Vertical Motion of Rockets in Presence of Gravity

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Abstract

The article derives the standard rocket equation for vertical motion in presence of gravity and finally moves on to show certain unphysical results. These results can occur only due to flaws in the classical theories.

Introduction

Vertical motion of rockets in presence of gravity has been investigated to derive certain results which point to errors in the classical theories.

Rocket Equation in Gravity

We consider the vertical motion of a rocket under gravity. The rocket plus ejected mass is our system. The velocity of the rocket is u with respect to earth. The velocity of the ejected mass is a constant v in the downwards direction with respect to the rocket.

$$\frac{(m-|dm|)(u+du)+(u-v)|dm|-mu}{dt} = -mg; dm < 0 \Rightarrow dm = -|dm|, du > 0, v > 0 \quad (1)$$

[if u is small $u - v$ will be negative[downward direction with respect to earth]; if u is large $u - v$ will be positive[upward direction]; u is in the upward direction away from the earth which has been reckoned positive.]

$$\frac{mu + mdu - u|dm| - |dm|du + u|dm| - v|dm| - mu}{dt} = -mg$$

$$\frac{mdu - v|dm|}{dt} = -mg$$

$$m\frac{du}{dt} - v\frac{|dm|}{dt} = -mg \quad (2)$$

$$du - v\frac{|dm|}{m} = -gdt$$

$$du = v\frac{|dm|}{m} - gdt \quad (3)$$

$$du = -v\frac{dm}{m} - gdt; dm < 0$$

$$du > 0 \Rightarrow v\frac{|dm|}{m} > gdt \Rightarrow \frac{|dm|}{dt} > \frac{mg}{v}$$

If $v = constant$ then b y integration we have the standard rocket equation[1],

$$u - u_0 = -vln\frac{m}{m_0} - \int_{t_0}^{t} gdt; v > 0 \quad (4)$$

$$u - u_0 = vln\frac{m_0}{m} - \int_{t_0}^{t} gdt; v > 0 \quad (5)$$

Vertical Thrust

From (3) $du = v\frac{|dm|}{m} - gdt$

$$\frac{du}{dt} - \frac{v}{m}\frac{|dm|}{dt} = -g$$

$$\Rightarrow \frac{1}{m}(F - mg) + g = \frac{1}{m}v\frac{|dm|}{dt}$$

where F is the vertical thrust due to ejection of fuel

$$F = v\frac{|dm|}{dt} = -v\,\frac{dm}{dt} \quad (6)$$

Work Energy Theorem extended to variable mass Systems

We deduce the work energy theorem for variable masses

$$dW = \vec{F}.d\vec{r} = \frac{d\vec{p}}{dt}.d\vec{r} = \frac{d(m\vec{u})}{dt}.d\vec{r} = \frac{d(m\vec{u})}{dt}.\frac{d\vec{r}}{dt}dt = \left[m\frac{d\vec{u}}{dt} + \vec{u}\frac{dm}{dt}\right].\vec{u}dt$$

$$dW = m\frac{d\vec{u}}{dt}\vec{u}dt + \vec{u}.\vec{u}\frac{dm}{dt}dt$$

$$= \frac{1}{2}m\frac{d(\vec{u}.\vec{u})}{dt}dt + u^2\frac{dm}{dt}dt$$

$$dW = \frac{1}{2}mdu^2 + u^2\frac{dm}{dt}dt \quad (7)$$

For constant mass $dm = 0$ and we have the differential expression for the standard work energy theorem[2]: $dW = \frac{1}{2}mdu^2 \Rightarrow W = \frac{1}{2}mu_f{}^2 - \frac{1}{2}mu_i{}^2$

If mass decreases $\frac{dm}{dt} < 0$, dm negative for positive dt. If dm is zero we obtain the conventional work energy theorem for affixed mass.

