Energy and the tessellated 3-sphere

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Abstract

The tessellation of space is considered for both the 2-sphere and the 3-sphere. As hypothesized in an earlier work, it is found that there is an energy associated with the 3-sphere.

1 Curvature and energy

For a method of calculating the curvature of triangle meshes and tetrahedron meshes, please see [1]. Unlike in [1], the tessellations in this paper will rely on pseudorandomly placed vertices, rather than the vertices placed by Marching Cubes and Marching Hypercubes. Also unlike in [1], we will not be compensating for the variation in simplex extent (e.g. do nothing special even where there are sliver simplices). The vertex count is \( N \). Note that the Planck energy \( E_P = 1.0 \), and so the fundamental constants \( c = G = \hbar = 1.0 \) as well.

On one hand, it is found that for a tessellated 2-sphere, the local curvature vanishes when the tessellation is made up of finer and finer triangles. That is, the more vertices \( N \) used in the tessellation, the less the local curvature is:

\[
\lim_{N \to \infty} K(N) = 0.0.
\]

On the other hand, it is found that for a tessellated 3-sphere, the local curvature does not vanish when the tessellation is made up of finer and finer tetrahedra. The curvature settles around

\[
\lim_{N \to \infty} K(N) = 0.284.
\]

Unexpectedly, this is in line with the matter density measure \( \Omega_M \) used in the xCDM models \([2,3]\) – it is unknown if this is merely a coincidence. If it is not just a coincidence, then this is direct evidence of the discrete nature of space, based on a few simple, first principles. Note that curvature is proportional to energy:

\[
K \propto E.
\]

See Fig. 1 for a 3-sphere edge length histogram, where vertex count \( N = 1,000,000 \). Also see Table 1 for a list of properties of the histograms where the vertex count \( N \) is variable. A C++ code for generating the tessellated 3-sphere can be found at [4]. The code requires the qhull executables for mesh generation, the OpenCV library for plotting histograms, and the OpenGL / GLUT library for visualizing the vertices.
Figure 1: 3-sphere edge length histogram, where vertex count $N = 1,000,000$. Max = 0.0565194, mode = 0.012455. curvature $K = 0.28452$.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$K$</th>
<th>Max</th>
<th>Mode</th>
<th>Max / Mode</th>
</tr>
</thead>
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<tr>
<td>1,000</td>
<td>0.29473</td>
<td>0.405105</td>
<td>0.132555</td>
<td>3.05612</td>
</tr>
<tr>
<td>10,000</td>
<td>0.28821</td>
<td>0.215664</td>
<td>0.0619268</td>
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<tr>
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<td>0.28413</td>
<td>0.113452</td>
<td>0.0268951</td>
<td>4.21831</td>
</tr>
<tr>
<td>1,000,000</td>
<td>0.28452</td>
<td>0.0565194</td>
<td>0.012455</td>
<td>4.53788</td>
</tr>
</tbody>
</table>

Table 1: Properties of the histograms where vertex count $N$ is variable.
References

https://vixra.org/abs/1812.0423


https://github.com/sjhalayka/4d_universe