more in detail. This happened in 2006, for example, when the metal value of the coin went above the notional ("symbolic") value of the coin. Some people then started to melt coins. This was stopped by the Department of the Treasury and the US Mint, which enforced strong penalties for doing so, see the Appendix. During the hyperinflation of the Weimar Republic in the 1930s, the value of German monetary bills had less value than the energy in the bills, so some used the bills to heat their homes or even ovens for baking bread.

In a similar way, Haug [6] has recently discussed how electronic money is not exempt from this possibility. If the energy value of electronic money should be higher than the notional ("symbolic") value of the money, then even the electronic money system would be arbitraged. In this paper, we have extended this idea further, to binary numbers themselves.

Conclusion

We have shown that the standard binary system is far from optimal if one considers numbers as energy units. If 1 is represented by an energy unit, and 0 is represented by no energy, then three will be more valuable than eight in a standard binary system. We have introduced an energy-linked binary system where higher numbers have higher values than lower numbers for a receiver of the numbers, and a similar system for the sender of binary numbers. We have also suggested several other number systems where higher numbers are always more valuable (in terms of the energy they contain) than lower numbers. Only when the symbolic value of the number is considerably above the energy value can the most modern number systems be used without running the risk of being arbitraged.

References

[1] Fuxi. *I-Ching*. Likely published first around 800 B.C., A series of modern English translations exist, one of them is the version by editor Hellmut Wilhelm, Princeton University Press (2011), 800.

[2] E. C. Shannon. *A Symbolic Analysis of Relay and Switching Circuits*. Master Thesis, Massachusetts Institue of Technolog, 1936.

[3] D. R. Lande. Development of the binary number system and the foundations of computer science. *The Mathematics Enthusiast*, 11(3).

[4] G. W. Leibniz. *Explanation Of Binary Arithmetic, Which Uses Only The Characters 0 And 1, With Some Remarks On Its Usefulness, And On The Light It Throws On The Ancient Chinese Figures Of Fuxi*. Memoires de l'Academie Royale des Sciences http://www.leibniz-translations.com/binary.htm, 1703.

[5] E. G. Haug and J. Stevenson. Options embedded in physical money. *Wilmott Magazine*, 2009.

[6] E. G. Haug. The absolute smallest possible money unit! when money crashes into the laws of physics. *Submitted to journal, in for review*, 2020.

Appendix

This appendix is a reprint of the statement given by the United States Mint to limit melting and exporting of certain circulating coins.

December 14, 2006 United States Mint Moves to Limit Exportation & Melting of Coins

Interim Rule Goes Into Effect Immediately

WASHINGTON – The United States Mint has implemented regulations to limit the exportation, melting, or treatment of one-cent (penny) and 5-cent (nickel) United States coins, to safeguard against a potential shortage of these coins in circulation. The United States Mint is soliciting public comment on the interim rule, which is being published in the Federal Register.

Prevailing prices of copper, nickel and zinc have caused the production costs of pennies and nickels to significantly exceed their respective face values. The United States Mint also has received a steady flow of inquiries from the public over the past several months concerning the metal value of these coins and whether it is legal to melt them. “We are taking this action because the Nation needs its coinage for commerce,” said Director Ed Moy. We don’t want to see our pennies and nickels melted down so a few individuals can take advantage of the American taxpayer. Replacing these coins would be an enormous cost to taxpayers.?

Specifically, the new regulations prohibit, with certain exceptions, the melting or treatment of all one-cent and 5-cent coins. The regulations also prohibit the unlicensed exportation of these coins, except that travelers may take up to $ 5 in these coins out of the country, and individuals may ship up to $ 100 in these coins out of the country in any one shipment for legitimate coinage and numismatic purposes. In all essential respects, these regulations are patterned after the Department of the Treasury’s regulations prohibiting the exportation, melting, or treatment of silver coins between 1967 and 1969, and the regulations prohibiting the exportation, melting, or treatment of one-cent coins between 1974 and 1978.

The new regulations authorize a fine of not more than $ 10,000, or imprisonment of not more than five years, or both, against a person who knowingly violates the regulations. In addition, by law, any coins exported, melted, or treated in violation of the regulation shall be forfeited to the United States Government.

The regulations are being issued in the form of an interim rule, to be effective for a period of 120 days from the time of publication. The interim rule states that during a 30-day period from the date of publication, the public can submit written comments to the United States Mint on the regulations. Upon consideration of such comments, the Director of the United States Mint would then issue the final rule.

Those interested in providing comments to the United States Mint regarding this interim rule must submit them in writing to the Office of Chief Counsel, United States Mint, 801 9th Street, N.W., Washington D.C. 20220, by January 14, 2007. The interim rule appears on the United States Mint website at www.usmint.gov. The United States Mint will make public all comments it receives regarding this interim rule, and may not consider confidential any information contained in comments.