Denial of the alleged Łukasiewicz nightmare in system Ł₄

Abstract: The alleged Łukasiewicz nightmare of $(◊p & ◊q) \rightarrow ◊(p \& q)$ is not tautologous in Prover9; however, the equation recast in one variable as $(◊p & ◊\neg p) \rightarrow ◊(p \& \neg p)$ is tautologous. For the alleged nightmare as a consequent, Prover9 coerces it into the disavowed theorem: $\neg(◊p=◊q) \rightarrow ((◊p & ◊q) \rightarrow ◊(p \& q))$. In Meth8/VŁ₄, both propositions are tautologous. This speaks for Meth8/VŁ₄, based on the corrected modern Square of Opposition as an exact bivalent system, as opposed to Prover9, based on the uncorrected modern Square of Opposition as an inexact probabilistic vector space. These results form a non tautologous fragment of the universal logic VŁ₄.

We assume the method and apparatus of Meth8/VŁ₄ with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). For results, the 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Let $p, q$: Schrödinger's cat is alive; Schrödinger's cat is dead;
$\neg$ Not, ¬; $+$ Or, $\lor$; $\neg$ Not Or; $\&$ And, $\land$; $\neg$ Not And;
$>$ Imply, greater than, $\rightarrow$, $\Rightarrow$; $<$ Not Imply, less than, $\in$;
$\equiv$ Equivalent, $\equiv$, $\vDash$; $\neq$ Not Equivalent, $\neq$;
% possibility, for one or some, $\exists$, ◊, $M$; # necessity, for every or all, $\forall$, $\Box$, $L$;
$(p=p)$ T as tautology; $(p\oplus p)$ F as contradiction;
$(◊p<#p)$ C as contingency, $\Delta$; $(◊p>#p)$ N as non-contingency, $\nabla$;
$\neg( y < x) \ ( x \leq y), \ ( x \leq y)$.


Remark: The term "nightmare" was attributed to J-Y. Béziau in 2011 for the purpose of a four-valued schema of paraconsistent logic to evaluate systems based on numeric values such as -x, +x, -y, +y. The motivation was to discount the fact that the Ł4 logic system was provably bivalent (James, 2010, Estoril), and hence it was not mappable into a vector space, the continuing definition of paraconsistent logic.

This proposition is supposed to be egregious to logic system Ł₄: $(◊p & ◊q) \rightarrow ◊(p \& q)$. (1.0)

If possibly the cat is alive and possibly the cat is dead, then possibly both the cat is alive and the cat is dead. (1.1)

$(◊p & ◊q) >◊(p \& q)$; TTTT TTTT TTTT TTTT (1.2)

Assumptions: ((exists(p) & exists(q))).
Goals (exists(p & q)). Exhausted. (1.3)

Prover9 invalidates Eq. 1.0 to show Ł₄ is untenable as an alethic logic.

If we preload $p=\neg q$ as the antecedent to Eq. 1.0, then:

If possibly the cat is alive is equivalent to Not (the cat is dead), then if possibly the cat is alive and possibly the cat is dead, then
possibly both the cat is alive and the cat is dead.  \hspace{1cm} (2.1)
\[
%(p=-q)>(%(p&q)>(%p&%q)) \ ; \ \text{THTT TTTT TTTT TTTT} \quad (2.2)
\]
Assumptions: \ (exists(p<>-q)).
Goals: \ (exists(p)&exists(q)) \rightarrow (exists(p&q)). \ \text{Exhausted.} \quad (2.3)

Prover9 invalidates Eq. 2.0 to show \(\mathcal{L}_4\) is untenable as an alethic logic.

**Remark 2.3:** Eq. 2.3 shows Prover9 does not distribute the existential quantifier.

We rewrite Eq. 2.1 using one variable and its negation as respectively *alive* and *not alive*:
\[
(\Diamond p \& \Diamond \sim p) \rightarrow \Diamond (p \& \sim p). \quad (3.0)
\]

*If possibly the cat is alive and possibly the cat is not alive, then possibly both the cat is alive and the cat is not alive.  \hspace{1cm} (3.1)*
\[
%(p \& \%\sim p)>(%p \& \sim p) \ ; \ \text{TTTT TTTT TTTT TTTT} \quad (3.2)
\]
Assumptions: \ (exists(p)&-exists(p)).
Goals: \ (exists(p&-p)). \ \text{Theorem.} \quad (3.3)

Prover9 validates Eq. 3.0 to show \(\mathcal{L}_4\) is tenable as an alethic logic.

We explain Eqs. 1.2, 2.2, and 3.2 as rendered as tautologous in Meth8, but 1.3 as exhausted in Prover9 in this way. For more than one variable, the vector space for arity with Prover9 diverges from the bivalence inherent in \(\mathcal{V}\mathcal{L}_4\), in which modal operators and quantifiers are distributive. This speaks to Meth8/\(\mathcal{V}\mathcal{L}_4\), based on the *corrected* modern Square of Opposition as an exact bivalent system, opposed to Prover9, based on the uncorrected modern Square of Opposition as an inexact probabilistic vector space.

**Remark 3.2:** Meth8/\(\mathcal{V}\mathcal{L}_4\) distinguishes between Eqs. 2.0 and 3.0 by protasis and apodosis as:
\[
%p \& %q \ ; \ \text{CCCC CCCC CCCC CCCC} \quad (1.2.1.2)
\]
\[
%(p \& q)=(p=p) \ ; \ \text{CCCC CCCC CCCC CCCC} \quad (1.2.2.2)
\]
and
\[
%p \& %\sim p \ ; \ \text{CCCC CCCC CCCC CCCC} \quad (3.2.1.2)
\]
\[
%(p \& \sim p)=(p=p) \ ; \ \text{CCCC CCCC CCCC CCCC} \quad (3.2.2.2)
\]

We further note the following peculiar behavior of Prover9 which coerces and recasts what Prover9 disavows in Eq. 1.3 now as a theorem.

 Possibly the cat is alive is not equivalent to possibly the cat is dead implies that if possibly the cat is alive and possibly the cat is dead, then possibly both the cat is alive and the cat is dead. \hspace{1cm} (4.1)
\[
%(p@%q)>(%(p&%q)>(%p&%q)) \ ; \ \text{TTTT TTTT TTTT TTTT} \quad (4.3)
\]
Assumptions: \ -(exists(p) <-> (exists(q))
Goal: \ (exists(p)&exists(q)) \rightarrow (exists(p&q)). \ \text{Theorem.} \quad (4.2)