Rate of Ejection

For rockets, considering the rocket only [and not the ejected mass]

$$dW = \frac{1}{2}mdu^2 + u^2dm = mudu + u^2dm = (F - mg)dr; dm < 0, du > 0$$

$$dW = mudu + u^2dm = (F - mg)dr \quad (8)$$

Since $dm = -|dm|$we may write

Dividing both sides of (8) by dt,

$$mu\frac{du}{dt} - u^2\frac{|dm|}{dt} = (F - mg)u$$

$$\Rightarrow m\frac{du}{dt} - u\frac{|dm|}{dt} = (F - mg\ ) \quad (9)$$

But according to (6)

$$F = v\frac{|dm|}{dt}$$

From (6) and (9)

$$m\frac{du}{dt} - u\frac{|dm|}{dt} = \left(v\frac{|dm|}{dt} - mg\right)$$

$$m\frac{du}{dt} = (u + v)\frac{|dm|}{dt} - mg \quad (10)$$

Using (4) with (10) we obtain: $u - u_0 = -vln\frac{m}{m_0} - \int_{t_0}^{t} gdt; v > 0$

$$m\frac{du}{dt} = \left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt + v\right)\frac{|dm|}{dt} - mg \quad (11)$$

By differentiating (5)

$$\frac{du}{dt} = mv\frac{m_0}{m}\frac{1}{m_0}\frac{dm}{dt}$$

Therefore (11) reduces to

$$mv\frac{m_0}{m}\frac{1}{m_0}\frac{dm}{dt} = \left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt + v\right)\frac{|dm|}{dt} - mg$$

$$-v\frac{|dm|}{dt} = \left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt + v\right)\frac{|dm|}{dt} - mg$$

$$\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right)\left|\frac{dm}{dt}\right| + 2v\left|\frac{dm}{dt}\right| = mg$$

$$\left|\frac{dm}{dt}\right|\left[\left(u_0 - vln\frac{m}{m_0} + \int_{t_0}^{t} gdt\right) + 2v\right] = mg$$

$$\frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right]|dm| = g|dt| \quad (12)$$

Integrating the above and using (4) we obtain,[remembering $dm = -|dm]|$

$$-\int \frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right]dm = \int gdt$$

Again from (5)

$$\int gdt = u_0 - vln\frac{m}{m_0} - u \quad (13)$$

Therefore,

$$-\int \frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right]dm = u_0 - vln\frac{m}{m_0} - u \quad (14)$$

Differentiating both sides of the above with respect to m we have,

$$-\frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right] = \frac{d}{dm}\left[u_0 - vln\frac{m}{m_0} - u\right]$$

$$-\frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right] = -v\frac{m_0}{m}\frac{1}{m_0} - \frac{du}{dm}$$

$$\frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right] = -v\frac{1}{m} - \frac{du}{dt}\frac{dt}{dm}$$

$$\frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right] = -v\frac{1}{m} - \frac{1}{m}(T - mg)\frac{dt}{dm} \quad (15)$$

$$\frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right] = -v\frac{1}{m} - \frac{1}{m}\left(-v\frac{dm}{dt} - mg\right)\frac{dt}{dm}$$

$$\frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right] = g\frac{dt}{dm}$$

$$\frac{dm}{dt} = \frac{g}{\frac{1}{m}\left[\left(u_0 - vln\frac{m}{m_0} - \int_{t_0}^{t} gdt\right) + 2v\right]} \quad (16)$$

But $\frac{dm}{dt}$ is negative

The formula was quite natural had it not been for the following fact

Setting g=0

$$\frac{dm}{dt} = 0 \quad (17)$$

Equation (17) holds for u changing with time: even for an accelerating rocket[in the absence of gravity].That is quite unphysical.

An Unphysical Result Again!

We recall equations(11) and (2) [rewriting them with their old numbering]

$$m\frac{du}{dt} = (u + v)\frac{|dm|}{dt} - mg \ (11)$$

$$m\frac{du}{dt} - v\frac{|dm|}{dt} = -mg \quad (2)$$

From (2)

$$m\frac{du}{dt} = v\frac{|dm|}{dt} - mg \quad (18)$$

From (11)

$$m\frac{du}{dt} - u\frac{|dm|}{dt} = v\frac{|dm|}{dt} - mg \ (19)$$

From (18) and (19) we have $u\frac{|dm|}{dt} = 0$

For non zero u, we have the strange , unphysical result

$$\frac{|dm|}{dt} = 0 \ (20)$$

Something is wrong with the classical laws

Conclusions

As claimed earlier we have arrived at results that indicate towards errors in the classical theories

References

1. Wikipedia: Tsiolskovsky Rocket Equation

https://en.wikipedia.org/wiki/Tsiolkovsky_rocket_equation#Derivation

2. MIT, Open Course Ware, equation (14)

https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-07-dynamics-fall-2009/lecture-notes/MIT16_07F09_Lec14.pdf