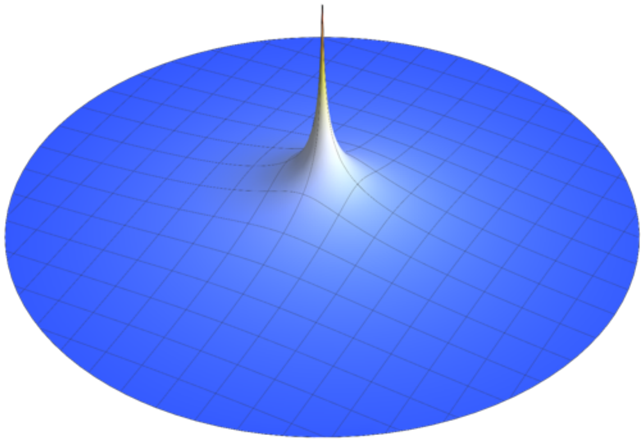
STATISTICAL PRINCIPLES OF NATURAL PHILOSOPHY

Guo Tao



2020-2-16

SIMM

Shanghai · China

Title

Statistical Principles of Natural Philosophy

Author:

Tao Guo[a,*]

Affiliation:

[a]Center for Drug Delivery System, Shanghai Institute of Materia Medica, Chinese Academy of Sciences, 501 Haike Road, Shanghai 201210, China

***Corresponding Author:**

Center for Drug Delivery System, Shanghai Institute of Materia Medica, Chinese Academy of Sciences, 501 Haike Road, Shanghai 201210, China; Tel: +86-18602131982; E-mail: gotallcn@gmail.com (Tao Guo)

Abstract

Currently, the natural philosophy (Physics) is short of a most basic model and a whole set of self-consistent explanations, for which my article attempts to discuss some related issues. Starting from the most basic philosophical paradoxes, here I deduced a physical model (the natural philosophy outlook) to describe the running law of the universe. Based on this model, a mathematical model was established to describe the general diffusion behavior of moving particles, of which the form without external field was simply verified. For the first time, the gravitational force and Relativistic effect was interpreted as the statistical effect of randomly moving particles in this article. Thus, the gravitational force and Special Relativistic effect were actually integrated into the equation (achieved by selecting the initial wave-function with specific norm when solve it), and the cause of stable particle formation was also revealed. The derived equation was self-consistent with the hypotheses stated in physical model, which also proved the reliability of physical model to some extent. Some of these ideas may have potential reference values for understanding the essence of Quantum Mechanics, Relativity and Superstring Theory, as well as for further understanding of nature and the manufacture of quantum computers.

1. Introduction

"Birds flock and sing when the wind is warm, Flower-shadows climb when the sun is high"[1], our home is overflowing with vigor! However, light years away, it seems like a dead silence; Human beings sit on the vast earth, but in the solar system it is just a "little blue dot"…What force are these mysterious phenomena which are as far apart as heaven and earth in our eyes arranged by? How huge is the universe? Why is it like this? Which mechanism does it run under? Is there a beginning or an end? Where does the huge energy come from? Will it run out? How do the concepts of time, space and speed come into being? Will the total entropy in the universe continue to increase? …All along, these are more difficult questions to answer. "Know the enemy and know yourself, and you can fight a hundred battles with no danger of defeat"[2], to explore the origin of the universe is the only way for human beings to conquer nature.

Since ancient times, human beings have gradually deepened their understanding for the laws of nature and the universe, which could be roughly divided into the following three stages:

In the initial period of Aristotle, Ptolemy, Copernicus, Kepler and so on, people's explorations for nature were restricted not only by the level of productivity development at that time, but also by various political conditions[3]. The explorations of nature and the universe were slow and the understandings were also relatively shallow. By the time of Galileo and Newton, productivity was greatly improved and people also had relatively strict logics and scientific thinking methods. Under the guidances of Newton Mechanics and Calculus, the levels of understanding nature had been greatly improved. However, Newtonian Mechanics held that gravitation was generated directly by mass and was not affected by motion or energy. The regulations on gravitation, inertia and acceleration were all based on the simple rules of experience from the perspective of philosophy (scilicet axioms. Although the definition of universal gravitation came from Newton, Galileo had set up the empirical rules according to the observation; the essence of inertia or acceleration was not clear), and the whole universe was relatively static.

In modern times, Einstein's General Relativity came into being, and people's ability to understand natural laws and predict natural phenomena had improved tremendously. According to General Relativity, gravitation or space-time field is affected by matter, energy and motion, which leads to kinds of magic changes in motion. On this basis, the existences of black holes and other celestial bodies is predicted[3,4]. With the rapid development of Quantum Mechanics, people's understanding of the micro scale had been greatly improved, resulting in a new era of Philosophy (Copenhagen School's interpretation of Quantum Mechanics) as well as a large number of modern technical means[3].

However, what are the physical principles behind Quantum Mechanics? How to perceive quantum entanglement and Wheeler's delayed-choice experiment, and whether the Dirac equation with Special Relativity effect is correct or not in essence? What is the more essential reason behind the curved space-time and the principle of the Special Relativity effect? What's more, how can dark matter, dark energy and inexplicable repulsion, which are often mentioned in modern cosmology, be explained? …

All this time, people have not made efforts to explore the answers behind these substantive questions (without establishing a more basic physical model), but stayed on the superficial surface of quantum physics. Based on classical physics (such as Newton Mechanics), the formulas were deduced from a mathematical point of view, and the conclusions of Special Relativity or the constraints of Lorentz covariant were added to various equations, which seemed to be very fragmented (such as Dirac equation and Quantum Field Theory). All these practices have led to the emergence of various theories, but have not fundamentally solved the problem[3,5]. The whole physics building seems to have improved under some explanations such as the so-called Standard Model and Superstring Theory, but none of them is completely satisfactory. The Standard Model and the Grand Unified Theory etc. only integrate the previous models from the views of the mathematics and the surface of physical phenomena, so they cannot perfectly cover the gravitational effect (irreducible normalization after introducing gravitation). The Superstring Theory seems to cover all the known successful theories because it includes more degrees of freedom (higher dimensions). However, higher dimensions cannot solve more practical problems. On the contrary, because of a lot of additional false possibilities which makes the equations extremely difficult to solve, the requirements for mathematical skills have reached an amazing level. Moreover, "string" is not and should not be a most basic physical morphology. In addition, the theory of Loop Quantum Gravity is not perfect, and there seem to be more difficulties than can be solved. In view of the above problems, it is necessary to further understand the essence behind physical phenomena or physical constraints and to establish a more basic physical model.

Starting from the most basic philosophical paradoxes, this article probed into a series of even deeper and more essential problems in physics, and tried to establish a most basic physical model to describe the running law of the universe. Based on this, a self-consistent mathematical equation was established in a more concise form. This equation might have unified Quantum Mechanics and (General and Special) Relativity and solved the problem that it could not be renormalized when integrating Quantum Mechanics with General Relativity. Furthermore, the cause of stable particle formation was also revealed. The frameworks of physical and mathematical models derived in this article may provide guidance for the interpretation and prediction of other natural phenomena. However, many viewpoints in this article were put forward for the first time, and there must be some immature ideas even defects. I heartily beg that my readers may read these with candour, not too reprehend the un-precise or individual mistakes in some more complicated details but willing to bestow constructive suggestions for some rough and flawed points in the main idea of this article.

2. Methods

Based on philosophical paradoxes, this article expounds many ideological experiments. Mathematica 12.0 for Mac and Linux (*Wolfram Research Inc.*) were used for all of the mathematical calculation and the operating system were macOS High Sierra 10.13.6 and Red Hat Enterprise Linux Server (Release 6.3 Kernel Linux 2.6.32-279.el6.x86_64). The solutions to each specific problem could be find in the Supplementary Information. If no specific parameter method was specified, the default value in software system had been used. The effective numbers of numerical methods were not less than 6. In addition, it should be noted that some "abnormal" parameter configurations or script details in Supplementary Information of this article were actually helpless actions, of which the aim was to deal with software bugs (For instance, in Fig. 5, different settings of line widths were used for different midlines, such as the lines at $x = 0$, in the same graphics. Only in this way could the midlines look beautiful and equal in width). If the results in this article are to be reproduced completely, the software version used must be identical with that of this article.

3. Results and Discussions

3.1 Can the World be Understood?

The innate knowledges possessed by the human beings are the perceptual knowledges that can correspond to the external stimulations, which are preserved through the long interaction and internalization between organism and natural environment[6,7]. Therefore, the innate knowledges are excellently reliability. The acquired knowledge or experience obtained by human beings in a model of innate cognitive (even if such a cognitive model has more or less subjective factors) should still be reliable and applicable in the same cognitive model if practiced in the same cognitive model. In addition, in view of the relative stability and repeatability of some external conditions (i.e. the translation invariance of time and space), the innate knowledge and acquired experience possessed by human beings should also be reliable in the whole range of human practice.

Therefore, the theories established by human beings, even if they only recognize from the perspective of human beings on the earth as cosmic dust, and even if there are all kinds of narrow or mistakes (this "mistake" is relative, which means that the appearance and form of things reflected in human consciousness are not the original appearance and form of things), as long as they can effectively explain and predict the phenomena we observed, are successful theories, although we cannot confirm whether they are completely correct truth[6].

3.2 The World in the View of Philosophical Paradoxes

The reason why the world has infinite energy and runs endlessly is that there must be a series of philosophical paradoxes restricting each other[8,9]. Only with these contradictory constraints can the world become balanced and logical (self-consistent). Under the guidance of this view, this article summarized three axioms as follows:

AXIO 1: There are substances in the world.

It is a very old philosophical topic whether there are substances or not in the world. However, it is the original basis for rational inference and logical expansion in this article. There are only two possible situations in this world: either there is substance or there is no substance at all in the world. The fact is obvious: there are substances existence in this world. On average, however, these substances are so thin that they are almost nonexistent[10]. As a result, the world (or at least within the range of human observation) is as thin as it is without substance.

AXIO 2: The substances are inhomogeneous.

If the world is full of substance, there are only two possibilities for its distribution: It's either homogeneous or inhomogeneous. Obviously, the distribution of substance in this world is inhomogeneous within a certain range that we have observed. However, there is no reason for these substances to be "favor one more than another", that is, there are not more opportunities to be distributed here and less opportunities to be distributed there. Therefore, it should be considered that the probability of the distribution of substances in every place (not limited to 3-dimensional) is equal or it is homogeneous from the view of large-scale[11,12]. Meeting both distribution inequality and probability equality, the substances in this world must exist in quantum form. This fact does not need to be discussed for that it has been verified by various physical experiments. There is no reason for the world to be "favor one more than another" and the probability between "quantum dots" should be the same. Meeting both the above two statements of "distribution inequality and probability equality" also determines that the world is a paradox body with uniform probability but inhomogeneous micro (or several dimensions) performances.

AXIO 3: These substances are moving.

The substances observed in the world are moving, or from the perspective of human understanding the substances in the world are moving. In any case, the world can be interpreted as dynamics rather than static. So, which is the more reasonable moving pattern?

The current understandings are as follows: Photons without stationary mass are the fastest substances in the universe. It is impossible to accelerate species with stationary masses (such as an electrons) to the speed of light. When they reach this speed, their masses will become infinite and their energy consumptions will become infinite (according to the conclusion of Relativity). Therefore, there is no species faster than the speed of light, even if there is, it cannot transmit information. However, from this point of view, the essence of quantum entanglement cannot be understood, the phenomenon of Wheeler's delayed-choice experiment is amazing, and the essence that gravitation can spread out from black hole is not easy to explain…

Therefore, this view is broken through here, and it is considered that photons are the fastest species that can transmit information found or perceived by human beings at present, and photons have light mass (see Section **3.7**: "Speculation on photonic structure"), and the motion cannot substantially change the physical mass. Particles with a smaller mass-level than photons, even if they can transmit information, cannot be perceived (or consciously perceived) by human beings at present and the limiting speed of which is faster. Therefore, when the particle speed is fast to a certain extent, it must "split" into particles of lower mass-level until the speed reaches infinity and the mass becomes infinitesimal (in the framework of "the substances in this world must exist in quantum form", Section **3.3.5.2** will confirm they are possible that "the particles of lower mass-level can form the particle of higher mass-level" and the opposite process).

From this point of view, the whole universe will show the motion spectacle as follows: For a particle with infinitesimal mass, its velocity can reach infinity. So, no matter how large the space is, this infinitesimal particle can exist at any position in an instant. Therefore, it can be everywhere, and the large space is an infinitesimal space without the concept of time or distance for it, and such particles are infinitely great relative to such space or the space cannot perceive the motion at all. For infinitesimal particles, there is no concept of space or time, so there is no concept of energy. If the universe is composed of infinitely many such moving particles (they are infinitesimal particles, so there is no collision between them), it will not consume the so-called energy and can run forever. Once the particle of larger mass-level (particle swarm of infinitesimal particles) was observed, the speed of which would decrease (the relationship between the mass of the undisturbed particle swarm and their average speed obeys the law of Maxwell distribution, see Part 1 of Supplementary Information for details). Simultaneously, the concepts of time, space, speed, mass and energy will arise. Therefore, there is no concept of time, space or speed, and no concept of mass or energy in the universe, all of which are caused by the presentations observed from different perspective. Although the particle is infinitesimal, it is infinite relative to the universe; though the universe is infinitely great, it is infinitesimal relative to the infinitesimal particles. As the velocity of the particle approaches infinity, it would lose the concept of motion. The universe is both large and small, in which the substances move and do not move, and the concepts of time, space, speed, quality and energy are both existence and absence, which are several pairs of mutual-constraining paradoxes.

The validities of the above three axioms are obvious, and their existences depend on the constraints of the other ends of the mutual-constraining paradoxes. Here, only the meaningful ends are selected for working on. In addition, under the constraints of logic, the concepts derived from the 3 axioms are also contradictory constraints. Only the logics which are constrained by paradoxes are complete and self-consistent. On the basis of the above three axioms, this article makes a reasonable inference and extracts the following 3 hypotheses:

HYPO 1: The universe is composed of infinitely many uniform particles with infinite speed and infinitesimal mass.

The concepts of "infinity" and "infinitesimal" are equivalent to that in Mathematical Analysis. That the masses of the particles are uniform is relative to its standard deviation, and the concept of "uniform particle" mentioned below is the same meaning.

HYPO 2: The speeds of infinitesimal particles in 3-dimensional space are equal and the directions of the motion are random.

As mentioned in **AXIO 2**, these infinitesimal particles are formed according to the same law, so the masses and velocities (or momentums) between them should be equal or be equal relative to their standard deviations (the concepts of equal masses and speeds mentioned below are the same meanings), and the probabilities of directions in each dimension are also equal, there is no reason to be uneven.

HYPO 3: There is no interaction between infinitesimal particles.

In the world we observed, interaction force exists everywhere. However, this is not necessary between infinitesimal particles. For infinitesimal particles, it is assumed that there is no traditional interaction between them (such as gravitation), and the macroscopic force (or interaction) is caused by the statistical effect of the infinitesimal moving particles. This assumption will not conflict with the concept of classical force, but also be helpful to establish the equation and expand the self-consistent range.

These are the 3 basic characteristics (hypotheses) extracted from the 3 basic axioms of the world. Next, the model will be built on the basis of the 3 assumptions.

3.3 Model Building Based on Philosophical Paradoxes

On the basis of the above 3 axioms and 3 hypotheses, this article infers that there are only four possibilities for the scale (big or small) of space (in any dimension) and the number (many of few) of particles in any local domain, and the four possibilities are independent in different local domains. That is because the world is motional (in infinite dimensions) and inhomogeneous, and motion and non-homogeneity are two independent quantities. Because of the movement of particles, when a certain number of particles are observed without spatial difference, the concept of velocity will be generated in the world. If it is in infinite dimensions, the concept of velocity will characterize by the concept of time and distance; Due to the inhomogeneous of distribution of particles, when a certain number of particles are observed with spatial difference, the concept of density will be generated in the world. If it is in infinite dimension, the concept of density will characterize by the concept of the scale (distance of another degree of freedom in disguise) and the number of particles with spatial difference (another single degree of freedom different from distance). If the latter two degrees of freedom are fixed (that is to say, the two degrees of freedom or the entities they represent are used as reference to determine the inspection object), they will characterize by the degrees of freedom of distance in the other two dimensions. Therefore, there are four independent dimensions in this world, which are four dimensions (three dimensions characterize by the concept of space in our consciousness, one dimension characterizes by the concept of time in our consciousness). In principle, 3-dimensional space and 1-dimensional time coordinate can describe all of the natural phenomena. The method in so-called multidimensional space of String Theory solves the problem of 4-dimensional space-time finally.

In order to understand the world more easily and intuitively, people are used to putting various abstract results and conclusions into the world we are familiar with. In principle, if 4-dimensional variable coordinate system is adopted by coordinate transformation, the operation may be simple, but it will bring difficulties for understanding the problem. Einstein's General Relativity uses a 4-dimensional variable coordinate system (space-time), which is an "immersion perspective" with a sense of participation. Although individual immersive physical events (such as the constant speed of light) are more consistent with physical observation, it will eventually bring difficulties to understand the essence of physical problems; In absolute space-time, the coordinate system of 3-dimensional space and 1-dimensional time is "God perspective", which is helpful for people to look at and understand problems from a macroscopic perspective. Of course, no matter which perspective, it does not affect the description for physical phenomena in 4-dimensional space-time. Finally, the evolution of various phenomena would eventually be measured and understood in the flat coordinate system that we are familiar with at present.

It should be emphasized that the "God perspective" (or "absolute space-time") mentioned here also has relativity. When the absolute space-time is used as the reference system, the particles in it should meet the conditions given by **HYPO 1–3** (it can also be considered to be the classical "inertial reference system" here). This means that if the particle swarm moves as a whole set, the absolute space-time will also move with it; It is meaningless that the absolute space-time (or the corresponding absolute coordinate system) does not follow the whole movement of the particle swarm.

Since our goal is to understand and grasp the world, it is unnecessary to use a relatively variable view of space-time. Sometimes, the concept of absolute space-time is more advantageous for building models and understanding laws. In view of the above analysis, the physical and mathematical models will be established in the 4-dimensional (3-dimensional space plus 1-dimensional time) absolute coordinate system.

3.3.1 Physical Model

Here, **HYPO 1–3** are summed up as the physical model of this article: The universe is composed of infinitely many uniform particles with infinite speed and infinitesimal mass; The speeds of infinitesimal particles in the 3-dimensional space are equal and the directions of the motion are random; There is no interaction between infinitesimal particles. No more rules are needed.

3.3.2 Special Relativistic Effect of Infinitesimal Particles

It will be proved that there is (Special) Relativistic effect in the above mentioned physical model (Section **3.3.1**). Once again, it is emphasized that the speeds of these particles (In this article, the "infinitesimal particles" described in the above physical model are called "particles", "1-order particles" or "tiny particles", while the larger mass-level particles composed of k particles are called "k-order particles") are exactly the same (or $\sigma \ll c$, where c is the mean value of particle speeds, and σ is their standard deviation), and the directions of the motions are random in the 3-dimensional space. So, that is the random vectors with equal norm in Euclidean space. In this article, the statistical method will be used to prove that there is the Special Relativistic effect in the vector swarm composed of such a group of vectors. When a group of particles in the same 3-dimensional space move in one direction on average (or their centroid move in one direction), they will lose some moving probability in other directions for the statistical effect, or the moving trend in other directions will decrease, so there will be Special Relativistic effect. This phenomenon will be explained quantitatively in detail below.

