

Are there any imaginary pi formulas?

Yes, one using the cosine. The most famous pi formula was written by Leibniz.

$$\sum_{n=0}^{\infty} \frac{-1^n}{2n+1} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \cdots = \frac{\Pi}{4} \quad (1)$$

The imaginary version of this formula generates the same output, so it is also a pi formula.

$$\sum_{n=0}^{\infty} \frac{(i^n + i^{-n})/2}{n+1} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \cdots = \frac{\Pi}{4} \quad (2)$$

The table below illustrates the numerator in the above imaginary formula (it is the cosine).

$n$	0	1	2	3	4	5	6	7
$i^n$	1	$i$	-1	$-i$	1	$i$	-1	$-i$
$i^{-n}$	1	$-i$	-1	$i$	1	$-i$	-1	$i$
$(i^n + i^{-n})$	2	0	-2	0	2	0	-2	0
$(i^n + i^{-n})/2$	1	0	-1	0	1	0	-1	0

The last line of the table shows how the cosine deletes every other term, and rotates the sign. This pattern can be seen in the imaginary formula above. The pattern can also be seen in the  $\cos(x)$ .

$$\sum_{n=0}^{\infty} \frac{(i^n + i^{-n})}{2} \times \frac{x^n}{n!} = \frac{1}{1} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \dots \quad (3)$$