Energy and the tessellated 3-sphere

S. Halayka *

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Abstract
The tessellation of space is considered for both the 2-sphere and the 3-sphere. As hypothesized in an earlier work, it is found that there is an energy associated with the 3-sphere.

1 Curvature and energy

For a method of calculating the curvature of triangle meshes and tetrahedron meshes, please see [1]. Unlike in [1], the tessellations in this paper will rely on pseudorandomly placed vertices, rather than the vertices placed by Marching Cubes and Marching Hypercubes. The vertex count is \( N \).

On one hand, it is found that for a tessellated 2-sphere, the local curvature vanishes when the tessellation is made up of finer and finer triangles. That is, the more vertices \( N \) used in the tessellation, the less the local curvature is:

\[
\lim_{N \to \infty} K(N) = 0.0.
\] (1)

On the other hand, it is found that for a tessellated 3-sphere, the local curvature does not vanish when the tessellation is made up of finer and finer tetrahedra. The curvature settles around

\[
\lim_{N \to \infty} K(N) = 0.284.
\] (2)

Unexpectedly, this is in line with the measure \( \Omega_m \) used in the \( w\)CDM model [2] – it is unknown if this is just a numerology. Where curvature is proportional to energy,

\[
K \propto E,
\] (3)

there is an energy because of this non-vanishing curvature. See Fig. 1 for a 3-sphere edge length histogram, where vertex count \( N = 1,000,000 \). See Table 1 for a list of properties of the histograms where the vertex count \( N \) is variable. A C++ code for generating the tessellated 3-sphere can be found at [3]. The code requires the qhull executables for mesh generation, the OpenCV library for plotting histograms, and the OpenGL library for visualizing the vertices.

\*sjhalayka@gmail.com
Figure 1: 3-sphere edge length histogram, where vertex count \( N = 1,000,000 \). Max = 0.0565194, mode = 0.012455. curvature \( K = 0.28452 \).

<table>
<thead>
<tr>
<th>Vertex count ( N )</th>
<th>( K )</th>
<th>Max</th>
<th>Mode</th>
<th>Max / Mode</th>
</tr>
</thead>
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<tr>
<td>1,000</td>
<td>0.29473</td>
<td>0.405105</td>
<td>0.132555</td>
<td>3.05612</td>
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<tr>
<td>10,000</td>
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<td>0.215664</td>
<td>0.0619268</td>
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<td>0.113452</td>
<td>0.0268951</td>
<td>4.21831</td>
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<tr>
<td>1,000,000</td>
<td>0.28452</td>
<td>0.0565194</td>
<td>0.012455</td>
<td>4.53788</td>
</tr>
</tbody>
</table>

Table 1: Properties of the histograms where Vertex Count is variable.

References