As already mentioned, it is assumed that the speed of the particle is c ($c > 0$), and the direction is evenly distributed in the 3-dimensional space. In the system composed of moving particles, the system with an average velocity being 0 (i.e. the "absolute space-time" mentioned earlier) is called the stationary reference system (namely $\mathcal{R}_0$), and the 3-dimensional Cartesian (rectangular) coordinate system $Oxyz$ is established for it; The particle swarm formed by some particles in a certain period of time and moving at an average velocity u is called the moving reference system (namely $\mathcal{R}_u$). Let the direction of the velocity of $\mathcal{R}_u$ parallel to the z-axes in the direction of increasing z. Then, the mean value of the velocity component of particles in (along) z-axis of $\mathcal{R}_u$ must be u. Assuming that all particles in $\mathcal{R}_u$ are vectors with starting point at the origin of the coordinates, and taking point $(0, 0, u)$ as the dividing point of z-axis, the vectors in $\mathcal{R}_u$ can be resolved into two parts: the components of the vectors above it and the components of the vectors below it. These vectors randomly enter (are extracted into) $\mathcal{R}_u$ from $\mathcal{R}_0$ with equal probability. Therefore, the distribution of vectors in $\mathcal{R}_u$ can be thought of as the mixture distribution of the vector distribution of the components above the dividing point and the vector distribution of the components below the dividing point. When the mean value of the components in the z-axis of the mixture distributions is u, the mixture weight w can be determined. With this value as the reference, the distribution of these vectors forming the mixture distribution in the x-axis (or y-axis) can be determined, so their standard deviation σ_u can be obtained also. Once σ_u is determined, according to the Central Limit Theorem, the distribution of the components in the x-axis (or y-axis) of the sum for these vectors can be determined—approximately follows the Normal distribution, and its standard deviation depends on the number k of vectors, namely

$$\sigma_{u,k} = \frac{\sigma_u}{\sqrt{k}}. \tag{1}$$

If the three Normal distributions are the same and independent in the three coordinate axes (x, y, z), the velocity of the particles (Suppose its mass is μk, where μ is the mass of single particle, the same below) with larger mass-level composed of these particles follow the Maxwell distribution with scale parameter $\sigma_{u,k}$; Even if they are not independent to each other, as mentioned above, for such a system, the corresponding relationship still holds. So $\sigma_{u,k}$ is directly proportional to the average speed $\overline{v}_{u,k}$ of the larger mass-level particle (μk), namely

$$\overline{v}_{u,k} = 2\sqrt{\frac{2}{\pi}}\sigma_{u,k}. \tag{2}$$

To insert Equ. 1 into Equ. 2, we obtain

$$\overline{v}_{u,k} = 2\sqrt{\frac{2}{\pi}} \cdot \frac{\sigma_u}{\sqrt{k}}. \tag{3}$$

The distribution of the vectors is relatively simple in $\mathcal{R}_0$. Supposing that the standard deviation of their components in the x-axis (y- or z-axis) is σ_0, similarly, the average velocity of the particles (μk) with larger mass-level formed by them is

$$\overline{v}_{0,k} = 2\sqrt{\frac{2}{\pi}} \cdot \frac{\sigma_0}{\sqrt{k}}. \tag{4}$$

When particles of the same mass-level are formed in $\mathcal{R}_u$ and $\mathcal{R}_0$ respectively, the ratio between their average speeds (Equ. 3 to Equ. 4) is

$$\frac{\overline{v}_{u,k}}{\overline{v}_{0,k}} = \frac{\sigma_u}{\sigma_0}. \tag{5}$$

Therefore, the ratio of σ_u to σ_0 is the ratio between the speeds of particles with higher mass-levels in $\mathcal{R}_u$ and $\mathcal{R}_0$ respectively. More detailed introduction will be presented in the following.

As mentioned above, in the 3-dimensional Cartesian coordinate system which has been built in the stationary reference system $\mathcal{R}_0$, if the moving reference system $\mathcal{R}_u$ moves along the z-axis at the velocity u, then the x- and y-coordinates are equivalent, so only the x-coordinate is considered in the following. In $\mathcal{R}_0$, if the components of these vectors are uniformly distributed in the z-axis in the interval $[-c, c]$, then the probability density in the x-axis is:

$$\mathcal{D}(\theta,\eta) = c \cdot \cos\theta \cdot \sin\cos^{-1}\eta, \tag{6}$$

where, the random variables $\Theta \sim U(-\pi, \pi)$, $H \sim U(-1, 1)$. In this article, the random variables (vectors) are expressed in capital letters, and the values of the random variable (vectors) are expressed in corresponding lower-case letters. The component distribution of the vectors whose components are above $(0, 0, u)$ in the x-axis is recorded as $\mathcal{D}_1$, and its probability density is written as

$$\mathcal{D}_1(\theta,\eta) = c \cdot \cos\theta \cdot \sin\cos^{-1}\eta, \tag{7}$$

where, the random variables $\Theta \sim U(-\pi, \pi)$, $H \sim U(\frac{u}{c}, 1)$; Correspondingly, the component distribution of these vectors in the z-axis is recorded as $\mathcal{D}_3$, namely $\mathcal{D}_3 \sim U(u, c)$. The component distribution of the vectors whose components are below $(0, 0, u)$ in the x-axis is recorded as $\mathcal{D}_2$, and its probability density is written as

$$\mathcal{D}_2(\theta,\eta) = c \cdot \cos\theta \cdot \sin\cos^{-1}\eta, \tag{8}$$

where, the random variables $\Theta \sim U(-\pi, \pi)$, $H \sim U(-1, \frac{u}{c})$; Correspondingly, the component distribution of these vectors in the z-axis is recorded as $\mathcal{D}_4$, namely $\mathcal{D}_4 \sim U(-c, u)$. When the mean value of the components of the mixture distribution for $\mathcal{D}_3$ and $\mathcal{D}_4$ in the z-axis is u, their mixture weights are $\frac{c+u}{2c}$ and $\frac{c-u}{2c}$ respectively. The mixture distribution of $\mathcal{D}_1$ and $\mathcal{D}_2$ can be calculated according to their weights (the analytical form of the mixture distribution cannot be given in this article temporarily), thereby the standard deviation of the velocity components in the x-axis of the particles in $\mathcal{R}_u$ is

$$\sigma_u = \frac{\sqrt{c^2 - u^2}}{\sqrt{3}}. \tag{9}$$

Evaluating the ratio between Equ. 9 and the standard deviation for the velocity components in the x-axis of the particles in $\mathcal{R}_0$, we can get the scale factor, namely

$$\boxed{\frac{\sqrt{c^2-u^2}}{c}}. \tag{10}$$

It is also the Lorentz factor. Obviously, the ratio of the standard deviation for the velocity components in the y-axis is still the Lorentz factor shown in Equ. 10. It (Equ. 10) can also be obtained by evaluating the ratio of the standard deviation of the velocity components in the z-axis of the mixture distribution in $\mathcal{R}_u$ to the standard deviation of the velocity components in the z-axis in $\mathcal{R}_0$. The detailed Mathematica code of the above calculation process can be found in Part 2 of Supplementary Information. This means that when part of the particles in the reference system $\mathcal{R}_0$ composed of particles with the same speed (such as c) and random direction form the reference system $\mathcal{R}_u$ moving at the speed u, the speed of the moving aggregations with higher mass-level in $\mathcal{R}_u$ will relatively decrease, and the degree of deceleration is the value determined by the scale factor given by Equ. 10.

In this article, we will not do more discussion about the Special Relativistic effect (such as time expansion, length contraction, etc.) under this logic. Obviously, with the effect of deceleration, other phenomena all work.

The above mentioned prove that the vectors with equal norms in Euclidean space possess the Special Relativistic effect. In a stationary (inertial) reference system, particles with different mass-levels are moving according to the relationship determined by Equ. 40 below, and the Maxwell distribution considers that their average velocities are different. When the average velocity of a larger mass-level particle composed of $\mathcal{K}$-order particles is measured in a moving reference system $\mathcal{R}_u$ with velocity u, the degree of their deceleration is determined by the average speed $c_\mathcal{K}$ of $\mathcal{K}$-order particles according to the scale factor $\frac{\sqrt{{c_\mathcal{K}}^2-u^2}}{c_\mathcal{K}}$; And when the average velocity of a larger mass-level particle composed of $\mathcal{L}$-order particles is measured, the degree of their deceleration is determined by the average speed $c_\mathcal{L}$ of $\mathcal{L}$-order particles according to the scale factor $\frac{\sqrt{{c_\mathcal{L}}^2-u^2}}{c_\mathcal{L}}$. If the moving species in the moving reference system $\mathcal{R}_u$ consists entirely of photons (the energy group of photons), the degree of their average velocity reduction of these substances is calculated by the Lorentz factor given in Equ. 10 (or determined by Special Relativity). At present, human beings only detect the photons and the photon-level formations that can be detected (such as electromagnetic wave, atomic clock, etc.), from this point of view, the quantitative relationship given by Special Relativity is extremely accurate! It is also noticed that in $\mathcal{R}_u$ the three axes slow down to the same extent. This means that there is no difference in physical laws can be perceived between $\mathcal{R}_u$ and the stationary reference system $\mathcal{R}_0$. Therefore, when another moving reference system $\mathcal{R}_{u'}$ appears in $\mathcal{R}_u$, $\mathcal{R}_u$ can also be treated as a stationary reference system, which is a good feature. This reveals that any reference system that meets the conditions given in **HYPO 1–3** can be regarded as a stationary reference system, regardless of whether it is an absolutely stationary reference system. The Special Relativistic effect is the statistical effect of moving particles. If the equation established in this article can contain the statistical effect of moving particles, then it also possesses the (Special) Relativity effect.

3.3.3 Establishment of Classical Diffusion Equation

To describe the above physical model comprehensively, we should establish a four-parameter equation including time for the motion law of each particle, such as $\wp(x, y, z, t)$. For a system with n particles, it is necessary to establish an equation with $3n + 1$ degrees of freedom in the same time-dimension, where $n \to +\infty$. This is obviously extremely unrealistic. Establishing equations according to this idea will only increase the complexity of solution. It's easy to establish equations with tens or even tens of degrees of freedom, such as Superstring Theory. Although it can be "all inclusive" to the greatest extent, the difficulty of solution is unimaginable, and gave the euphemistic name "String theory is 21st century physics that fell accidentally into the 20th century"[5]. The equation established according to the above thought may be difficult to solve in the 210th century!

In this article, we take the second best. We do not expect to describe all the motion characteristics of all particles, but only want to describe the laws of particle motion succinctly and practically: to establish an equation that does not lose the motion characteristics of particles and can be described (solved) actuality as much as possible. It may be a more appropriate way to deal with it by using Statistics, that is, to establish a mathematical model with statistical characteristics on the basis of physical model.

Theoretically, infinitely many aggregations of any degree of particles can be found in infinite 4-dimensional space-time. Yet the farther the difference between the aggregation degree and the total average density in space-time, the lower the formation probability of the particles, and the more unstable in the time dimension. However, it is difficult to describe this situation with a certain function. Therefore, this article does not cast about for functions at the level of micro or uncertain cases, but cast about for statistical description functions of relatively certain by expanding the visual field to cover a larger range of enough cases. It is noticed that no matter how these particles move in 3-dimensional space, their trajectories are continuous, which will lead to diffusion (or agglomeration) behavior that is the general diffusion of randomly moving particles. Here, each moving particle is regarded as a vector whose direction is the same as the moving direction of this particle and the norm is the speed of that. Therefore, the general diffusivity of randomly moving particles is also the general diffusivity of (direction) random vectors. The "random vector" and "randomly moving particle (or velocity)" mentioned below are the same meaning. Considering the same mass and speed of these particles, the general diffusivity of random vectors is the general diffusivity of random momentums (which are also vectors). It is considered that the scale of "general diffusivity of vectors" is just a good one, which is most suitable for describing the invariable law of randomly moving particles. More information will be lost if the scale is a little more macroscopic (e.g. the scale can be approximately described by real diffusion), and there will be no invariable statistical law to follow if the scale is a little more microcosmic (for example, the scale described at the beginning of this paragraph). At this scale, the external performance of the vectors in a tiny space cannot be considered as isotropic. When the randomly moving particles are not disturbed, according to the Maxwell distribution, the total vector in a certain domain always points to an uncertain direction, and the norm is directly proportional to $\sqrt{k}$, where k is the number of vectors (See Part 1 of Supplementary Information for details). Although Maxwell distribution cannot determine the direction of total vector in a tiny space, we hope to use appropriate constraints to obtain the distribution rule of the norm and direction of total vector at any position of the space.

First of all, we determine the constraints of spatial vectors. Let the density of vector sum of some point in space be $\boldsymbol{\mathcal{X}}$, which is the function of position and time, namely $\boldsymbol{\mathcal{X}}(x, y, z, t)$. It is defined as: At a certain time t, let $\boldsymbol{\mathcal{Y}}(\mathcal{V})$ be a function of the sum of all vectors in the closed domain $\mathcal{V}$ containing $\mathcal{P}(x, y, z)$, then $\boldsymbol{\mathcal{X}}(x,y,z,t)=\lim_{\mathcal{V}\to\mathcal{P}}\frac{\boldsymbol{\mathcal{Y}}(\mathcal{V})}{\mathcal{V}}$) (The following $\boldsymbol{\mathcal{X}}$ is also the function of spatial coordinates (x, y, z) and time coordinate t). The situation when particle position aggregation is dominant will be studied in the following.

$\mathcal{X}$ is a statistical average vector. When it is undisturbed, the relationship between $\mathcal{X}$ and the number of vectors follows the Maxwell distribution. As is illustrated in Fig. 1a, it is assumed that there are two micro-domains $\mathcal{V}_A$ and $\mathcal{V}_B$ with the same size along the normal direction on both sides of the segmentation surface Φ. If the sum of all vectors in $\mathcal{V}_A$ is $\overrightarrow{OA}$, and the sum of all vectors in $\mathcal{V}_B$ is $\overrightarrow{OB}$, then their sum is $\overrightarrow{OC}$, and their difference is $\overrightarrow{AB}$. Let the sum and difference vectors intersect at point M (Fig. 1b). In view of the previous assumption that the domains of $\mathcal{V}_A$ and $\mathcal{V}_B$ on both sides of Φ are equal, and there is no need to consider the statistical effect before particles move. Due to the characteristic that the distribution of the velocity directions is homogeneous, both the vectors must tend to approach their average value $\overrightarrow{OM}$, that is, both $\overrightarrow{OA}$ and $\overrightarrow{OB}$ tend to form $\overrightarrow{OM}$. In this way, the change rate of $\mathcal{X}$ along the normal direction at somewhere should be related to the time-dependent change rate of $\mathcal{X}$; This time-dependent rate of change is also affected by another inherent factor (i.e. the velocity of the particle forming $\mathcal{X}$), and the concrete value of which is temporarily uncertain. Therefore, the above two change rates should be in directly proportional without considering the difference between particles caused by density (including position aggregation and direction aggregation).

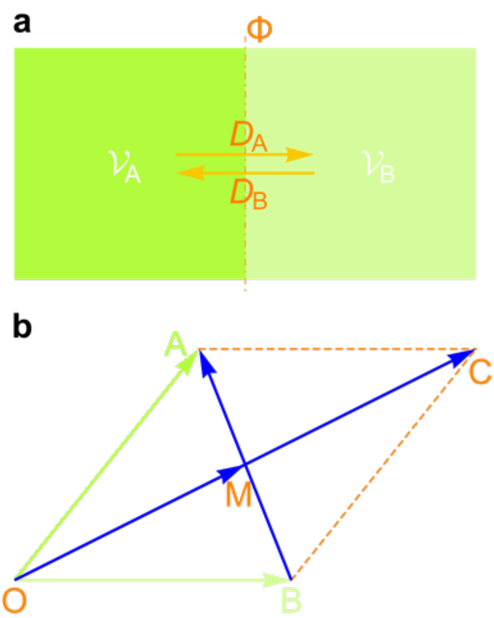


Figure 1 | Illustration of the production principle of mutual diffusion potential at micro-domains $\mathcal{V}_A$ and $\mathcal{V}_B$.

In view of the similar calculus properties of vector and quantity, the derivation method of real diffusion is imitated here. If domain $\mathcal{W}$ is enclosed by a closed surface

Σ, then in the infinitesimal period $\mathrm{d}t$, the direction derivative $\dfrac{\partial \boldsymbol{\mathcal{X}}}{\partial \boldsymbol{N}}$ of $\boldsymbol{\mathcal{X}}$ along the normal direction of an infinitesimal area element dS on the surface Σ is directly proportional to the vector $\mathrm{d}\boldsymbol{\mathcal{X}}$ flowing through dS along the normal direction in the closed domain $\mathcal{W}$ enclosed by Σ (Fig. 2), assuming that the coefficient is a positive real number D.

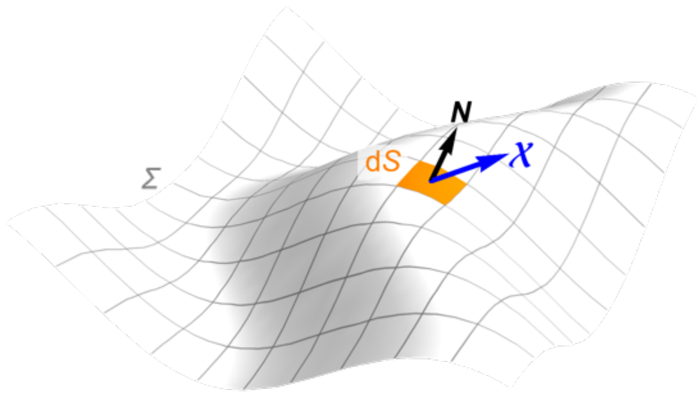


Figure 2 | Illustration for diffusion of vector sum density $\boldsymbol{\mathcal{X}}$

From the time t_1 to t_2, when the influence of vector density on D is not considered (i.e. there is the same diffusion coefficient at every position), the variation of vector sum $\boldsymbol{\mathcal{A}}$ in closed surface Σ is

$$\delta \boldsymbol{\mathcal{A}} = \int_{t_1}^{t_2} \left(\oiint_{\Sigma} D \frac{\partial \boldsymbol{\mathcal{X}}}{\partial \boldsymbol{N}} \mathrm{d}S \right) \mathrm{d}t. \tag{11}$$

According to Gauss formula, the right-hand side of Equ. 11 can also be written in the form

$$\int_{t_1}^{t_2} \left(\iiint_{\mathcal{W}} D \Delta \boldsymbol{\mathcal{X}} \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \right) \mathrm{d}t, \tag{12}$$

where Δ is the Laplace operator, describes the second derivative with respect to position (x, y, z). And the left-hand side of Equ. 11 (namely $\delta \boldsymbol{\mathcal{A}}$) can be written as

$$\delta \boldsymbol{\mathcal{A}} = \iiint_{\mathcal{W}} \left(\int_{t_1}^{t_2} \frac{\partial \boldsymbol{\mathcal{X}}}{\partial t} \mathrm{d}t \right) \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z. \tag{13}$$

Let Equ. 13 equal to Equ. 12, and transform the order of integration, then we can obtain

$$\int_{t_1}^{t_2} \iiint_{\mathcal{W}} \frac{\partial \boldsymbol{\mathcal{X}}}{\partial t} \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \, \mathrm{d}t = \int_{t_1}^{t_2} \iiint_{\mathcal{W}} D \Delta \boldsymbol{\mathcal{X}} \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \, \mathrm{d}t. \tag{14}$$

Noticing that t_1, t_2 and the domain $\mathcal{W}$ are all arbitrary, there is the following equation

$$\frac{\partial \boldsymbol{\mathcal{X}}}{\partial t} = D\Delta\boldsymbol{\mathcal{X}}. \tag{15}$$

Obviously, the above conclusion still holds when $\boldsymbol{\mathcal{X}}$ is dominated by the aggregation of velocity direction rather than position of the particles. That is because this situation is also a statistical characteristic of a large number of particles, and the diffusion has no discrimination to these two types of aggregations.

