Refutation of the incremental model in HOL4, recast as mechanized overloading

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Abstract: We evaluate the incremental model in prover HOL4 as not tautologous. The model also relies on induction, the weakest form of inference. That the mechanized overloading of constants somehow avoids the incremental model does not follow because of reliance on induction which is an incremental approach. Hence the conjecture of mechanized overloading serves to recast and exasperate the refuted incremental model, forming a non tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨, ∪; - Not Or; & And, ∧, ∩, ·, ⊓; \ Not And;
> Imply, greater than, →, ⇒, ≻, ⊃, ↦, ≲; < Not Imply, less than, ∈, ⊆, ⊂, ⊢, ⊭, ⊑; = Equivalent, ≡, ⇔, ↔, ≈, ≃; @ Not Equivalent, ≠, ⊤;%
% possibility, for one or some, ∃ ∃◊, M; # necessity, for every or all, ∀ □ L;
(z=z) T as tautology, T, ordinal 3; (%z=z) F as contradiction, Ø, Null, ⊥; zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∨, ordinal 2;
~( y < x) ( x ≤ y), ( x ⊆ y), ( x ⊂ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.


Remark 0: Our emphasis is underlined.

Abstract Isabelle/HOL augments classical higher-order logic with ad-hoc overloading of constant definitions—that is, one constant may have several definitions for non-overlapping types. In this paper, we present a mechanised proof that HOL with ad-hoc overloading is consistent. All our results have been formalised in the HOL4 theorem prover.

2.3 Inference system … We define ⊢ as an inductive relation comprised of the standard inference rules of higher-order logic, plus whatever axioms are present in the theory.

4 Soundness … The proof comprises around 700 lines of HOL4, and is a mostly straightforward induction on the derivation of the ⊢ judgement.

5 Model construction In the previous section, we showed that all provable sequents of a theory are satisfied in all models of the theory. This section tackles the missing puzzle piece before we can consider consistency: to show that if the theory is constructed by definitional extension of the predefined initial contexts … , then a model exists. … In the absence of overloading, the model can be constructed incrementally: when theory extension cannot change the meaning of previously introduced types and constants, any model (δ,γ) of the old theory can be updated to create a model of the new theory, e.g., (δ(⌜ty→x⌝), γ(⌜c→y⌝)) where x, y models the new types and constants ty, c. This has the very pleasant consequence that, when augmenting a model to accommodate a theory update, the details of how exactly the previous model was constructed can be ignored. Hence there is no need to explicitly write down a model for the whole theory: it suffices to prove its existence by induction on the context.
LET \( p, q, r, t, x, y: c, \delta, \gamma, t, x, y. \)

\[(q \& r) = ((q \& ((t \& y) > x)) \& (r \& (p > y))) ; \]

\[
\begin{array}{cccc}
TTTT & TTT & TTT & FTTF \quad (32) \\
TTTT & TTTT & TTTT & TTTF \times (16) \\
TTTT & TTFF & TTTT & TFFT \\
TTTT & TTTT & TTTT & TTTT (16)
\end{array}
\]

**Remark 5.1.2:** Eq. 5.1.2 as rendered is *not* tautologous. This refutes the incremental model in Hilbert-style HOL4 which relies on induction as the weakest form for inference, if even valid. That the mechanized overloading of constants somehow avoids an incremental model does not follow because of reliance on induction which is an incremental approach. Hence the conjecture of mechanized overloading serves to recast and exasperate the refuted incremental model.