To make it easy for vector resolution in the following, the 3-dimensional vector is needed to convert into the plane vector. Next, we determine the constraints of the plane vector. Although the operation in Equ. 15 uses 3-dimensional vectors, when differential operation is performed on spatial vectors, the sum and difference operations are always taken from the vector at two points with infinitesimal distance, then all 3-dimensional vectors can only exhibit the relative 2-dimensional characteristics. After solving the differential equation, only 2-dimensional constraints can be obtained. Therefore, only the derivatives of the plane vectors are needed to act as the derivatives of the 3-dimensional vectors (In this case, plane vectors can retain the important information such as norm and included angle between them). Moreover, the function of plane vectors obtained by solving the partial differential equation of plane vectors is also unique, and can correspond to the 3-dimensional vectors obtained by the same form of differential equation. It is assumed that the function of the plane vectors describing the density of the vectors or the momentums is $\boldsymbol{\mathcal{M}}(x, y, z, t)$, which corresponding to $\boldsymbol{\mathcal{X}}$ at the point of (x, y, z, t) (Without special explanation, $\boldsymbol{\mathcal{M}}$ in the following is the function of spatial coordinates (x, y, z) and time coordinates t). The above-mentioned $\boldsymbol{\mathcal{X}}$ can be replaced with $\boldsymbol{\mathcal{M}}$. After replacement, the norm of the plane vector will not change obviously in such an operation, but the direction will be reallocated. At least, Equ. 15 can be written as:

$$\left|\frac{\partial \boldsymbol{\mathcal{M}}}{\partial t}\right| = D\left|\Delta\boldsymbol{\mathcal{M}}\right|. \tag{16}$$

Followed that, the constraints of the direction of the plane vector $\boldsymbol{\mathcal{M}}$ are determined. In view of the continuity of the trajectories of infinitesimal particles, $\boldsymbol{\mathcal{M}}$ is also characterized by the statistical properties of a huge number of particles, so it should be smooth. According to the Theory of Plane Curve, the first derivative and the second derivative of plane vectors in any direction of space are vertical. If the equation between them was established following the above derivative relationship (Equ. 16), the direction needs to be adjusted to be consistent, and their relationship can be written in the form

$$\frac{\partial \mathcal{M}}{\partial t} = \boldsymbol{i} D \Delta \mathcal{M}. \tag{17}$$

where $\boldsymbol{i}$ is the imaginary unit. To multiply both sides of Equ. 17 by $\boldsymbol{i}$, the form of Schrödinger equation can be obtained

$$\boldsymbol{i}\frac{\partial \mathcal{M}}{\partial t} = -D \Delta \mathcal{M}. \tag{18}$$

Equ. 18 describes the distribution of the moving particle swarm (including the direction of movement) in space following the same diffusion coefficient, namely the classical diffusion equation. However, when the particle swarm moves faster or there are more particles gathering in a certain micro-domain, the effect on diffusion is not clear. To describe this kind of diffusion process (which is called general diffusion) more comprehensively, further analysis is needed.

3.3.4 Construction of General Diffusion Equation

In order to construct the general diffusion equation, we need to take in account many aspects, including whether the general diffusion coefficient $Đ$ should be variable, and how to describe it to include the effects of General Relativity (gravitation) and Special Relativity, etc.

The classical view is that no matter how large the target norm of vector is, they follow the diffusion equation with the same diffusion coefficient (Schrödinger equation). However, this article does not think so: $Đ$ should be variable with the value of the target vector. As mentioned above, when the influence of the vector sum density (including the aggregation density of position and direction, the same below) on D is not considered, the diffusion equation of vectors conforms to the form of Schrödinger equation. However, when the density of vector sum is large, the effect on D cannot be ignored. As is illustrated in Fig. 1a, the density of vector sum at the micro-domain $\mathcal{V}_A$ are greater than those at $\mathcal{V}_B$. If they are in the same background field, there is a cost for the high density at $\mathcal{V}_A$. If it keeps a high density at the next moment (in terms of probability, more uncertainty is introduced into the unit volume), which will inevitably consume the (average) moving speed of the particles, the comprehensive moving speed of the particles at $\mathcal{V}_A$ will decelerate (Section **3.3.2**). As mentioned above (or Equ. 31 below), the speed of particle is the determinant of D, so the law of diffusion rate towards the right (D_A) is not equal to the law of the diffusion rate at $\mathcal{V}_B$ towards the left (D_B) (assuming that $Đ$ is the combination of D_A and D_B). Therefore, it is necessary to change the general diffusion coefficient with the density of vector sum in time to reflect this inequality.

In view of the above considerations, choosing the appropriate quantitative function to describe the phenomenon is the main problem to solve in this article. First of all, the momentum vector in the micro-domain is resolved.

3.3.4.1 Vector Resolution

Let us determine the distribution function of particles with equal probability in a certain domain in the following: Suppose that the whole domain contains n particles in total. For the convenience of description, the whole domain is also partitioned into n boxes with equal size. The gap between boxes and wall thickness are both 0. Now, let us determine the probability of k ($k \in \mathbb{N}$, the same below) particles in the local area containing $\mathcal{M}$ boxes (Suppose the particles are small enough to fall into the box, not the wall). In view of the foregoing, the probability of particles in each domain is the same. So, the total possible cases that n particles are randomly distributed in n boxes is n^n; there are $\binom{n}{k}$ cases in total by randomly extracting k particles from n particles; there are $\mathcal{M}^k$ cases in total by putting the k particles extracted each time into $\mathcal{M}$ boxes; There are $(n-\mathcal{M})^{n-k}$ cases in total by putting the remaining $n-k$ particles randomly into $n-\mathcal{M}$ boxes. Therefore, the probability $P(\mathcal{M}, k)$ of k particles in $\mathcal{M}$ boxes can be expressed as

$$P(\mathcal{M},k) = \frac{\binom{n}{k}\mathcal{M}^k(n-\mathcal{M})^{n-k}}{n^n}. \tag{19}$$

Supposing that the number n of particles in the whole domain is infinite, as $n \to +\infty$, take the limit of Equ. 19, then

$$P(\mathcal{M},k) = \frac{e^{-\mathcal{M}}\mathcal{M}^k}{k!}, \tag{20}$$

where $\mathcal{M}$ denotes the number of boxes covered by a local domain (It is the size of volume in 3-dimensional space); k denotes the number of particles in that domain (including $\mathcal{M}$ boxes), and P denotes the probability of k particles in that domain (including $\mathcal{M}$ boxes). Equ. 20 is the (position-based) Poisson distribution.

It is considered that it is the most appropriate method to partition the whole domain (the domain can be the whole universe, or a broader range including the objects of investigation) into the uniform boxes with the same number of particles. In addition to reducing parameters and facilitating discussion, the reasons are as follows: If the box is a little larger, it will not ensure the accuracy of the following vector resolution; if it is a little smaller, it will not reflect the grouping effect of particles. Therefore, in this article, the whole domain is divided into the same number of uniform boxes as the number of particles it contains, based on of which, the following contents are discussed. In this article, the whole (environment) domain is called the T-domain, and part (target) of which is called the S-domain; all particles contained in the T-domain is called T-particle swarm, and part of which is called S-particle swarm.

Followed that, we will investigate the equiprobability distribution of the static particle swarm in the above mentioned S-domain $\mathcal{V}$. In Equ. 20, $\mathcal{M}$ denotes the number of boxes (volume) covered by some S-domain (where the object particles distribute). Another way, when the T-domain is partitioned into uniform boxes according to the above method, $\mathcal{M}$ can also denote the average relative density of particles in the S-domain $\mathcal{V}$, and the referenced density is the average density of the T-particle swarm in the T-domain. $\mathcal{M}$ denotes the multiple of the average density, k denotes the number of particles in one box, and P is the probability of k particles in that box. The distribution of S-particle swarm in $\mathcal{V}$ is Poisson distribution with density intensity $\mathcal{M}$. Next, we will analyze the Poisson distribution formula given in Equ. 20. Actually, it is the weight of each term determined by k to the value of $e^{\mathcal{M}}$ after $e^{\mathcal{M}}$ is expanded by power series. The meaning here is that it is also the ratio of the number of boxes containing k particles to the total number of boxes in $\mathcal{V}$ when the S-particle swarm with the relative density $\mathcal{M}$ is distributed in the reference boxes which is determined by the above criteria and covered by the S-domain $\mathcal{V}$ (suppose that the number of boxes covered by $\mathcal{V}$ is large enough). According to the Mathematical Analysis, we can see that the expansion of power series of this case is unique, and obviously, this ratio distribution of the number is also unique. If the right-hand side of Equ. 20 is multiplied by k, which is recorded as $R(\mathcal{M}, k)$, namely the following form:

$$R(\mathcal{M},k) = \frac{e^{-\mathcal{M}}\mathcal{M}^k}{(k-1)!}. \tag{21}$$

In that way, its termwise addition specifies a possible resolution form after resolving

$\mathcal{M}$ into infinite items. Seeing that the expansion of power series in the foregoing is unique, so the resolution of this form is also unique. According to the physical meaning of previous statement, the meaning of Equ. 21 is the contribution of the relative density apportioned by the particles in the boxes containing *k* particles to the total relative density $\mathcal{M}$ (This is the average relative density in $\mathcal{V}$) after the particles with relative density $\mathcal{M}$ is dispersed into the (infinitely many) reference boxes covered by $\mathcal{V}$ in equal probability. Multiplying Equ. 21 by the number of boxes contained in $\mathcal{V}$ is the total number of particles in the boxes containing *k* particles. Since the distribution of particles in this form is definite—following the Poisson distribution, from this point of view, the resolution of the relative density $\mathcal{M}$ in this form is also unique.

If $\boldsymbol{\mathcal{M}}$ is a complex number (or plane vector), Equ. 21 can be written in vector form

$$R(\boldsymbol{\mathcal{M}},k)=\frac{e^{-\mathcal{M}}\boldsymbol{\mathcal{M}}^{k}}{(k-1)!}. \tag{22}$$

The form of dividing Equ. 22 by *k* is still the ratio of the complex number determined by *k* after $e^{\boldsymbol{\mathcal{M}}}$ is expanded by power series to the complex $e^{\boldsymbol{\mathcal{M}}}$. There is one more direction here, and the expansion of power series is unique. Similarly, the termwise addition of Equ. 22 is also a resolution of vector $\boldsymbol{\mathcal{M}}$. The resolution form of the power series is also unique.

Now we study the distribution of the velocity of the moving S-particle swarm in the above-mentioned S-domain $\mathcal{V}$. If the particles of T-particle swarm move randomly in the T-domain, the distribution of the S-particle swarm on a time slice in a small enough S-domain (when the speed of the particle is fast enough) can also be approximately regarded as the equiprobability. In the scale of human being, we can think that the number of S-particle in almost every "micro-domain" of the universe is huge, so the distribution of the moving S-particle swarm in a certain micro-domain $\mathcal{V}$ can be described by Equ. 20. The moving particles in each type of boxes partitioned by *k* in one S-domain $\mathcal{V}$ can form a component-vector which is added together by *k* as the total vector in $\mathcal{V}$. If the total 3-dimensional vector $\boldsymbol{\mathcal{Y}}$ of the moving S-particle swarm in $\mathcal{V}$ are determined, then the norm of each component-vector should be directly proportional to the number of particles forming it, that is to say, the S-particle swarm are scattered into various boxes (partitioned by *k*) covered by $\mathcal{V}$, of which the particle number in each *k* type boxes is directly proportional to each term (determined by *k*) given in Equ. 21. That is because: According to the Central Limit Theorem, when samples are taken from the population with determined distribution, it is approximately proportional between the mean value of these samples and the sample size (See Part 3 of Supplementary Information for details). It should be noted that even if $k = 1$, the number of samples in $\mathcal{V}$ should be very huge. So far, the distribution of both the number of particles and the ratio between norms of component-vectors of $\boldsymbol{\mathcal{Y}}$ in $\mathcal{V}$ follow the same Poisson distribution which is fixed form and which is partitioned by k, and they correspond to each other.

We will determine the directions of the above-mentioned component-vectors in the following. When $\boldsymbol{\mathcal{Y}}$ in $\mathcal{V}$ is determined (i.e. the state of the system is determined), the directions of component-vectors according to the above-mentioned resolution form should also be determined. This article studies the limit value of the quotient of $\boldsymbol{\mathcal{Y}}$ and $\mathcal{V}$ (namely $\boldsymbol{\mathcal{X}}$), which can still be considered as the sum of 3-dimensional vectors in the S-domain $\mathcal{V}$. When the 3-dimensional component-vectors of the 3-dimensional vector $\boldsymbol{\mathcal{X}}$ are mapped to the 2-dimensional component-vectors of the plane vector $\boldsymbol{\mathcal{M}}$, in fact, the direction should also be unique. The ratio between the 2-dimensional component-vectors of $\boldsymbol{\mathcal{M}}$ and the 3-dimensional component-vectors of $\boldsymbol{\mathcal{X}}$ should be the same—follow the Poisson distribution. However, the specified value of the norm and direction of each component-vector of $\boldsymbol{\mathcal{X}}$ have not be determined. Fortunately, Equ. 22 exactly shows a set of component-vectors whose norm ratio conforms to Poisson distribution and whose sizes and directions are affirmatory. In view of the discussion in the preceding paragraph, each norm of component-vector is directly proportional to the number of moving particles forming it. Therefore, that the norm ratios of the component-vectors determined by Equ. 22 are consistent with that of the 2-dimensional component-vectors of $\boldsymbol{\mathcal{M}}$ (or the 3-dimensional component-vectors of $\boldsymbol{\mathcal{X}}$, i.e. the velocities in various boxes) means that the numbers of moving particles in various boxes implied in Equ. 22 are consistent with that of moving particles which form the 2-dimensional component-vectors of $\boldsymbol{\mathcal{M}}$. So, it can also be thought of as that such a set of component-vectors were generated respectively by the particles (with the same particle number ratio, mapped to the plane) in various boxes partitioned by k, that is to say, it is possible that $\boldsymbol{\mathcal{M}}$ can be resolved into the termwise addition form of $R(\boldsymbol{\mathcal{M}}, k)$ given in Equ. 22. The resolution of such a form of plane vector $\boldsymbol{\mathcal{M}}$ may be unique, but this conclusion will not be proved here for the time being. In any case, it will not affect the discussion of the following issues. That is because, even if such a resolution is not in accordance with the established combined method of boxes, but scattered in each box, as long as it is partitioned according to the patten of Poisson distribution, still follows the same law of motion (deceleration or diffusion), and it can be thought of as that there is such a resolution covering various boxes. The resolution form given in this article is one of the equivalent cases (No matter whether the moving particle is in the dominant of position or direction aggregation, as long as their vectors are equal, their influences on diffusion are the same. Therefore, when the direction aggregation is dominant, it can be equally interpreted as position aggregation being dominant). In summary, the plane mapping of the sum of all vectors in the boxes with the same particle number k can be thought of as the component-vector determined by k in Equ. 22. When k is given all the values in $\mathbb{N}$, the termwise addition of these terms is at least an equivalent resolution of $\boldsymbol{\mathcal{M}}$, namely

$$\boldsymbol{\mathcal{M}} = \sum_{k=1}^{\infty} \frac{e^{-\mathcal{M}} \boldsymbol{\mathcal{M}}^{k}}{(k-1)!} \tag{23}$$

The resolution form in Equ. 23 is independent of the number of vectors forming the total vector in the S-domain, but dependent of the value of relative total vector. As mentioned above, $\mathcal{M}$ denotes the relative density of particles in the S-domain $\mathcal{V}$, it is a concept of multiple. Therefore, $\boldsymbol{\mathcal{M}}$ is also a relative vector, which is the multiple of the speed of the single particle in each partitioned box after the T-particle swarm are averagely put in the T-domain; Meanwhile, it is also the multiple of the number of boxes covered by the infinitesimal S-domain (Scilicet, the relative density of the number of particles at a target point). And the direction is the same as that of the sum of the absolute vector. Therefore, $\boldsymbol{\mathcal{M}}$ in Section **3.3.3** should be the relative density of vector sum, exactly. As mentioned above, the sum and difference operations between two spatial vectors are performed in the plane. In this plane, they can be resolved to the sum of plane vectors as described in Equ. 23. Therefore, the two sets of plane component-vectors can also act as their respective spatial vectors to correspondingly perform sum-difference or derivative operations.

3.3.4.2 Description for Diffusion

As mentioned earlier, particles with a higher mass-level composed of k particles are called k-order particles. Then, the velocity of the k-order particle is the velocity of the mass center of the total k particles, which is the average of the velocity vectors of all these particles. As mentioned above, each particle possesses the same speed and random direction in space. Then, the projectile of the velocity vector of the k-order particle on one of the three equivalent coordinate axes of the 3-dimensional Cartesian coordinate system is the mean value of the projectile (on the same axis) of the velocity vectors of the 1-order particles (forming the k-order particle) which follow the same distribution, so it approximately follows the Normal distribution (Central Limit Theorem). There are three equivalent (approximate) Normal distributions on the three axes respectively, which are not completely independent. However, James Clerk Maxwell[13] has proved that they were in fact equivalent to the effect of complete independence. This is because that to take a vector randomly is equivalent to determine a three-axis coordinate randomly; moreover, from the point of view of momentum transfer of gas molecules under random impaction, it is also equivalent to the problem discussed in this article. In this way, the speed of k-order particles follows the Maxwell distribution. Suppose that the standard deviation of the projectile (as a random variable, the same below) of the velocity of any one of the k equivalent particles forming the k-order particle in each equivalent coordinate axis is σ. Then, the standard deviation of the projectile of the velocity of k-order particles in each equivalent coordinate axis is $\frac{\sigma}{\sqrt{k}}$. Therefore, it (the projectile in each coordinate axis) follows the Normal distribution with standard deviation $\frac{\sigma}{\sqrt{k}}$. As a result, the speed of k-order particles follows the Maxwell distribution with scale parameter $\frac{\sigma}{\sqrt{k}}$ (In this case, it is unnecessary to consider the situation that the direction aggregation is dominant. The diffusion coefficient is the inherent statistical effect of the system, and only the average speed needs to be calculated according to its definition). Then, the average speed of k-order particles is

$$\bar{v} = 2\sqrt{\frac{2}{\pi}} \cdot \frac{\sigma}{\sqrt{k}}. \tag{24}$$

For k_1- and k_2-order particles, the ratio of their average speeds is

$$\frac{\bar{v}_1}{\bar{v}_2} = \frac{\sqrt{k_2}}{\sqrt{k_1}}. \tag{25}$$

Because that the size or mass of the 1-order particles are the same, if the masses of the

k_1-order particle and the k_2-order particle are m_1 and m_2 respectively ($m \propto k$), according to the relationship shown in Equ. 25, the ratio of their average speeds can be written as

$$\frac{\bar{v}_1}{\bar{v}_2} = \frac{\sqrt{m_2}}{\sqrt{m_1}}. \tag{26}$$

See Part 1 of Supplementary Information for the detailed calculation and derivation process. According to Equ. 26, when the mass of k-order particle is m, comparing with the 1-order particle, the average speed is

$$\bar{v} = \frac{\kappa_1}{\sqrt{m}}, \tag{27}$$

where κ_1 is the coefficient constant.

Diffusion coefficient is defined as: under the condition of per unit time and per unit concentration gradient, it is the mass or mole number of the substance diffused vertically through the unit area along the diffusion direction. Therefore, it is believed that the real diffusion in the traditional view is consistent with the essence of vector diffusion described here. According to Einstein-Brown's displacement equation, the diffusion coefficient

$$D = \frac{\bar{x}^2}{2t}, \tag{28}$$

where $\bar{x}$ is the average displacement of k-order particles along the direction of x-axis. To replace the average displacement of $\bar{x}$ in Equ. 28 with the average velocity (namely $\bar{V}$) of k-order particles, the diffusion coefficient can be transformed into

$$D = \frac{\bar{V}^2}{2} t^1. \tag{29}$$

The dimension of the diffusion coefficient D is $m^2 \cdot s^{-1}$. In combination with the Equ. 28 and Equ. 29 (where t^1 and t implicated in $\bar{V}^2$ are consistent, so t^1 = 1 s), the above-mentioned diffusion coefficient can also be regarded as: In unit time, the average area of k-order particles spread on the plane. This average area is related to the speed of single k-order particle. If the (average) speed of single k-order particle is $\bar{v}$, then the statistical average speed of these particles in one direction is

$$\bar{V} = \frac{\bar{v}}{2}. \tag{30}$$

The k-order particle swarm spread in the plane at this rate. To substitute Equ. 30 into

Equ. 29 and combine $t^1 = 1$ s into the coefficient, then replacing it with κ_2, we can obtain

$$D = \kappa_2 \overline{v}^2, \tag{31}$$

where κ_2 is the constant coefficient in seconds (s).

By substituting Equ. 27 into Equ. 31, the diffusion coefficient of the (k-order) particle swarm with the (average) mass m is obtained

$$D = \kappa_2 \left(\frac{\kappa_1}{\sqrt{m}} \right)^2 = \frac{\kappa_1^{\ 2} \kappa_2}{m}. \tag{32}$$

The above equation (Equ. 32) can also be thought of as the apparent diffusion rate constant of particles with mass m described by the 1-order particle swarm (which form the particle with mass m after collapse) without Relativistic effect. Moreover, in the Schrödinger equation without Relativistic effect (i.e. the case of the apparent diffusion described by the 1-order particle swarm), the specific form of this coefficient has been given. Comparing it with Equ. 32, the following relationship can be obtained immediately:

$$\kappa_1^{\ 2} \kappa_2 = \frac{\hbar}{2} \tag{33}$$

where $\hbar$ is the reduced Planck's constant.

3.3.4.3 Assembly of General Diffusion Equation

The previous assumption declares that there is no interaction between infinitesimal particles. Even if there is interaction between particles of larger mass-level (the "interaction" is produced by statistical effect in this article), there is a process of continuous disappearance and generation of particles, that is, there is no interaction in fact. In addition, seeing that the essence of these "interactions" is gravitation (that is, the statistical effect of moving particles, other forces can be treated equally), the concept of no interaction between particles of various mass-levels is equivalent. On a time slice of a micro-domain, the resolution of velocity determined by Equ. 23 must be exhibited, and the boxes containing the same number of particles in different micro-domain of possessing the different density of vectors are equivalent. That is because there should be no more differences between boxes of the same type (including the same particle number) when Poisson distribution determines the number of different type of boxes in different micro-domain of possessing the different density of vectors. Although the moving particles are distributed in the micro-domain with the same probability, when k particles are counted, their average speed will inevitably slow down. The particles in various boxes partitioned by k move at their average speed (the centroid for box containing k particles is at the center of each box on average). Among the boxes (including k particles) of the same type, the average speed of each k-order particle is the same, which must conform to the diffusion form of Schrödinger equation determined by the diffusion coefficient of this type. Therefore, according to the particle number of k in the previously partitioned boxes, from 1 to ∞, respectively study its corresponding term $R(\mathcal{M}, k)$ which is the component-vector of $\mathcal{M}$. Firstly, we investigate the individual diffusion and then add them together.

Here, all the particles in each box containing k particles are regarded as a k-order particle of larger mass-level, and the total k-order particles in all boxes containing k particles in the micro-domain $\mathcal{V}$ is called the k-order particle swarm. Based on the above discussion, it can be considered that the average speed of each (k-order) particle in k-order particle swarm is the same when there is no external disturbance, and all of them follow the same diffusion coefficient. According to the relationship given by Equ. 32 (the diffusion coefficient is inversely proportional to the mass of k-order particles or the number of 1-order particles forming k-order particles), if the diffusion coefficient of the 1-order particle swarm is D_1, then the diffusion coefficient of k-order particle swarm is

$$D_k = D_1 \cdot \frac{1}{k}, \tag{34}$$

where $\frac{1}{k}$ is called the diffusion coefficient factor.

When it is no need to consider the influence of the deceleration effect of the statistical speed caused by particle aggregation on diffusion, it is the diffusion of the 1-order particle swarm, which is consistent with the description of diffusion by Schrödinger equation. Therefore, the diffusion coefficient

$$D_1 = -\frac{\hbar}{2m}. \tag{35}$$

The diffusion equation determined by this coefficient describes the dynamics of the probabilistic diffusion of the target object (or the aggregation after collapse) with the mass m on the basis of the apparent diffusion rate (after deceleration) determined by the 1-order particles forming it (before collapse), but the distribution characteristic of the target object in its dispersion space is determined by the diffusion behavior of the

1-order particles in the background field. When $k > 1$, according to the above discussion, the diffusion coefficient of k-order particle swarm can be obtained by substituting Equ. 35 into Equ. 34, namely

$$D_k = -\frac{\hbar}{2m} \cdot \frac{1}{k}. \tag{36}$$

This is equivalent to the proportional decline of the apparent diffusion rate of the target object (or the aggregation after collapse) whose mass is m due to the slowdown of the speed of k-order particles forming the target object. The meaning of the diffusion equation determined by this diffusion coefficient is similar to the case of the 1-order particles above, and that is: the dynamics of the probabilistic diffusion of the target object (or the aggregation after collapse) with the mass m on the basis of the apparent diffusion rate (after deceleration) determined by the k-order particles forming it (before collapse), but the distribution characteristic of the target object in its dispersion space is determined by the diffusion behavior of the k-order particles in the background field.

Finding the second partial differential of $R(\mathcal{M}, k)$ (It is the plane vector sum $R(\mathcal{M}, k)$ in the boxes containing k moving particles, namely k-order particle swarm, which is one of the component-vectors in the whole micro-domain $\mathcal{V}$) with respect to position (x, y, z) and noticing the intermediate variable $\mathcal{M}$, we obtain the following form:

$$\frac{\partial^2 R(\mathcal{M},k)}{\partial \mathcal{M}^2} \cdot T^2(\mathcal{M}) + \frac{\partial R(\mathcal{M},k)}{\partial \mathcal{M}} \cdot \Delta\mathcal{M}, \tag{37}$$

where $T^2(\mathcal{M}) = \left(\frac{\partial \mathcal{M}}{\partial x}\right)^2 + \left(\frac{\partial \mathcal{M}}{\partial y}\right)^2 + \left(\frac{\partial \mathcal{M}}{\partial z}\right)^2$. It should be emphasized that the absolute sizes of the two (infinitesimal) micro-domains $\mathcal{V}_1$ and $\mathcal{V}_2$ which are taken in comparing difference are equal when calculating the differential of vector $\mathcal{M}$. After multiplying Equ. 37 by the diffusion coefficient of each-order particle swarm (Equ. 36), and then adding the products of all-orders together, the whole general diffusion expression (including coefficient) can be obtained:

$$-\frac{\hbar}{2m}\sum_{k=1}^{\infty}\left[\frac{1}{k} \cdot \frac{\partial^2 R(\mathcal{M},k)}{\partial \mathcal{M}^2} \cdot T^2(\mathcal{M}) + \frac{1}{k} \cdot \frac{\partial R(\mathcal{M},k)}{\partial \mathcal{M}} \cdot \Delta\mathcal{M}\right]. \tag{38}$$

In this way, the calculated diffusion is the general diffusion from the whole (infinitesimal) micro-domain $\mathcal{V}_1$ to $\mathcal{V}_2$. Equ. 38 can be simplified as follows:

$$-\frac{\hbar e^{-\mathcal{M}}}{2m}\left[\Delta\mathcal{M} - T^2(\mathcal{M})\right]. \tag{39}$$

By combining the left-hand side of Equ. 18 with Equ. 39, a complete expression of vector general diffusion equation for vectors is obtained:

$$\boxed{\boldsymbol{i}\frac{\partial \mathcal{M}}{\partial t}=-\frac{\hbar e^{-\mathcal{M}}}{2m}\left[\Delta\mathcal{M}-T^{2}(\mathcal{M})\right]}. \tag{40}$$

Therefore, the expression of general diffusion coefficient with Relativistic effect (including gravitation) is

$$\boxed{Đ=-\frac{\hbar e^{-\mathcal{M}}}{2m}}. \tag{41}$$

The diffusion coefficient here is not a constant, but a natural exponential function varying with the density of relative vector of moving particles. Hereto, the general diffusion equation and the coefficient *Đ* of vectors have been determined. According to the Maxwell distribution, the norms of the spatial vectors can be determined in an undisturbed micro-domain; while according to Equ. 40, the norms and directions of the spatial vectors (the values in the complex plane) can be determined. So far, the basic and effective information of the spatial (moving) particle swarm has been mastered.

Going back to the vector resolution in Section **3.3.4.1**, we now prove that Equ. 23 is the unique form of the plane resolution when the 3-dimensional vector $\mathcal{X}$ is mapped to the plane vector $\mathcal{M}$. It can be seen from the above that the ratios between the norms of the plane component-vectors obtained by mapping follow the Poisson distribution, while the direction is unknown. Suppose that there are many sets of directions conforming to this distribution of norms of the plane component-vectors. In fact, the result of general diffusion is definite, so these sets of directions should be equivalent. After they are operated in the way shown in Equ. 38, it can be seen that the identical Equ. 39 cannot be obtained after substituting the component-vectors with different sets of directions. Therefore, Equ. 23 given in this article is the unique resolution result satisfying the condition.

For Equ. 39, the overall diffusion potential has been reflected in the term $\Delta\mathcal{M}$ without considering its own "gravitation" (particle agglomeration), and the rest terms are caused by Relativistic effect. This is because the Special Relativistic effect mentioned in Section **3.3.2** only evaluate the scene in $\mathcal{R}_u$ from $\mathcal{R}_0$. If the particles in $\mathcal{R}_u$ are observed together with that in $\mathcal{R}_0$, the velocity sum of some particles in $\mathcal{R}_u$ will be slightly different from the situation described in the previous Special Relativistic effect. At this point, all moving particles in $\mathcal{R}_u$ will have an additional velocity component u along the z-axis. However, the diffusion of these particles still follows the rule in $\mathcal{R}_0$. Obviously, when such a moving particle swarm (sum of the vector swarm) is resolved into plane component-vectors, the same rule is followed. The vector resolution in Equ. 23 is unique. For the total vector in the boxes containing k vectors, it is the sum of the vectors in $\mathcal{R}_0$, which is converted from the vectors in $\mathcal{R}_u$. The statistical effects of these moving particles can be incorporated into Equ. 38 by multiplying the second derivative of the component-vectors (after comparative treatment) by different diffusion coefficients according the classification standard of k and adding them up. It can be clearly seen from the above proof process of Special Relativity that the principle of Special Relativistic effect of moving particles in space is the statistical effect of randomly moving particles. More precisely, it is the statistical effect of the direction aggregation being dominant. It should be noted: When the direction is dominant, it shows the Special Relativistic effect. While in general, the aggregation effect also includes the situation that the position aggregation is dominant. Here, the two effects are collectively called the General Relativistic effect. From this perspective, the two aggregation effects are unified and both conform to the law given by Equ. 10. Therefore, the proof process of Special Relativistic effect is also the proof process of General Relativistic effect. The essence of these two effects is the statistical effect of moving particles. Obviously, the treatment in this article (Equ. 40) can also cover all of the aggregation effects, that is, all Relativistic effects have been included. However, the equations with the constraints of Lorentz covariant (such as Dirac equation and Quantum Field Theory) are not enough to reflect all of the Relativistic effects.

3.3.5 Further Study on Equ. 40

3.3.5.1 The Relationship with Schrödinger Equation

To expand the right-hand side given in Equ. 40 according to the power series of $e^{-\mathcal{M}}$, we can get the following form

$$\begin{aligned} \boldsymbol{i}\frac{\partial \boldsymbol{\mathcal{M}}}{\partial t} &= -\frac{\hbar}{2m}\left(1-\boldsymbol{\mathcal{M}}+\frac{\boldsymbol{\mathcal{M}}^2}{2}+\cdots\right)\left[\Delta\boldsymbol{\mathcal{M}}-T^2(\boldsymbol{\mathcal{M}})\right], \\ &= -\frac{\hbar}{2m}\left[\Delta\boldsymbol{\mathcal{M}}-T^2(\boldsymbol{\mathcal{M}})-\boldsymbol{\mathcal{M}}\cdot\Delta\boldsymbol{\mathcal{M}}+\boldsymbol{\mathcal{M}}\cdot T^2(\boldsymbol{\mathcal{M}})+\cdots\right]. \end{aligned} \tag{42}$$

When only the first term of the right of the equal sign of the second line in Equ. 42 is taken, this equation is the form of Schrödinger equation without external field. Therefore, in the form, Equ. 40 is the result of adding several corrections to Schrödinger equation. When the norm of the wave-function $|\mathcal{M}| \to 0$, obviously, $T^2(\mathcal{M})$ is an infinitesimal of higher order than $\Delta\mathcal{M}$ (This is similar to the case of the sine and cosine wave-functions when the velocity is small but the acceleration is large). Moreover, the terms after $-T^2(\mathcal{M})$ on the right of the equal sign of the second line in Euq. 42 are all related to $\mathcal{M}$, and the product of each term and $\mathcal{M}$ is also an infinitesimal of higher order than $\Delta\mathcal{M}$. Therefore, when $|\mathcal{M}|$ is smaller, Equ. 40 can be approximated to the form of Schrödinger equation without external field; when $|\mathcal{M}|$ is larger, the Relativistic effect (the statistical effect of moving particles) in Equ. 40 is palpable and can not be replaced by Schrödinger equation.

3.3.5.2 Nondispersive Particle Swarm

The operators of generation and annihilation for particle have been included in Quantum Field Theory, but such descriptions are rigid. The equation (Equ. 40) in this article naturally contains the process of generation and disappearance for particles, and even can give their half-life (we will not study this problem in detail here). Equ. 40 is the equation describing the general diffusion of particle swarm. When

$$\Delta\mathcal{M} - T^2(\mathcal{M}) = 0, \tag{43}$$

$\mathcal{M}$ does not change with time t, And the particle swarm that meets this condition is a nondispersive particle swarm. This particle swarm can also be regarded as a particle with higher mass-level, which is composed of a series of particles with lower mass-level which obey the statistical law.

Assuming $\mathcal{M}$ is only a function of position (x, y, z), the general analytical solution containing 9 arbitrary constants (Equ. 44) can be obtained by solving Equ. 43 with the method of separating variables:

$$\mathcal{M}(x,y,z) = -\frac{1}{2}\ln\frac{C_1^2 + C_2^2 - \left[C_1\cos\left(\sqrt{C_7}\,x\right) + C_2\sin\left(\sqrt{C_7}\,x\right)\right]^2}{C_7}$$
$$-\frac{1}{2}\ln\frac{C_3^2 + C_4^2 - \left[C_3\cos\left(\sqrt{C_8}\,y\right) + C_4\sin\left(\sqrt{C_8}\,y\right)\right]^2}{C_8} \tag{44}$$
$$-\frac{1}{2}\ln\frac{C_5^2 + C_6^2 - \left[C_5\cos\left(\sqrt{C_9}\,z\right) + C_6\sin\left(\sqrt{C_9}\,z\right)\right]^2}{C_9}.$$

Based on the consideration of the equivalence of three coordinates, C_7, C_8 and C_9 are complex constants and not equal to 0, in which the sum of any two are equal to the opposite number of the third, so there should be $C_7 : C_8 : C_9 = 1 : (-1)^{\frac{2}{3}} : (-1)^{\frac{4}{3}}$. However, it is very difficult to eliminate all the arbitrary constants under certain initial conditions, which is not the focus and interest of this article. Therefore, without affecting the discussion of the problem, the concrete form of the actual analytical solution will not be explored here, and the numerical solution will be adopted instead.

In order to investigate the shape of the nondispersive particle swarm in detail, it is still assumed that $\mathcal{M}$ is only a function of position (x, y, z) and in the case of the position aggregation for moving particles being dominant. In 3-dimensional space, Equ. 43 is specified the following initial conditions:

$$\begin{cases} \mathcal{M}(0,0,0) = 1 + 2\boldsymbol{i}, \\ \mathcal{M}(x,y,z) = 0,\, x^2 + y^2 + z^2 = 4^2. \end{cases} \tag{45}$$

To numerically solve the simultaneous equations of Equ. 43 and Equ. 45 (See the drawing process of Fig. 3 in Part 8 of Supplementary Information for the detailed Mathematica code of solution process), the distribution of mass density ($\left|\mathcal{M}\right|^2$) can be obtained, which is illustrated in Fig. 3:

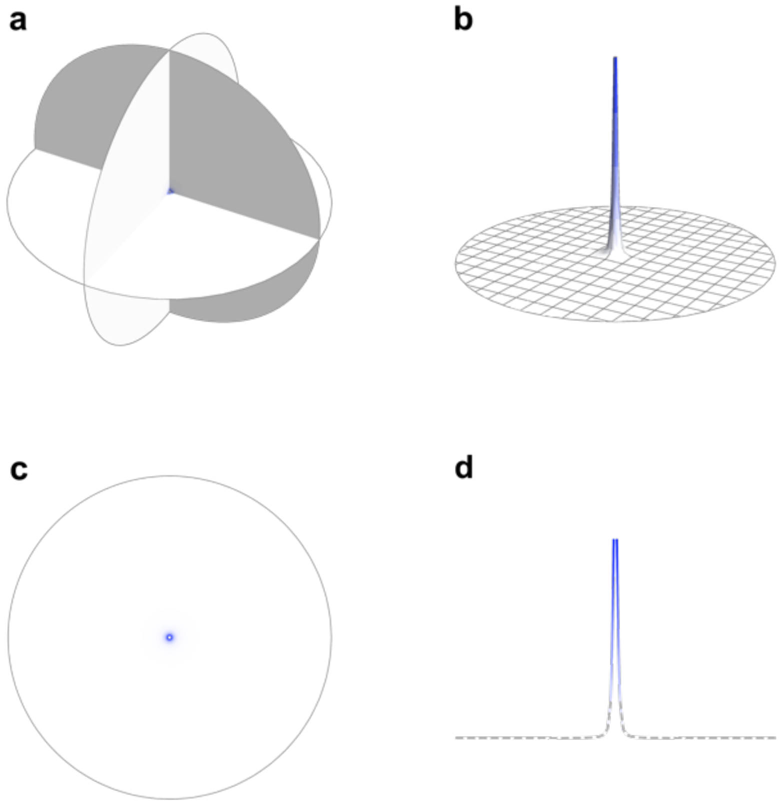


Figure 3 | Distribution of mass density for particle swarm meeting the conditions of Equ. 43 and Equ. 45 (shown from different angles). **a**, 3-dimensional density distribution; **b**, 2-dimensional density distribution at z = 0; **c**, 2-dimensional density distribution on the plane at z = 0; **d**, 1-dimensional density distribution at y = 0 and z = 0. For the convenience of comparison, the three (two) coordinate axes of each figure are displayed in a scale of 1:1.

It can be seen from the figure that the mass of stable particle is almost concentrated in a small spherical area near the spherical center, and the rest area is very spacious (very low mass density), which is very similar to the atomic structure. This result further qualitatively shows that we can study not only the distribution of electron but also the distribution of nuclear mass by solving Equ. 40.

It should be noted that for the boundary condition $\mathcal{M}(0,0,0)=1+2\boldsymbol{i}$ given in Equ. 45, the value on the sphere of $x^2+y^2+z^2=0.04^2$ is assigned 1 + 2 $\boldsymbol{i}$ in the solution process to approximate this condition. It can be inferred that when the radius of this (inner boundary) sphere approaches infinitesimal, what shown in Fig. 3 is still similar to this shape. At the same time, the equations of 2-dimensional case under the same conditions have also been solved in this article. See Part 3 of Supplementary Information for details. Without affecting the discussion of the problem, only a small value ($\sqrt{5}$) and a relatively larger radius (0.04) of inner boundary sphere are taken for the norm of the initial condition. If the norm is further increased or the radius of the inner boundary sphere is further reduced, there will be more obvious contrast of mass density, but the difficulties of solving the equation and drawing the graph will be greatly increased at this time. In addition, the value of the function on the sphere with radius being 4 is 0 in the above boundary conditions, which is also an approximation of the actual situation. In the realistic state, the environment of the mass density around the research object is complex. Even if this complex environment is not considered, the objects will be in the background field with a mass density of not 0. In this case, the outer boundary condition should be a constant value close to 0 or a wave-function with the norm close to 0 at infinity (When $\mathcal{M}$ is a function of position (x, y, z) and time t).

Based on the analysis of the above equation (Equ. 40), the formation mechanism of the particles with large mass-level in the universe could be estimated: In the process of randomly fluctuating movement of the particles with lower mass-level in this universe, if they meet the appropriate external conditions, they will have the chance to form many standing waves with the same distribution. When the external conditions change, the general diffusions of these standing waves will occur with time. Some of them disappear; some of them form particle swarms that basically meet the above conditions. These particle swarms would become larger mass-level particles with extremely slow decay (the decay rate depends on the value of $e^{-\mathcal{M}}$ in Equ. 40 and the conformity of the particle shape with the condition of Equ. 43), and be retained stably in the universe for a long time (if the external or boundary conditions do not change significantly). In different positions and different external conditions, standing waves with different densities may form. Once the above conditions are basically met, these standing waves can be retained for a long time, thus forming more stable particles with different mass-levels. So, it can be concluded that the concept of macroscopic mass is the characterization of the agglomerate number of lower mass-level particles in certain domain, while the concept of macroscopic energy is the characterization of the non-agglomerate number of lower mass-level particles in certain domain. Moreover, the boundary between the two concepts are very blurred. It should be noted that due to the limitation of computing scale, it is impossible to simulate or be watched the generation process of particles from uniformly distributed energy or other conditions in this article, so the above possible generation process is envisaged, and its authenticity remains to be examined.

3.3.5.3 Acquisition Method of the Initial Wave-function

Obviously, the initial conditions of the solution to Equ. 40 have constraint on the norm of wave-function. The following gives the acquisition method of the initial wave-function when the position aggregation of lower mass-level particles is dominant (mostly in this case). To eliminate *D* by solving the simultaneous equations of Equ. 29, Equ. 32 and Equ. 33, we can obtain

$$\bar{V}^2 t^1 = \frac{\hbar}{m}. \tag{46}$$

Noticing $t^1 = 1$ s in Equ. 46, we ignore it for now. By substituting *m* in Equ. 46 with the mass $\mathfrak{M}$ in certain domain $\mathcal{V}$ and extracting the roots of both sides of the equation, we can get the quantity expression of the average velocity $\bar{V}$ for the particle swarm in this domain after finding the norm of both side of the equation (Euq. 47).

$$\left|\bar{V}\right| = \sqrt{\frac{\hbar}{\mathfrak{M}}}. \tag{47}$$

From a statistical point of view, the norm of the vector sum in certain domain is $\left|\boldsymbol{\mathcal{X}}\right| = \left|\bar{\boldsymbol{\mathcal{X}}}\right| \cdot k$ in the system with the same norm and the same opportunity for the directions in space (it is different from the problem described in Section **3.3.4.1** when the norm of the vector sum is fixed. See Part 1 of Supplementary Information for details). Where $\left|\bar{\boldsymbol{\mathcal{X}}}\right|$ is the average contribution of each particle to the total norm of vector sum in the domain, and *k* is the number of vectors. If these random vectors are regarded as the random movement of small particles with the same speed in space, then the total momentum of particle swarm in domain $\mathcal{V}$ or the sum of the total velocity from a statistical point of view in domain $\mathcal{V}$ is

$$\left|V\right| = \left|\bar{V}\right| \cdot k, \tag{48}$$

where *k* is the number of particles in domain $\mathcal{V}$. By substituting Equ. 47 into Equ. 48 and replacing *k* with $\frac{\mathfrak{M}}{\mu}$, we obtain

$$\left|V\right| = \sqrt{\frac{\hbar}{\mathfrak{M}}} \cdot \frac{\mathfrak{M}}{\mu} = \frac{\sqrt{\hbar \mathfrak{M}}}{\mu}, \tag{49}$$

where μ is the mass of single particle.

From the perspective of Max Born's interpretation for wave-function, after the wave-function of a system is normalized (assume it is ψ_1), the mass density of the wave-function everywhere it reaches is as follows

$$\rho_m = |\psi_1|^2 \cdot m, \tag{50}$$

where m is the mass of the target object (the same as m given in Equ. 40). In fact, even if not from the perspective of Max Born's interpretation for the wave-function, but from the perspective of Statistics according to the logic of this article, the square of the speed or the square of the norm of the wave-function is also directly proportional to mass. See Part 1 of Supplementary Information for details.

Since the wave-function represents the velocity or velocity density in unit volume, if use ψ_0 to denote the wave-function at a point, by substituting Equ. 50 into Equ. 49, the norm of the wave-function at a point is

$$|\psi_0| = \frac{\sqrt{\hbar m}}{\mu} \cdot |\psi_1|. \tag{51}$$

In view of the above discussion in Section **3.3.4.1**, further operation for ψ_0 is needed to get the relative wave-function $\mathcal{M}_0$ (divide ψ_0 by the speed of single particle and the number of particles in unit volume, and assign $\mathcal{M}_0$ to the same direction as ψ_1). If the system is composed of particles at the photon-level, $\mathcal{M}_0$ can be written as

$$\boxed{\mathcal{M}_0 = \lambda \cdot \frac{\sqrt{\hbar m}}{c \cdot \bar{\rho}_{m,0}} \cdot \psi_1}, \tag{52}$$

where c is the speed of light; $\bar{\rho}_{m,0}$ is the average mass density over a larger range (background field) than $\mathcal{V}$, which is generally accepted that $\bar{\rho}_{m,0} = 2 \times 10^{-28}\,\mathrm{kg} \cdot \mathrm{m}^{-3}$; λ is the unit coefficient, whose value is $1\,\mathrm{m}^{-3} \cdot \mathrm{s}^{-\frac{1}{2}}$. The intention of this coefficient is mainly to correct the dimensional difference caused by the conversion of diffusion coefficient into velocity and to compensate the denotation of unit volume implied in the conversion relationship. The above metioned is the acquisition method of the initial condition for Equ. 40.

In the case of low mass density (such as solving the problem of electron distribution outside the nucleus), the norm $|\mathcal{M}|$ of wave-function is extremely small in the initial condition obtained from Equ. 52 (the electrostatic interaction is not considered in the initial condition. Even if the electrostatic interaction with the nucleus is considered in the calculation process, the norm of wave-function is still a small one. see Section **3.6** for details). As already mentioned before, in this case, Equ. 40 is almost the same as Schrödinger equation. That is to say, Equ. 40 will reduce to Schrödinger equation when solving the electron distribution outside the nucleus, while the case of applying external electricity and magnetic field to the atomic system needs to be investigated separately. It should be noted that, as mentioned above, when the target system (background field) is composed of particles at the photon-level, c in Equ. 52 denotes the speed of light; while if the target system is composed of particles at other mass-levels, c denotes the speed of particles at that mass-level. The background domain here can be the whole universe or a small range of covering research objects. When the background domain is defined, the corresponding average mass density $\bar{\rho}_{m,0}$ of background can be determined. In addition, as can be seen from Equ. 40 and the acquisition method of the initial wave-function, only when both the position aggregation and direction aggregation reach maximum, $e^{-\mathcal{M}}$ is infinitesimal and particle swarm ($|\mathcal{M}|^2$) that do not satisfies the conditions in Equ. 43 can be completely nondispersive. In other words, the particle swarm with only direction or position aggregation being dominant can not completely prohibit diffusion when its shape does not satisfy the conditions described in Equ. 43.

3.3.5.4 Other Discussions

The way in which the initial wave-function $\mathcal{M}_0$ is acquired from Equ. 52 reflects the way in which the wave-function at a point is calculated. Therefore, to judge whether the wave-function $\mathcal{M}$ at a point changes with the reference system or the selection of the minimum reference particles, only examine whether the acquisition method of the initial wave-function $\mathcal{M}_0$ has changed. In view of the discussion in Section **3.3.2**, in any stationary (inertial) reference system, due to the synchronous change of movement and time that measures movement, from which the light speed c obtained and the velocity determining $\mathcal{M}$ are all constant; In addition, the light speed is included in Equ.

52 and other parameters are not limited by the reference system. Therefore, in any reference system, as long as the condition **HYPO 1**–**3** are satisfied, Equ. 40 deduced in this article and Equ. 52 which is the assignment method of initial wave-function are workable. Moreover, the case of particles of different mass-levels as the minimum (infinitesimal) reference particles in the same reference system will be investigated in the following. It is no need to consider the mass of the infinitesimal particle in the assignment method of initial wave-function; No matter which mass-level particle is the minimum reference particle, the synchronous change of movement and time that measure movement, from which the light speed *c* obtained and the velocity determining $\mathcal{M}$ are all constant; In addition, the other parameters in the assignment method of initial wave-function $\mathcal{M}_0$ are not limited by the selection of minimum reference particle. Therefore, no matter how large particles are regard as the minimum ones, Equ. 40 reduced in this article and Equ. 52 which is the assignment method of initial wave-function are also workable. To sum up, the gravitational effect between various particles can be regarded as the statistical effect of moving particles; It can be regarded as there is no interaction between particles of any mass-level. This is self-consistent with the hypothesis stated in **HYPO 3**. In this way, particles can be reunited step by step, and particles of high mass-level can form particles with higher mass-level under appropriate conditions. The whole universe is quantized no matter from which mass-level, and each mass-level is also equivalent. This is self-consistent with that "substance in the world is quantized" which is the axiomatic inference (or hypothesis) derived from **AXIO 2**.

When the direction aggregation is dominant, the form of Equ. 43 allows the velocities of some particles to be extremely fast, while the velocities of other particles not far away from them to decrease rapidly. The particles with very fast velocity can also have a higher mass density than the surroundings, and under certain conditions, the mass and velocity can mutually transform (as long as the conditions of Equ. 43 is met). The above conclusion is consistent with the hypothesis of "high-speed and random motion of particles in the universe" mentioned above.

Summarizing the results of the above stated, in any reference system that satisfies the conditions of **HYPO 1**–**3**, no matter what mass-level the basic (infinitesimal) reference particle in this artilcle is actually, and no matter how slow the "absolute" moving speed of that particle, from the perspective of human understanding, the particle mass at this level is infinitesimal, and the speed is infinite (the expansion of self-consistent range). At the same time, it also gives the legitimacy of the resolution of the vector in the (infinitesimal) micro-domain $\mathcal{V}$ in Section **3.3.4.1** and the viewpoint that "the absolute coordinate system needs to move following the whole particle swarm". In this way, Equ. 40 reduced in this article and Equ. 52 which is the assignment method of initial wave-function can be workable not only in a local space, but also in a wide space (or under various inertial reference systems); And they can be workable in not only in low mass-level particle system, but also the high mass-level particle system (or you can treat low mass-level particles as infinitesimal particles or high mass-level particles as infinitesimal particles)

3.4 A Simple Verification for the Mathematical Model

It can be seen from the above discussion that Equ. 40 can completely describe all things and phenomena in nature, and the situation described by Equ. 40 is logically self-consistent with the physical model (hypotheses) at the beginning of this article, while how reliable it is should be further tested in real situations. In this article, the time-dependent diffusion appearance of 1-dimensional Gaussian wave-packet without external field with even parity along the x-axis and the initial condition e^{-2x^2} is solved to compare with that of known theories, which is taken as a case for further discussion. For the convenience of operations, all evaluations in this Section adopt the Natural System of Units (i.e. $\hbar = c = 1$) and set $m = 1$ eV, while other Sections still adopt the International System of Units.

As mentioned above, to correctly solve Equ. 40, it is necessary to give the equation an initial condition with appropriate norm according to Equ. 52, which is different from solving Schrödinger equation. In the following, the average mass density of electrons outside the nucleus of hydrogen atom is taken as a reference to determine the norm of the wave-function in initial condition of Gaussian wave-packet e^{-2x^2} in time-dependent diffusion. It is assumed that the essence of these two kinds of problems is the same, both of them are particle movement at the photon-level. Let $m = 9.109\,389\,7(54) \times 10^{-31}$ kg, which is the electron mass, and evaluate the coefficient pre the normalized wave-function ψ_1 given in Equ. 52, of which the value is about 1.63×10^{-13}; The normalized norm of the above Gaussian wave-packet is $\sqrt{\frac{2}{\pi}}$. So, the approximate real value $\mathcal{M}_0(x,0) = 10^{-13} e^{-2x^2}$ of the same order of magnitude is taken as the initial condition without affecting the discussion of the problem (After verification, when the pre e^{-2x^2} coefficient is less than 10^{-3}, the maximum relative deviation of the contour for the wave-packet obtained by the two methods is less than 1.14% in all ranges, see Part 6 of Supplementary Information for details). As a comparison, the case of larger norm in the initial conditions is also evaluated (such as $\mathcal{M}_0(x,0)=e^{-2x^2}$). At the same time, Schrödinger and Dirac equations are used to solve the time-dependent diffusion appearance of this wave-packet. For Dirac equation, the case of the two components of the wave-function are equal (i.e. $\chi_1(x,0)=\chi_2(x,0)=\frac{\sqrt{2}}{2}e^{-2x^2}$) is taken as the initial value here (See the drawing process of Fig. 4 in Part 8 of Supplementary Information for the detailed Mathematica code of solution process).

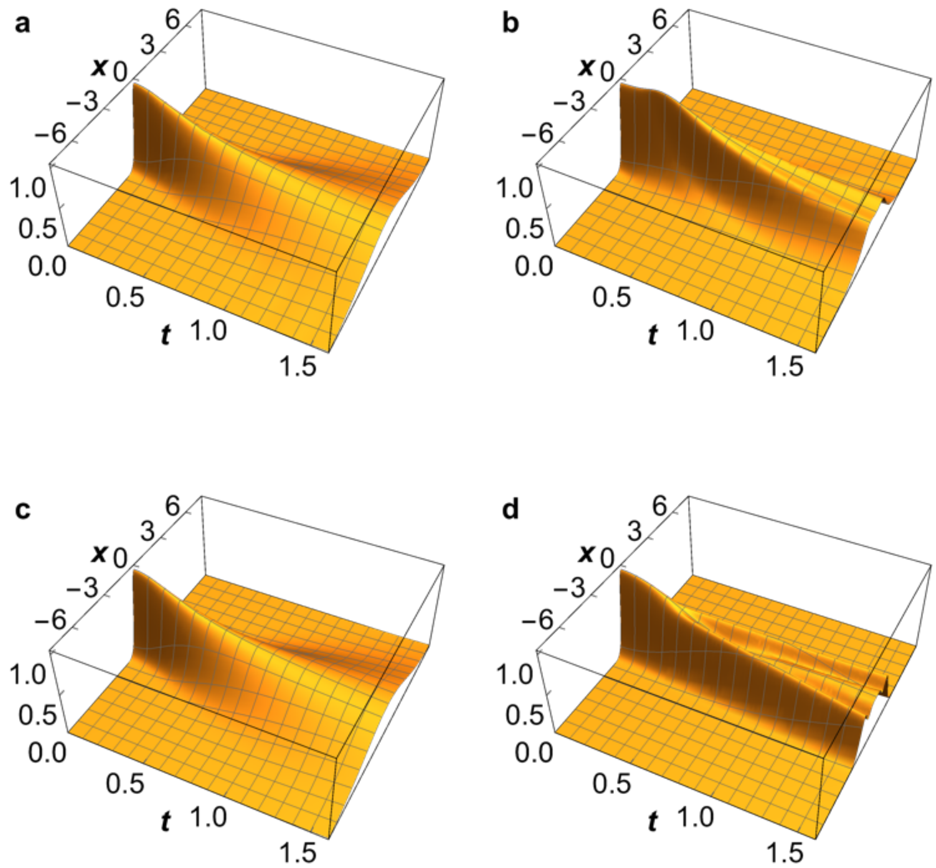


Figure 4 | 1-dimensional time-dependent diffusion graphics of Gaussian wave-packet e^{-2x^2} obtained by different methods under the Natural System of Units. **a**, Computation result of Equ. 40 when the initial condition is $\mathcal{M}_0(x,0)=10^{-13}e^{-2x^2}$. The norm has been magnified ($\times 10^{13}$) for the facilitation of the shape comparison; **b**,

Computation result of Equ. 40 when the initial condition is $\mathcal{M}_0(x,0)=e^{-2x^2}$; **c**, Computation result of Schrödinger equation; **d**, Computation result of Dirac equation.

As is illustrated in Fig. 4, there is almost no difference between the wave-packet time-dependent diffusion graphics obtained by Equ. 40 under an appropriate initial condition $\mathcal{M}_0(x,0)=10^{-13}e^{-2x^2}$ (Fig. 4a) and that obtained by the Schrödinger equation (Fig. 4c) (Note: For convenience, the norms of wave-functions are discussed in this Section, not the squares of norms). This little difference has been illustrated detailly by the standard deviations of norms at different diffusion times in Fig. 5, in which the profiles of Gaussian wave-packets predicted by the two methods almost coincide at each time. It is further verified that the equation given in this article will approximate to Schrödinger equation (at least in the problem of Gaussian wave-packet) in the domain with extremely thin mass density (such as the domain of electron distribution outside the nucleus, excluding the influence of electric field by nuclear), which is consistent with the conclusion in Section **3.3.5.1** above. In Part 6 of Supplementary Information, it was further verified that the time-dependent diffusion graphics of Gaussian wave-packet obtained by Equ. 40 according the initial condition estimated by the product of Equ. 72 and norm $|\psi_1|$ of the normalized wave-function is still basically consistent with that obtained by Schrödinger in the presence of nuclear electric field.

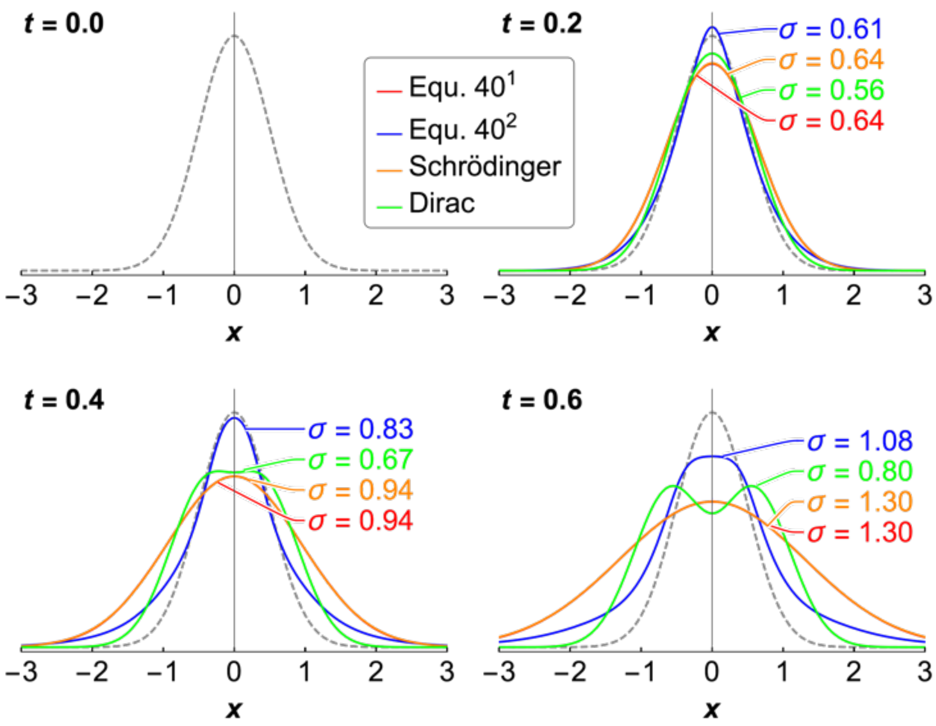


Figure 5 | Time-dependent diffusion trend graphics of Gaussian wave-packet (norm) predicted by four methods (Equ. 40[1]: initial condition is $\mathcal{M}_0(x,0)=10^{-13}e^{-2x^2}$; Equ. 40[2]: initial condition is $\mathcal{M}_0(x,0)=e^{-2x^2}$) in Natural System of Units at different moments (t = 0.0, 0.2, 0.4, 0.6 eV^{-1}).

If the norm of the wave-function in initial condition is large (such as $\mathcal{M}_0(x,0)=e^{-2x^2}$), the profile of wave-packet will show obvious bulge or particle (position of direction) aggregation near t = 0.3 eV^{-1} (Fig. 4b). Because of self-aggregation, the profile obtained by Equ. 40 is steeper along the direction of x-axis, which can be clearly seen from the standard deviation of the norm for Gaussian wave-packet in Fig. 5 at three moments. Under such initial conditions, the diffusion rate predicted by Equ. 40 is not as fast as that predicted by Schrödinger equation, and the main peak in the profile is unwilling to dissipate, which is closer to the situation described by the Dirac equation (Fig. 4d). It is speculated that it is mainly caused by the gravitation of the wave-packet itself. After t = 1 eV^{-1}, the main peak begins to split into two peaks (for the more obvious splitting, see the case the pre e^{-2x^2} coefficient is 1.4 in Part 7 of Supplementary Information); in the case described by the Dirac equation, serious splitting occurs after t = 0.5 eV^{-1} (the main peak splits into two secondary peaks, and then each secondary peak splits into two smaller peaks). It is considered that this phenomenon is caused by the fact that the gravitation of the wave-packet itself is not considered in Dirac equation and the correction to the real result is excessive. This is also confirmed by the standard deviation profiles illustrated in Fig. 5.

In addition, to study the influence of the norm in the initial condition on the diffusion of the wave-packet in detail, we also compare the shapes of the key parts of the profiles (the time-dependent trend of the norm for the wave-packet at $x = 0$ and the wave-packet at the maximum value of the norm) after assigning different initial conditions ($10^{-13}e^{-2x^2}$, e^{-2x^2}, $1.2e^{-2x^2}$ and $1.4e^{-2x^2}$) to Equ. 40 (See the drawing process of Fig. 6 in Part 7 of Supplementary Information for the detailed Mathematica code of solution process), which is illustrated in Fig. 6. It can be seen from the figure that when different initial conditions are given according to the norms from small to large, the wave-packet diffusion profiles predicted by Equ. 40 (at $x = 0$) is consistent with the Schrödinger equation at the beginning and gradually tends to continue to agglomerate near 0.3 eV^{-1} (the profile gradually bulged), and Fig. 6a shows this trend. In addition, Fig. 6b shows the shape of the wave-packet at the highest point (see Part 7 of Supplementary Information for the full spectrum waveform with the initial condition of $\mathcal{M}_0(x,0) = 1.2e^{-2x^2}$ and $\mathcal{M}_0(x,0) = 1.4e^{-2x^2}$, and the comparison with that by Dirac equation). It can be seen from the figure that with the gradual increase of the initial norm, the wave-packet will gradually shrink in the first period of time, and when the norm reaches the maximum, the waveform will gradually become steeper and steeper, and (presumably) it will gradually approach that by the function satisfying Equ. 43. It can also be seen from this trend that when the mass density of the wave-packet increases gradually, the attenuation speed of the wave-packet becomes slower and slower.

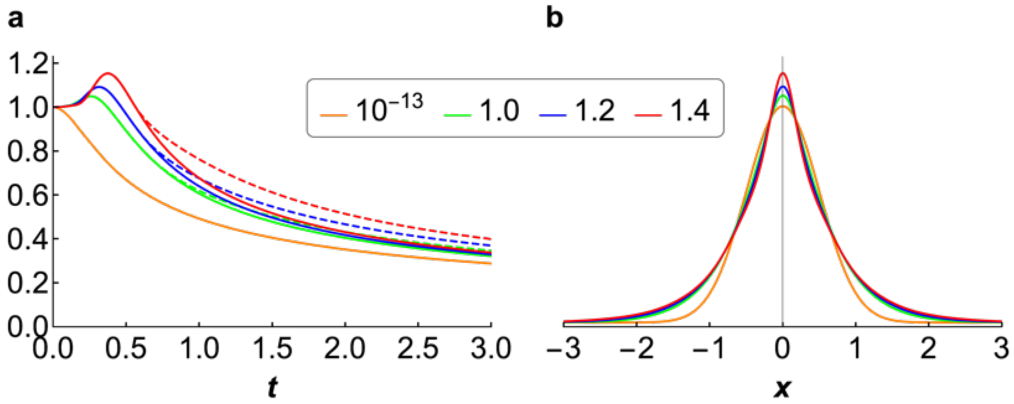


Figure 6 | Profile of the shape contrast of Gaussian wave-packet after assigning different initial conditions $\mathcal{M}_0 = (10, 1.0, 1.2, 1.4)\, e^{-2x^2}$ in Natural System of Units. **a**, Time-dependent trend of the norm for the wave-packet at $x = 0$ (solid line) or at the maximum value of the norm (dashed line with the same color); **b**, Profiles at the maximum values of the norms. The numbers in the legend are the pre e^{-2x^2} coefficients, which indicate different initial conditions. The initial norm at $t = 0$ eV^{-1} in each result is normalized for the facilitate of shape comparison.

3.5 Add External Fields to the Equation

Theoretically, the particle motion law of the whole system described by Equ. 40 is basically sufficient. All forces and happenings in nature are caused by the general diffusion of particles. However, when dealing with practical problems, to reduce computing scale etc., a local process is often studied. Therefore, it is necessary to add external fields to Equ. 40.

In Section **3.7**, we have speculated the essence of the four fundamental forces of nature that have been discovered by human beings. Since the strong force can be regarded as the energy interaction of single point to single point, it is no need to use the form of external fields. Therefore, it is easy to handle it directly with Equ. 40, and this kind of interaction is not considered here. Only the gravitational, electromagnetic and weak interaction fields are considered in the following. In view of the different forms (there is no repulsion in gravitation) of gravitation and other forces (such as electromagnetic force and weak force, etc.) and the understanding of gravitation in this article, it is considered that the external field should be divided into two situations: gravitational field and other potential field. The gravitational field is caused by the difference of spatiotemporal probability of spatial anisotropy of (randomly moving) larger mass-level particles (or tiny particles) forming by the local background density produced by the equiprobable distribution of randomly moving tiny particles in the space-time, which is a statistical effect dominated by position aggregation; Other fields are caused by the acceleration (or change of direction) of particles of higher mass-level (or tiny particles) forced by the spin field, which is a statistical effect dominated by direction aggregation. For the gravitational field, the acceleration effect can generally be ignored; for other potential fields, the gravitational effect can generally be ignored. Here is a detailed description:

First of all, the production principle of random spin of particles with large mass-level is described. If the Relativistic effect (statistical effect of moving particles) is not considered, the agglomeration of particles in space can be regarded as following the above Poisson distribution (Equ. 20). Here, we study the aggregation of particles in one box (or several adjacent boxes) (assuming that the aggregation is spherical, and there is no aggregation caused by Relativistic effect). This is the case when the volume of the particle is small enough and the density differences of the vectors are small enough. When the mass-level of the particle is determined, the more microscopic the investigated system is (the microscopic is relative), the closer it is to this situation. When a number of particles with the same speed and random direction agglomeration in 3-dimensional space, there must be a corresponding movement component to produce a spin effect on the overall centroid at a moment. To illustrate this problem, the particles with equal speed and random directions are still regarded as random vectors with equal norms, and it is divided into the following two steps: Firstly, the distribution of the norm for the angular velocity generated by single random vector to the total centroid is obtained, and then it is extended to the distribution of the norm for the angular velocity generated by a number of random vectors to their centroid.

At the beginning, the distribution of the norm $\left|\boldsymbol{\Omega}_{\mathrm{S}}\right|$ for the random angular velocity generated by the random vector $\boldsymbol{V}_{\mathrm{S}}$ with the equal norm (the linear velocity of the random points on the sphere) uniformly distributed on a unit sphere S is studied. The contribution of $\boldsymbol{V}_{\mathrm{S}}$ to the random angular velocity of the spherical center is possible in all directions of space. How to add multiple rotation contributions together to specify the overall rotation? It is noticed that the vector product $\boldsymbol{\omega}_{\mathrm{S}}$ of the linear velocity $\boldsymbol{v}_{\mathrm{S}}$ of a spherical point and the radius $\boldsymbol{r}$ of its unit sphere S can easily explain the total rotation contributions or explain the distribution of the contribution to the angular velocity by the single random vector (Here, $|\boldsymbol{r}|=1$. If $|\boldsymbol{r}|\neq 1$, then $\frac{\boldsymbol{\omega}_{\mathrm{S}}}{|\boldsymbol{r}|}$ is the contribution of the linear velocity $\boldsymbol{v}_{\mathrm{S}}$ to the angular velocity $\boldsymbol{\omega}_{\mathrm{S}}$), namely

$$\boldsymbol{\omega}_{\mathrm{S}}=\boldsymbol{r}\times\boldsymbol{v}_{\mathrm{S}}. \tag{53}$$

The above problems can be divided into two independent steps: The first step is to determine the direction of the unit vector $\boldsymbol{r}$ in space, that is, to determine the position of the end point of the vector $\boldsymbol{r}$ on the unit sphere S. This position is uniformly distributed on the whole sphere S, and is denoted by random vector $\boldsymbol{R}$. The second step is to determine the direction of the linear velocity $\boldsymbol{v}_{\mathrm{S}}$ of the point at this position, which is also uniformly distributed in the whole space, and it is denoted by the random vector $\boldsymbol{V}_{\mathrm{S}}$. Suppose that there is a sphere S' with the radius being $|\boldsymbol{v}_{\mathrm{S}}|$ at the end of $\boldsymbol{r}$ (it is the end on the sphere S). Then, the random vector $\boldsymbol{V}_{\mathrm{S}}$ is equivalent to the vector formed by the connection of uniformly distributed points on the sphere S' and the center of sphere S'. Noticing $|\boldsymbol{r}|=1$ and the definition of cross product, when the direction of each $\boldsymbol{r}$ is determined, $|\boldsymbol{r}\times\boldsymbol{v}_{\mathrm{S}}|$ is equivalent to the norm of the vector obtained from the connection between the center of sphere S' and the projectile of the uniformly distributed points on the sphere S' mapping along a direction paralleling to $\boldsymbol{r}$ onto a tangent disk D' which pass through the center of the sphere S' and is perpendicular to $\boldsymbol{r}$, which is recorded as $|\boldsymbol{\omega}'|$ (Fig. 7).

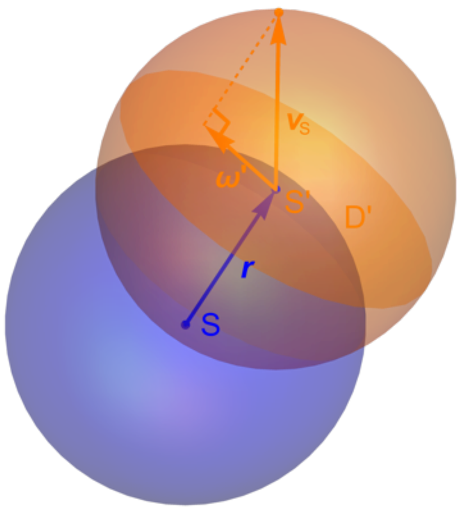


Figure 7 | Schematic diagram of generation way for vector $\boldsymbol{\omega}'$.

When the random vector $\boldsymbol{R}$ changes, it is equivalent to driving the tangent disk D' on the unit sphere S to move together. Since the sum of two uniform distributions is still uniform distribution, obviously, the result of $\boldsymbol{R}\times\boldsymbol{V}_{\mathrm{S}}$ can be regarded as the uniform distribution of the random vector $\boldsymbol{\Omega}'$ in the entire space. Find out the distribution of the norm $|\boldsymbol{\Omega}'|$ of the random vector on disk D', and assign the random direction of it in space to obtain the distribution of the norm $|\boldsymbol{\Omega}_{\mathrm{S}}|$ of the angular velocity generated by the contribution of $\boldsymbol{V}_{\mathrm{S}}$ to the center of the sphere S.

Assume the random variables $N_1 \sim N\ (0,\ 1)$, $N_2 \sim N\ (0,\ 1)$ and $N_3 \sim N\ (0,\ 1)$ are independent of each other, then one equivalent coordinate X of three coordinates of $\boldsymbol{R}$ which is the unit random vector can be written as[14,15]

$$X=\frac{N_1}{\sqrt{{N_1}^2+{N_2}^2+{N_3}^2}}, \tag{54}$$

of which the probability density is

$$f_X(x)=\begin{cases}\dfrac{1}{2}, & -1<x<1,\\ 0, & \text{other.}\end{cases} \tag{55}$$

We set up the 3-dimensional Cartesian coordinate system for S'. Suppose that the disk D' is perpendicular to the z-axis and the random variables $\Theta \sim U(-1,\ 1)$ and $H \sim U(-\pi,\ \pi)$. Then, the coordinates of the random vectors $\boldsymbol{\Omega}'$ obtained by projecting the points uniformly distributed on sphere S' onto disc D' are $X=c\cdot\sin\cos^{-1}\Theta\cdot\cos H$, $Y=c\cdot\sin\cos^{-1}\Theta\cdot\cos H$ and $Z=0$, of which the norm is

$$|\boldsymbol{\Omega}'|=\sqrt{X^2+Y^2+Z^2}=c\cdot\sqrt{1-\Theta^2}. \tag{56}$$

So, the probability of $|\boldsymbol{\Omega}'|$ is

$$|\boldsymbol{\omega}'|(x)=\begin{cases}\dfrac{x}{c\cdot\sqrt{c^2-x^2}}, & 0<x<c,\\ 0, & \text{other.}\end{cases} \tag{57}$$

Notice that the distribution of random variable ($|\boldsymbol{\Omega}_{\mathrm{S}}|$) is the random distribution generated by the random motion of $\boldsymbol{\Omega}'$ in space driven by $\boldsymbol{R}$. Therefore, by taking the product $F_X(x)\cdot\left|\boldsymbol{\Omega}'\right|(x)$ of random variables, we can get the probability density of one (X) of the equivalent coordinates which is the contribution of the random vector $\boldsymbol{V}_{\mathrm{S}}$ to the angular velocity $\boldsymbol{\Omega}_{\mathrm{S}}$, namely

$$\boldsymbol{\omega}_{\mathrm{S},X}(x)=\begin{cases}\dfrac{1}{2c}\sin^{-1}\dfrac{x}{c}+\dfrac{\pi}{4c}, & -c<x<0,\\ \dfrac{1}{2c}\cos^{-1}\dfrac{x}{c}, & 0\le x<c,\\ 0, & \text{other.}\end{cases}\tag{58}$$

The contribution to the angular velocity $\boldsymbol{\Omega}_{\mathrm{B}}$ of the random vector $\boldsymbol{V}_{\mathrm{B}}$ uniformly distributed in the whole unit ball enclosed by the sphere S is also magnified by the reciprocal value $\dfrac{1}{r}$ of the norm r of the radius $\boldsymbol{r}$ of where the starting point of this vector $\boldsymbol{V}_{\mathrm{B}}$ is in the ball. Therefore, the contribution of $\boldsymbol{V}_{\mathrm{B}}$ to one ($\boldsymbol{\Omega}_{\mathrm{B},X}$) of the equivalent coordinates of $\boldsymbol{\Omega}_{\mathrm{B}}$ is calculated as follows:

$$\boldsymbol{\Omega}_{\mathrm{B},X}(x,r)=\frac{1}{r}\cdot\boldsymbol{\Omega}_{\mathrm{S},X}(x).\tag{59}$$

Thereby, the new probability density is

$$\boldsymbol{\omega}_{\mathrm{B},X}(x,r)=\begin{cases}\dfrac{1}{2c}r\sin^{-1}\dfrac{rx}{c}+\dfrac{\pi r}{4c}, & -\dfrac{c}{r}<x<0,\\ \dfrac{1}{2c}r\cos^{-1}\dfrac{rx}{c}, & 0\le x<\dfrac{c}{r},\\ 0, & \text{other.}\end{cases}\tag{60}$$

Then, the distribution function $\boldsymbol{\Omega}_{\mathrm{B},X}(r)$ of contribution of $\boldsymbol{V}_{\mathrm{B}}$ to one of the equivalent coordinates of $\boldsymbol{\Omega}_{\mathrm{B}}$ is obtained. Next, $\boldsymbol{\Omega}_{\mathrm{B},X}(r)$ is integrated in the whole unit ball, namely

$$\int_0^1 4\pi r^2 \cdot \boldsymbol{\Omega}_{\mathrm{B},X}(r)\mathrm{d}r. \tag{61}$$

Equ. 61 describes the case that the particles are uniformly distributed in the ball. If the particles are distributed in the (nondispersive) shape described by Equ. 43, they should be integrated approximately according to the density function, but this analytical function is not available at present. Even if only the case of uniform distribution is considered, the discussion of the following problems will not be affected. For the case described by Equ. 43, it is a spherical particle swarm with uneven distribution of particle density along the radial direction. It can be inferred that it is still a similar case with rotation effect (In fact, for every sphere layer, a spin vector with random norm and direction can be generated. According to the Central Limit Theorem, the norm of the total vectors must follow the Maxwell distribution with certain parameter). The probability density of the contribution of $\boldsymbol{V}_{\mathrm{B}}$ in the whole unit ball to an equivalent coordinate X of angular velocity $\boldsymbol{\Omega}_{\mathrm{B}}$ can be obtained after finding the derivative of Equ. 61 with respect to x and normalizing it, namely

$$\boldsymbol{\omega}_{\mathrm{B},X}(x) = \begin{cases} \dfrac{9\pi c^3}{128x^4}, & x > c \vee x \le -c, \\ \dfrac{3\left(8x^4 \sin^{-1}\dfrac{x}{c} + 4\pi x^4 + U_1\right)}{64cx^4}, & -c < x < 0, \\ \dfrac{3\left(8x^4 \cos^{-1}\dfrac{x}{c} - U_1\right)}{64cx^4}, & 0 < x \le c, \\ 0, & \text{other}, \end{cases} \tag{62}$$

where $U_1 = x(2x^2 + 3c^2)\sqrt{c^2 - x^2} - 3c^4 \sin^{-1}\dfrac{x}{c}$. This is the case of random vector $\boldsymbol{V}_{\mathrm{B}}$ inside the unit spere S (including S).

Next, we extend it to the case that the radius of the ball is arbitrary value R. When the radius of the ball is R, that is, each R will scale the above situation to $\dfrac{\boldsymbol{\Omega}_{\mathrm{B},X}(x)}{R}$. In this way, the probability density of the contribution of the random vector $\boldsymbol{V}$ to the single equivalent coordinate X of angular velocity $\boldsymbol{\Omega}$ is

$$
\boldsymbol{\omega}_X(x)=\begin{cases}\dfrac{9\pi c^3}{128R^3x^4}, & x>\dfrac{c}{R}\vee x\le -\dfrac{c}{R},\\[2ex] \dfrac{3\left(8R^4x^4\sin^{-1}\dfrac{Rx}{c}+4\pi R^4x^4+U_2\right)}{64cR^3x^4}, & -\dfrac{c}{R}<x<0,\\[2ex] \dfrac{3\left(8R^4x^4\cos^{-1}\dfrac{Rx}{c}-U_2\right)}{64cR^3x^4}, & 0<x\le\dfrac{c}{R},\\[2ex] 0, & \text{other},\end{cases} \tag{63}
$$

where $U_2 = Rx(2R^2x^2+3c^2)\sqrt{c^2-R^2x^2}-3c^4\sin^{-1}\dfrac{Rx}{c}$, of which the standard deviation is $\dfrac{\sqrt{6}c}{3R}$. If take $R = 3$ and $c = 1$, the above probability density (Equ. 63) of angular velocity can be plotted as follows (Fig. 8):

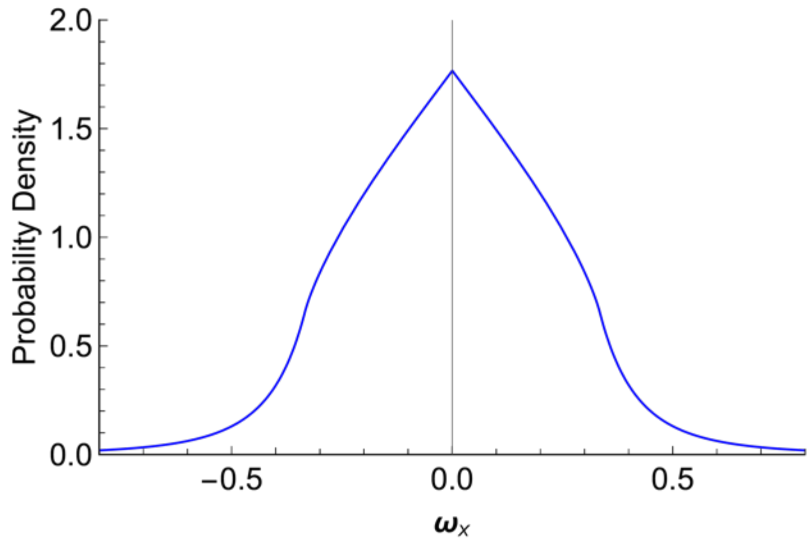


Figure 8 | When R = 3, c = 1, the contribution of random vector $\boldsymbol{V}$ to the distribution of single equivalent coordinate $\boldsymbol{\Omega}_X$ of angular velocity $\boldsymbol{\Omega}$.

Therefore, when k independent and identically distributed random vectors $\boldsymbol{V}$ move randomly in space, according to the Central Limit Theorem (when they are grouped together), the norm $|\boldsymbol{\Omega}|$ of contribution of the average angular velocity of the whole component to its centroid follows the Maxwell distribution with scale parameter $\dfrac{\sqrt{6}c}{3R\sqrt{k}}$. To verify this conclusion, $k = 10^3$, $R = 3$ and $c = 100$ are taken here, and it is compared with the simulation result with 10^6 samples of the same parameter. The result is illustrated in Fig. 9 (See the drawing process of Fig. 9 in Part 8 of Supplementary Information for the detailed working process of simulation). It can be seen from the figure that the analytical expression derived in this article is in good agreement with the simulated results.

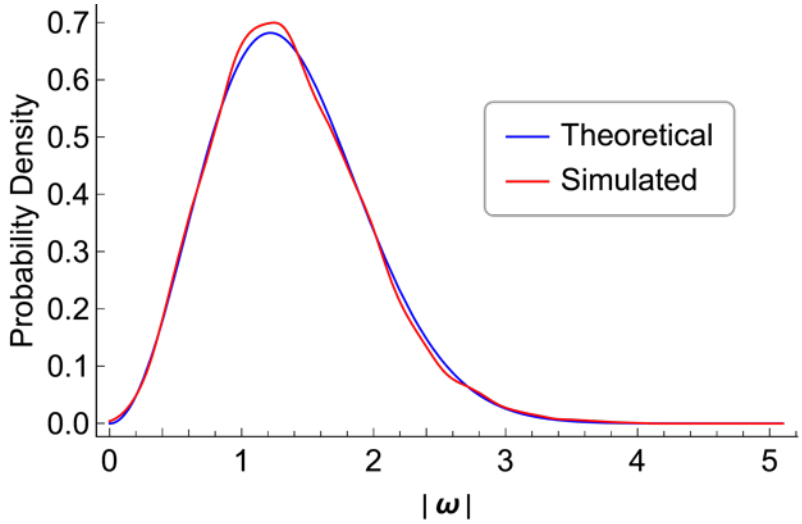


Figure 9 | When R = 3, the distribution of the norm $\left|\boldsymbol{\Omega}\right|$ of the total angular velocity when 10^3 random vectors with norm being 100 are grouped together.

See Part 4 of Supplementary Information for the detailed Mathematica code of the above calculation process. So far, this article proves that there are more or less rotational components in the agglomeration (even if there is no aggregation) generated by the aggregation of randomly moving particles in space, and the angular velocity of the agglomeration follows the Maxwell distribution with a certain parameter. If the strengthened of the statistical effect of moving particles to the agglomeration and rotation is added, spin is the same general law of the universe as aggregation.

It is believed that the acceleration in a certain direction that we can perceive is essentially the time-dependent change rate of the average velocity of the tiny particles in the target domain towards that direction (If it is only the average velocity, it has been proved in Section **3.3.2** that the target object will form an inertial system with no physical law difference from the previous state, and then the target object will lose the sense of acceleration). The essence of the gravitational acceleration is the time-dependent change rate of the average velocity of the tiny particles at the lower vector density to the higher vector density due to the statistical effect of the moving particles (e.g. when a particle agglomeration meeting the condition of Equ. 43 is in the gravitational field, the particles in it are replaced by the particles in the gravitational field with higher density, resulting in the deformation of the agglomeration and that the particles forming the agglomeration move weakly to the low-density domain and strongly to the high-density domain); The essence of other types of acceleration is to increase the time-dependent change rate of the average velocity in the opposite direction in the (moving) reference system composed of tiny particles. In essence, the causes of these forces or potentials are the same, but they belong to different types.

For uniform circular motion, the acceleration direction is perpendicular to the velocity direction, and the principle of acceleration generation is the same as above, but the form is special. When a spherical agglomeration domain of particles produces spin (which can be approximately thought of as uniform circular motion), ignoring the behavior of single particles and from the overall view, the matter spherical domain rotates continuously and has continuous centripetal force to change the direction of motion. If there is no spin, a particle $\mathcal{S}$ can be formed which satisfies the condition of Equ. 43; As the rotation intensifies, more centripetal force is needed to maintain the acceleration. In this case, $\mathcal{S}$ must satisfy the condition of Equ. 43 again by changing the previous shape, that is, by increasing the norm $|\mathcal{M}|$ of wave-function near the center of rotation (From another point of view, if the norm of the wave-function at each point near the edge of $\mathcal{S}$ becomes larger, $|\mathcal{M}|$ should be larger near the center to meet the condition of Equ. 43). That $|\mathcal{M}|$ is larger can not only act out that the particle concentration is larger, but also act out that the particle speed is faster. In this article, we only study the case that the faster speed results in the increase of $|\mathcal{M}|$. There are four cases that the particle speed is faster at the center of rotation, as shown in Fig. 10a–d.

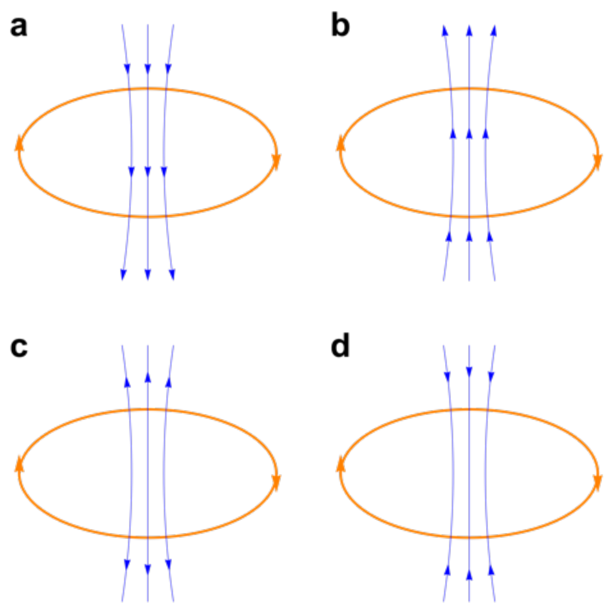


Figure 10 | Possible cases of faster velocity of particles at the center of rotation satisfying the condition of Equ. 43.

On the other hand, if S doesn't rotate, the number of tiny particles escaping along the radial direction is a certain value (It is in balance with the number of tiny particles absorbed by S); With the increase of rotation, according to the above analysis of acceleration, there may be more tiny particles escaping along the radial direction or almost no particles. Considering the gravitation and mass stability of the particle swarm (basically satisfy the condition of Equ. 43), if some particles escape, the whole particle swarm obviously needs to be replenished. This replenishment can only be provided in the axial direction perpendicular to the rotation plane. This cause that it is a suction state along the spin axis direction (excluding the possibility of Fig. 10c), and the particle swarm is more inclined to be in the state of Fig. 10d; When there is almost no particle escape, the particle swarm is more likely to be in the state of Fig. 10a and Fig. 10b. We will only study this case in the following. As a result, the spin results in a field of direction gathering in both the axial and radial directions (especially in the axial direction), resulting in a radial spin axis (similar to the formation mechanism of the solar wind[16]). When such a series of spin particle swarms come together, if the radiation axis is consistent with each other, there will be an obvious direction aggregation field of velocity (similar to the mechanism of passive magnetic generation of iron bars). In addition, particles of different mass-levels (larger particle groups) formed by the spin particles will form direction gathering fields with different intensity. It is considered that this kind of direction gathering field is other potential field (electromagnetic field or weak interaction field). When studying one particle of the spin particle swarm, it may be found strings similar to those in String Theory, and the length of the string should also be computable (this problem will not be studied in depth here). Other potential fields generated in this way only change the aggregation state of particles in space (including the aggregation of velocity direction), but not the original velocity of particles. Since the spin is also spontaneously formed and in the background field, the aggregation of velocity direction will also be limited by the background field and follows the statistical law.

It is believed that the reason for the change of the moving speed of the agglomerate caused by the gravitational potential energy is the accumulation of the statistical effects of position aggregation being dominant provided by the moving particles in the gravitational field; The reason that other potential energy causes the change of the moving speed of agglomerate is the accumulation of the statistical effects of direction aggregation being dominant provided by the moving particles in other potential field. The mechanism for gravitational field to realize the acceleration process is that the gradual aggregation of positions of particles (after being replaced by particles with position gathering effect) forming the target object leads to the unbalanced diffusion trend which promotes the accumulation of velocity; That the particles forming the target object are constantly replaced by the particles with velocity aggregation effect, which makes the velocity aggregation effect accumulate gradually, realizes the acceleration process for other potential field. Such a process can only form a trend at first, that is, the so-called force. Once target object moves along the trend line, it will realize the accumulation of the trend. In these potential fields, the particles that follow the statistical law to form the potential field will continue to follow the statistical effect together with the particles of the target object in the field (meet the general diffusion described by Equ. 40) . When the particles in the target object are studied separately, the phenomenon of continuous aggregation of the position or direction will appear. This phenomenon is characterized by the increase of the potential energy of the target particles. In Section **3.7**, we have deduced the particle structure of the matrix for electron. It is considered that other external fields (electromagnetic field and weak interaction field) contain similar structures. The existence of other external fields leads to the increase of potential energy of particles in this structure, while the existence of gravitational field leads to the increase of potential energy caused by the aggregation of particles in unit volume. In this way, most of the mass in the whole target particle can be accelerated by the external potential field. The point of view of the target wave-function is generally much smaller than the particle structure of the matrix for electron, and the point where the wave-function is located is generally among these larger particles, which can be regarded as equivalent in the gravitational field. The wave-function (norm) of one point before the action of the external field is much smaller than that after the action of the external field (If the wave-function caused by an external field, such as a gravitational field, is small, it can be ignored or studied together with the target object). The superposition of the wave-function at this point can be approximately regarded as characterizing only the speed of these larger particles after acceleration.

Therefore, the contribution of other external fields to the time-dependent rate of change of the density $\mathcal{M}$ of the total relative vector at one point is shown as two superposable parts. On the one hand, since the diffusion part of this (infinitesimal) micro-domain is still in the background field, no matter how the acceleration effect is affected, the diffusion effect of the affected vector is no different from that of the vector formed spontaneously when it is not affected by acceleration, and still follows the same general diffusion law; On the other hand, it shows the direction aggregation effect added by the accelerating field in the (infinitesimal) micro-domain, which needs to meet the changing part of the average potential energy, and the direction is approximately the direction of $\mathcal{M}$ at this point, which is an additional time-dependent change rate of $\mathcal{M}$ that cannot be changed by diffusion. The joint action of these two vectors determines that the time-dependent rate of change of total relative vector density at a certain point is the sum of these two terms. For the additional time-dependent rate of change of $\mathcal{M}$ caused by other potential fields, the potential field here can be understood as the additional diffusion coefficient of particles (Since the diffusion coefficient is related to the square of speed, and this other potential field changes the velocity of particle swarm. For example, in the gravitational field, $\frac{GM}{r}m=\frac{1}{2}mv^2$ and the diffusion coefficient $D=\frac{v^2}{2}t^1$, where t^1 = 1 s). In addition, the Relativistic effect does not need to be considered to determine the potential energy of large monopole particles, because whether there is Relativistic effect or not, $\rho_m v^2$ is a fixed value, which depends on the potential energy where the monopole particles located. From another angle, since the potential energy of a particle with a larger mass-level is only related to its position, no matter what its motion state is, whether it is affected by Special Relativistic effect or not, the potential energy will not change; Moreover, the existing particles must disperse in the whole field, so the change of particle potential energy will inevitably lead to the corresponding change of particle kinetic energy. For this additional diffusion behavior in the external field, in Fig. 1a, it can be seen that the wave-function at A is a certain value $\boldsymbol{\mathcal{M}}$ and at B is 0. The same wave-function ($\boldsymbol{\mathcal{M}}$) can be obtained by taking A and B as a point to find the second derivative. Therefore, the current wave-function is numerically consistent with the second derivative. And the corresponding diffusion coefficient is also consistent with the current coefficient. Therefore, no matter what their dimensions are, they do not affect the investigation of quantity value. Since $\boldsymbol{\mathcal{M}}$ is a plane vector, its direction should be perpendicular to $\frac{\partial \boldsymbol{\mathcal{M}}}{\partial t}$. Therefore, the product of other potential energy $\mathcal{E}$ as diffusion coefficient and $\boldsymbol{\mathcal{M}}$ at a certain point is directly proportional to the change of diffusion rate of micro vector. The specific form of this expression has been verified by Schrödinger equation, namely

$$-\boldsymbol{i} \cdot \frac{\mathcal{E}}{\hbar} \cdot \boldsymbol{\mathcal{M}}. \tag{64}$$

In summary, the final form of the equation with external fields to characterize the general diffusion of relative momentums (vectors) could be written as:

$$\boxed{\boldsymbol{i}\hbar \frac{\partial \boldsymbol{\mathcal{M}}}{\partial t} = -\frac{\hbar^2}{2m} e^{-\boldsymbol{\mathcal{M}}} \left[\Delta \boldsymbol{\mathcal{M}} - T^2(\boldsymbol{\mathcal{M}}) \right] + \mathcal{E} \cdot \boldsymbol{\mathcal{M}}}, \tag{65}$$

Here again, it is emphasized that Equ. 65 is not a final solution, it is only a compromise solution to reduce the computational scale in some cases. If the whole universe is the physical model established in this article, and considering the essential reasons of the four fundamental forces (speculation in Section **3.7**), the Equ. 40 or Equ. 65 derived in this article can even be called the real Theory of Everything. Since it describes the most basic laws of motion in nature, (if added spin description, the adding method is the same as the traditional method) it can reveal the essence behind all phenomena and characteristics. According to the logic of particle classification by mass in this article, there is no need to add spin description. Spin effect is also caused by the general diffusion behavior of statistical moving particles, so spin information can be obtained by solving the general diffusion behavior of particles with lower mass-level.

3.6 Preliminary Exploration on the Spin Magnetic Moment of Electron

Firstly, the expression of momentum operator under the logic of this article is determined as follows. To find the first derivation of Equ. 22 with respect to position (x, y, z), we can obtain

$$\nabla R(\mathcal{M},k). \tag{66}$$

In view of the statistical effect of moving particles, the actual performance of the momentum for each-order particle swarm will not be directly proportional to Equ. 66, but will be "weakened" by the diffusion coefficient factors $\frac{1}{k}$ of each-order. Therefore, to multiply Equ. 66 representing each complex value by $\frac{1}{k}$ and add each-order together, we can obtain

$$\sum_{k=1}^{\infty}\left[\frac{1}{k}\cdot\nabla R(\mathcal{M},k)\right]=e^{-\mathcal{M}}\cdot\nabla\mathcal{M}. \tag{67}$$

Therefore, the momentum operator in this logic can be expressed as

$$\boldsymbol{p}\rightarrow-\boldsymbol{i}\hbar e^{-\varphi}\nabla\varphi=\boldsymbol{i}\hbar\nabla e^{-\varphi}, \tag{68}$$

where φ is the applied quantity.

Based on the analysis for Equ. 46 and the discussion about Equ. 64 in Section **3.5**, the effect of external potential fields on the velocity of tiny particle swarm can be expressed in quantity as

$$\frac{\bar{V}^2}{2}t^1=\frac{\bar{\mathcal{E}}}{\hbar}, \tag{69}$$

where t^1 = 1 s. Here, we take the situation of electrons in the hydrogen atom as the basic reference (The same conclusion could be obtained with other atoms as reference) to construct the expression. The average electric potential energy of electron in hydrogen atom is

$$\bar{\mathcal{E}}=\frac{e_{\mathrm{g}}^{\ 2}}{4\pi\varepsilon_0\bar{r}}, \tag{70}$$

where $e_{\mathrm{g}}=-1.602\ 177\ 33(49)\times10^{-10}$ C is the electronic charge; $\varepsilon_0=\frac{10^7}{4\pi c^2}\ \mathrm{F\cdot m^{-1}}$ is the permittivity of vacuum. $\bar{r}$ is the average radius of the electron in the hydrogen atom, where Bohr radius is adopted, namely $\bar{r}=\frac{4\pi\varepsilon_0\hbar^2}{m_e e_g^{\ 2}}$. For the same reasons as Equ. 47, t^1 is temporarily ignored. By substituting Equ. 70 into Equ. 69, we can get the quantity expression of the norm of the average velocity $\bar{V}$ for particle swarm in a domain $\mathcal{V}$, namely

$$\left|\bar{V}\right|=\sqrt{\frac{2e_{\mathrm{g}}^{\ 2}}{4\pi\varepsilon_0\bar{r}\hbar}}=\alpha c\sqrt{\frac{2m_e}{\hbar}}, \tag{71}$$

where $\alpha=\frac{e_g^{\ 2}}{4\pi\varepsilon_0\hbar c}$ is the fine structure constant; m_e is the mass of electron; c is the speed of light. This average velocity is a constant in $\mathcal{V}$ that does not change with the number of particles. In the dispersion region of electron, the number of particles forming the matrix of electron is $\frac{m_e}{\mu}$, which is substituted into Equ. 48 together with Equ. 71 and divided by the speed and the number density of tiny particles to obtain the norm of the relative vector of the whole electron or the norm of the total wave-function in the dispersion region of electron, namely

$$\left|\boldsymbol{\mathcal{M}}_{\mathrm{A},e}\right|=\lambda_e\cdot\frac{\sqrt{2}\,\alpha m_e^{\frac{3}{2}}}{\bar{\rho}_{m,0}\sqrt{\hbar}}, \tag{72}$$

where $\lambda_e=1\,\mathrm{m}^{-2}\cdot\mathrm{s}^{-\frac{1}{2}}$ which is to correct the dimensional difference caused by the transformation between different physical quantities. The expression of the equivalent wave-function of the whole electron in the potential fields is determined above (The product of Equ. 72 and the norm of the normalized wave-function is the initial wave-function here). However, the contribution of mass density to the wave-function is not considered here for that it can be ignored comparing with the contribution of electric potential field.

Suppose the Hamiltonian of free electron can be written as

$$H=\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{p})^2}{2m_e}, \tag{73}$$

where $\boldsymbol{\sigma}$ is Pauli operator and $\boldsymbol{p}$ is electron momentum. Under the external magnetic field $\boldsymbol{B}=\nabla\times\boldsymbol{A}$ ($\boldsymbol{A}$ is the electric vector potential), H can be transformed into

$$H = \frac{\left[\boldsymbol{\sigma}\cdot\left(\boldsymbol{p}+e_{\mathrm{g}}\boldsymbol{A}\right)\right]^2}{2m_e}. \tag{74}$$

If $\boldsymbol{A}$ and $\boldsymbol{p}$ both commute with $\boldsymbol{\sigma}$, then

$$H = \frac{\left(\boldsymbol{p}+e_{\mathrm{g}}\boldsymbol{A}\right)^2}{2m_e} + \frac{\boldsymbol{i}}{2m_e}\boldsymbol{\sigma}\cdot\left[\left(\boldsymbol{p}+e_{\mathrm{g}}\boldsymbol{A}\right)\times\left(\boldsymbol{p}+e_{\mathrm{g}}\boldsymbol{A}\right)\right]. \tag{75}$$

After the above treatment, the interaction term between the orbital magnetic moment and the external magnetic field (the first term on the right-hand side of Equ. 75) and the spin term of electron (the second term on the right-hand side of Equ. 75) have been separated successfully. Next, we study the spin term separately.

In view of the fact that the spin magnetic moment of the electron measured in the experiment is the effect of the external magnetic field on the whole atom. Therefore, the whole wave-function of electron is of great significance here. In view of the discussion in Section 3.5, Equ. 72 is the intrinsic relative wave-function of the whole electron that does not change with time. Therefore, it only appears in the natural exponential term of Equ. 68 and is not functioned by Hamilton operator ∇; When the external magnetic field $\boldsymbol{B}$ is weak, its Relativistic effect on the electron momentum can also be ignored. Therefore, only $\mathcal{M}_{\mathrm{A},e}$ alone affects the natural exponential term in Equ. 68. If the distribution of the electron matrix outside the nucleus is regarded as a flat field, the wave-function only shows the increment of real part of complex. Since it is in the position of natural exponential term indicating the statistical effect which is independent on the particle location, the real wave-function can be superposed together as a particle and it can be expressed as real $\mathcal{M}_{\mathrm{A},e}$; Because of the special structure of electron (see the "speculation on electronic structure" in Section **3.7**), its diffusion trend is just opposite to the usual performance of the density gradient, so it must be negative, and the negative value $-\left|\mathcal{M}_{\mathrm{A},e}\right|$ of the norm of $\mathcal{M}_{\mathrm{A},e}$ is the best choice. On these grounds, the second term on the right-hand side of Equ. 75 can be transformed into

$$
\begin{aligned}
\frac{\boldsymbol{i}\, e_{\mathrm{g}}}{2m_e}\boldsymbol{\sigma}\cdot(\boldsymbol{p}\times\boldsymbol{A}+\boldsymbol{A}\times\boldsymbol{p}) &= \frac{\boldsymbol{i}\, e_{\mathrm{g}}}{2m_e}\boldsymbol{\sigma}\cdot(-\boldsymbol{i}\hbar\, e^{|\mathcal{M}_{\mathrm{A},e}|}\nabla\times\boldsymbol{A}), \\
&= \frac{e_{\mathrm{g}}\hbar}{2m_e}e^{|\mathcal{M}_{\mathrm{A},e}|}\boldsymbol{\sigma}\cdot\boldsymbol{B}.
\end{aligned}
\tag{76}
$$

Therefore, the expression of spin magnetic moment of electron derived in this article is

$$
\mu_{\mathrm{g}} = \frac{e_{\mathrm{g}}\hbar}{2m_e}e^{|\mathcal{M}_{\mathrm{A},e}|},
\tag{77}
$$

where $e^{|\mathcal{M}_{\mathrm{A},e}|}$ can be calculated by Equ. 72. If the average mass density $\bar{\rho}_{m,0} = 2\times10^{-28}\,\mathrm{kg\cdot m^{-3}}$ of a broader space is used, then $\mu_{\mathrm{g}} \approx 1.004\,38\mu_{\mathrm{B}}$, where $\mu_{\mathrm{B}} = \frac{e_g\hbar}{2m_e}$ is the Bohr magneto. However, when a larger value is used, such as the average mass density near the earth (When compared with the experimental value on earth, it is obvious that this value should be used, but the specific form of this value cannot be obtained at present, so we can only assume $\bar{\rho}_{m,\mathrm{E}} = 7.537\,648\,754\,4 \times 10^{-28}$ $\mathrm{kg\cdot m^{-3}}$), we can obtain $\mu_{\mathrm{g}} \approx 1.001\,159\,652\,2\,\mu_{\mathrm{B}}$. Therefore, in turn, the average mass density near the earth can be deduced from the spin magnetic moment of the electron located there.

It's amazing that small electrons can form such a huge spin magnetic moment. It is estimated that the spin magnetic moment is caused by total spin of the tiny particles of background in the whole atomic domain, and the motion details need to be further investigated by Equ. 40. There is no more in-depth exploration here.

3.7 Speculation Based on Physical Model

Under the logic of this article, the black hole in the universe should not be a very small singularity, which is considered by the traditional theory. It should have a certain or larger volume but not limited to a relatively larger density. The mass of photons is at a certain level, so they cannot escape from the Event Horizon. Not all substances cannot escape, at least gravitational effects can escape and be perceived (If there is no substance to convey this information, it is obviously impossible to be perceived). Therefore, there may be another world, even a dense life body or a civilized society formed by them, in some black holes. These civilized life bodies on black holes (if they are existence) may look at us as light, loose and meaningless as we look at clouds in the sky on earth. They can even communicate through particles of lower mass-level.

In the light of the model in this article, it is speculated that the outer space can be divided into interstellar space with various degrees of emptiness according to interplanetary, interstellar, intergalactic and galaxy intergroup (clusters), etc. The matter in these spaces becomes thinner and thinner until it reaches a certain degree, and even photons cannot be formed, so that light cannot spread through it. The broader outer space is an untouchable field by the electromagnetic wave detection technology currently mastered by human beings. But anyway, gravitation can travel through it.

If we think that the galaxy groups of lower connection with each other are multiple universes (the whole of these universes can still be regarded as a larger universe), to observe one of them, it will be born from a certain moment, and eventually die. The dead universe, like the dead stars, will eventually evaporate. If multiple universes (multiuniverses) are completely unconnected, there may be parallel universes; however, according to the logic of this article, there should be connections between these universes. In this way, they must be distributed symmetrically due to their mutual influence. Therefore, the parallel universe cannot exist, and only the symmetrical universe could appear.

We do not think there is the situation of Big Bang. The idea that the universe originated from an infinitesimal point is absurd. General Relativity has its own applying conditions and should not be extended unconditionally. For example, the earth or the sun are employed as a mass point to calculate gravitation in Newtonian Mechanics, but they are not infinitesimal points themselves. The sun is constantly radiating and spreading, and its main body will expand in the future, but no one thinks that the sun originated from an infinitesimal point. Therefore, according to the view of multiple universes, our universe should have a life cycle similar to that of stars. The universe that we can observe is constantly spread (which may be the cause of the repulsion between celestial bodies) or expanding, but the whole matter of the larger universe will not change, nor will the total entropy. Under the logic of this article the concept of the abundance ratio of hydrogen and helium in the universe is very easy to understand.

Speculation on photonic structure: It is believed that photons are composed of particles of lower mass-level (k-order) than photon energy agglomeration. In view of the above discussion, the average speed of these lower mass-level particles is a fixed value, and they can form structures with different spin periods ($\mathcal{S}_1$ or $\mathcal{S}_2$). Two kinds of photon structures with different spin periods (frequencies) are illustrated in Fig. 11:

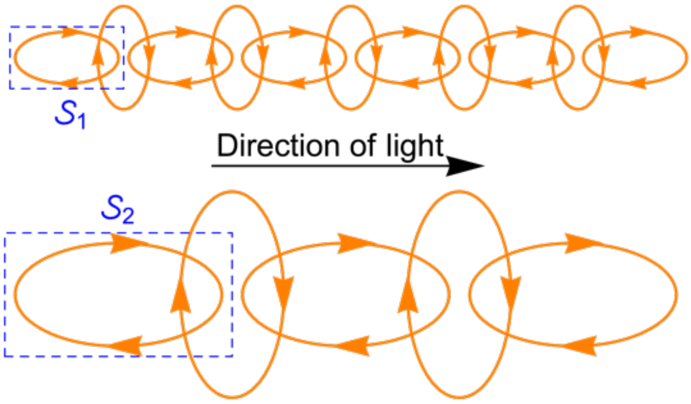


Figure 11 | Schematic diagram of two photon structures with different frequency. Ellipses with arrows in different directions represent rotations perpendicular to each other.

According to the logic of this article, the standard to measure the energy of a photon in one location is the density of *k*-order particles there or the number of *k*-order particles passing through there in unit time. Therefore, from the model structure of Fig. 11, it can be seen that frequency is directly proportional to energy. Since the speed of *k*-order particles is the same, when they form $\mathcal{S}_1$ and $\mathcal{S}_2$ structures which are different sizes, the spin periods of the two structures will be different, so photons of different frequencies will appear. However, in any case, it is obvious that the overall speed of *k*-order particle swarm is certain (it is $\frac{1}{\pi}$ times or in direct proportion to the speed of *k*-order particle). Therefore, although the particles have mass, photons of different frequencies formed by them still move at the same speed. The *k*-order particles do not move in a straight line, and they move faster than the speed of light. However, their group behavior is the speed of light. Therefore, we can think of their groups as larger particles (photons), whose speed is the speed of light. In this way, it will not affect the photon frequency as a measurement tool of the expansion of the universe, nor will it affect the way of taking the speed of light as the representative of the speed of particles at this mass-level. However, from the perspective of General Relativity, photons with different frequencies have different energies and different degrees of space-time distortion. So photons with high frequency or energy will move more slowly due to the influence of their own energy field; According to the logic of this article, we can get the same conclusion. However, due to its own energy is too low, the impact is negligible.

Understanding for the cosmic microwave background radiation: The background radiation is the measurable agglomeration produced by the tiny particles in the gravitational background field. According to the point of view presented in this article, gravitational background is everywhere, but in different interstellar space, its sparsity is different.

This article holds that the concept of "Ether" is necessary to return, but it should have different connotations. If the ether is regarded as "absolute space-time", it can be understood as follows: Ether is a gravitational background field composed of tiny particles moving randomly. The ether in the broadest space is the purest absolute space-time. The ether near the celestial bodies with different affiliations can be approximated to absolute space-time with varying degrees. For example, the ether near the earth can be approximated as absolute space-time; the ether near the sun can be approximated as absolute space-time to another extent. However, the approximation of the ether near the earth is not as high as that near the sun, because it is also affected by the ether near the sun.

To analyze the natural exponential term in the equation (Equ. 40) given in this article, when the momentum density reaches infinity (that is, the velocity density or the mass density or both of the two are infinity), the diffusion rate here is infinitesimal. If the speed v is infinite and the energy density is a certain value at one location, the mass density ρ_m here must be infinitesimal, and it is an infinitesimal of higher order than $\frac{1}{v}$. In this way, the momentum density $\rho_m v$ is infinitesimal, and there is no concept of momentum aggregation, so this situation is meaningless. However, if the mass density is relatively large, even if the mass distribution does not satisfy the condition in Equ. 43, the diffusion will be extremely slow and the distribution will be relatively stable in the perceptible time scale for human. This conclusion, together with the situation described by Equ. 43, explains the formation mechanism of "stable particles" in the universe. With such a formation mechanism, particles of different mass-levels will appear, and they are equivalent. In view of the above mentioned, there is no interaction between particles of different mass-levels and particles of different mass-levels are equivalent. Therefore, it is understandable that the universe has fractal law, and that the universe is a "big organism" also has theoretical basis.

The plane wave-function in Equ. 40 and Equ. 65 given in this article has requirements for direction. The plane wave-function is the simplification of the three-dimensional vector wave-function. For this reason, the uniqueness of the three-dimensional function determines the uniqueness of the plane wave-function described in this article. Therefore, this article does not support the idea that wave-function $\mathcal{M}$ should be equivalent in all directions of the plane. And as mentioned above, the wave-function given in this article also requires a suitable norm of wave-function in the initial condition.

Understanding for the wave effect of solid particles (such as protons, electrons, etc.): For instance, the matrix of electron (background field) is particle waves of low mass-level, which diffuses according to its laws. Electrons are produced by its matrix, and thus follow the dense way of matrix diffusion. The diffusion rate of electron can be thought of as the apparent diffusion rate according to the mass assignment under the diffusion law of the matrix wave-function.

Understanding for the process of electron escaping from atom: Since the electron have a aggregation degree of fixed mass, when it escapes by forming a fixed mass-level particles (at this time, the electron wave-function collapses, here the so-called wave-function collapse refers to the disturbed aggregation of the tiny particles forming the above-mentioned matrix), the left mass (atomic nucleus, etc.) will immediately follow the aggregation law described by Equ. 43 to another extent. For the spin, this is an unstable state. Due to the lack of the electron spin request, the energy released by the nuclear spin can only be spread out, thus showing a certain degree of exhibit electric field.

Speculation on electronic structure: As mentioned above, the electron is not a particle with a mass of $9.109\ 389\ 7(54) \times 10^{-31}$ kg, which is traditionally considered, but consists of its matrix (particles with a lower mass-level than the electron) dispersed in space. Now analyze the structure of the particle forming the matrix of electron (assume it is k_1-order particle). There is a strong interaction between the matrix of electron and the nucleus with positively charge, and the k_1-order particle forming the matrix must also be a negative monopole. A negative monopole with such a small mass can produce such a large force. It must have a spin acceleration structure as described in Section **3.5**. As a single spin accelerating structure, it cannot achieve such a function (omnidirectional attraction), so it is supposed to be a complex of multiple spin structures. A possible structure of spherical 3-dimensional composite is illustrated in Fig. 12 in the form of a plane.

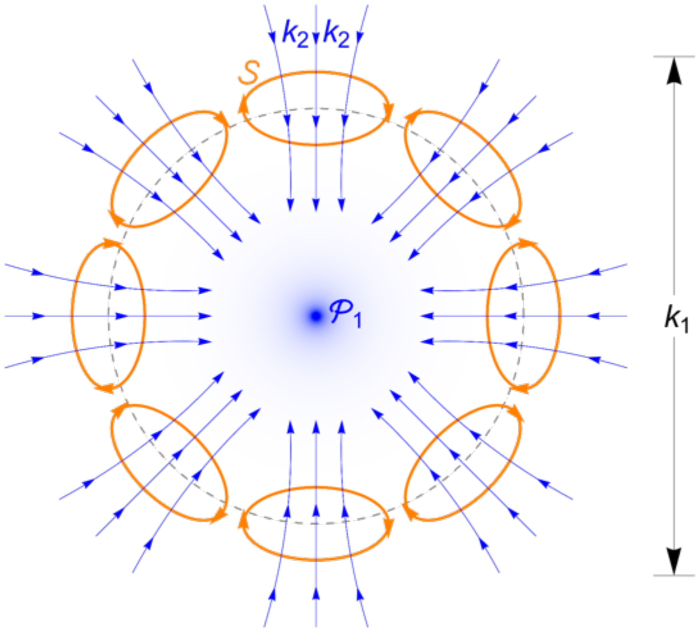


Figure 12 | Speculated schematic of the structure for k_1-order particle forming electron matrix. Here, we use the plane diagram to demonstrate the 3-dimensional spherical structure, where the blue line with arrow represents the track of k_2-order particles and the blue elliptical ring with arrow represents the $\mathcal{S}$-structure.

There is a black hole like structure (the blue diffusion point $\mathcal{P}_1$ in Fig. 12) with a great density of mass in the center of the k_1-order particle (composite structure), of which the outer sphere accumulates at least one layer of spin accelerating structure $\mathcal{S}$, the orange structures as shown in Fig. 12. On the one hand, these $\mathcal{S}$ are drawn by $\mathcal{P}_1$; On the other hand, because of the escape of the smaller particles (k_2-order particles) in the radial direction, these $\mathcal{S}$ will repel each other; The jet formed in the axial direction is just suctioned into $\mathcal{P}_1$. All of the restrictive mechanisms above make k_1-order particle a stable composite structure. Such a composite structure can continuously and rapidly suction k_2-order particles from the outside, thus forming a strong pulling force. Because of the Relativistic effect, the k_2-order particles suctioned by k_1-order paticle from the outside will gather together in the form of highly concentrated (similar to black holes), forming a stable suction structure ($\mathcal{P}_1$ and k_1-order particles) temporarily. But in the long run, the continuous suction of materials from the outside will inevitably lead to the instability of k_1-order particle structure. As a result, such a structure will continue to generate and collapse in a larger time scale, but the total number in a specific region will remain unchanged.

With the electronic matrix model, we can deduce the action principle of four known fundamental forces by the way: The strong interaction force is the statistical effect between a few of energy aggregations very close to each other, which belongs to the form of gravitation. This is a type of energy level force (for example, it is quite difficult to merge or separate two peaks in Fig. 4b). Such a force is strong and e^{-x} -like; The action principle of electromagnetic force is that both sides of the interaction are composed of structures of electronic matrix; The action principle of weak interaction force is that only one side of the interaction is composed of the structure of electronic matrix; The gravitation is that neither side of the interaction is composed of the structure of electronic matrix, or the structure of electronic matrix does not work, which belongs to the statistical effect between particle aggregations with a long distance.

Understanding for the antimatter: As mentioned above, the space is infinitesimal relative to the infinitesimal moving particles, and the particles in the space are as if there were nothing in the space. At this time, we can also think that for a particle, there are infinite spaces, such spaces move in the particles, and the particles are divided into entities with the same number of spaces. If the empty boxes in the space are taken as the research object, similarly, according to the way mentioned above, there will be antiparticle, white hole and negative energy. Our world has gathered matter, so antimatter is relatively small. In this way, the understanding of antimatter is complete and self-consistent.

Understanding for the quantum entanglement: If two particle states are entangled with each other, information must be transmitted between them. In this article, the way that different particles are classified according to their mass can explain this kind of over distance action well. Similarly, there is also a strange phenomenon of "photon delay selection". If we understand it according to the logic of this article, it is not a mysterious thing.

4. Conclusions

In this article, the physical model of the whole universe was constructed from the most basic philosophical paradoxes. Based on this model, a mathematical equation was established to describe the general diffusion behavior of moving particles, of which the form without external field was simply verified. For the first time, the Relativistic effect was interpreted as the statistical effect of moving particles in this article, which held that the higher the aggregation degree of particles (position or moving direction), the more they consumed their average velocity in other directions. Thus, the gravitational force and Special Relativistic effect were actually integrated into the equation (achieved by selecting the initial wave-function with specific norm when solve it), avoiding the problem of non-renormalization when gravitation was introduced into Quantum Mechanics. Further analysis showed that the gravitation between objects was also caused by the statistical effect of randomly moving particles. These particles could also form stable nondispersive particle swarms which as larger mass-level particle could reunite into stable nondispersive particle swarm…No matter which mass-level particles were regard as the infinitesimal particles, and no matter how slow the speed was regard as the infinite speed, the equations derived in this article were equivalent in the scale of human understanding. On the one hand, based on the hypotheses stated in **HYPO 1–3**, this article deduced the form of Schrödinger equation and the conclusion of Special Relativity, which further confirmed the rationality of these hypotheses for the universe. On the other hand, based on these assumptions, the derived equation contained the production conditions of stable particles, which in turn formed a logical self-consistency with the previous assumptions. Therefore, the basic physical model established in this article for the universe is a relatively reliable and complete logic model—The universe is likely to be the product of the movement of non-interacting random particles and obeys the mathematical equation given in Equ. 40.

Based on this physical model, we can answer the questions raised at the beginning of this article. The universe is big and small, and its size is only a relative logical concept. From this relative point of view, the universe is boundless. Now the appearance of the universe is only a stage of its evolution, and the evolution is a process without beginning or end. The non-stop random motion or general diffusion of particle swarm is its running mechanism, and there is no beginning and end point for this diffusion movement (But there is beginning and end in local space). The energy in the universe cannot be designated as there is or not and it is just a relative concept from the movement of infinitesimal particles. If we observe the group behavior of these particles, their average speed will drop down, and then we have the concepts of time, space, speed and energy. Therefore, these concepts (including force) are all statistical effects when observing these moving particles from different angles. Energy will never be exhausted, nor will it increase or decrease. According to this view, the total entropy in the whole universe will not increase or decrease.

However, due to various conditions, the viewpoints in some paragraphs of this article did not give more rigorous derivations and proof processes; The equations were not tested by more rigorous cases; Some conjectures at the end of this article were not based on more rigorous theories. In view of the above problems, more efforts are needed in the future to make it become a more mature theory.

Acknowledgements I thank the engineers in *Wolfram Inc*. for the technical support.

References

1 Harris, P. *Three Hundred Tang Poems*. (Everyman's Library, 2009).

2 Sun, T. *The art of war · Attacking by stratagem*. (Zhonghua Book Company, 2001).

3 Hawking, S. *A Brief History of Time*. (Bantam Dell Publishing Group, 1988).

4 Dirac, P. A. M. *General theory of relativity*. (Princeton University Press, 1975).

5 Kaku, M. *Hyperspace: A scientific odyssey through parallel universes, time warps, and the tenth dimension*. (Oxford University Press, 2016).

6 Kant, I. *Critique of pure reason*. (Cambridge university press, 1999).

7 Darwin, C. *On the Origin of Species*. (Routledge, 1859).

8 Marx, K. H. *Capital*. (Progress Publisher, Moscow, USSR, 1887).

9 Hegel, G. W. F. *The Science of Logic*. (Cambridge University Press, 1929).

10 Einstein, A. & de-Sitter, W. On the relation between the expansion and the mean density of the universe. *Proc. Natl. Acad. Sci.* **18**, 213–214 (1932).

11 Scaramella, R. *et al.* The ESO Slice Project [ESP] galaxy redshift survey⋆ V. Evidence for a D=3 sample dimensionality. *Astronomy and Astrophysics* **334**, 404–408 (1998).

12 Martínez, V. J., Pons-Bordería, M. J., Moyeed, R. A. & Graham, M. J. Searching for the scale of homogeneity. *Mon. Not. R. Astron. Soc.* **298**, 1212–1222 (1998).

13 Maxwell, J. C. *Illustrations of the dynamical theory of gases*. Vol. 20 (The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 1860).

14 Marsaglia, G. Choosing a point from the surface of a sphere. *Ann. Math. Stat.*

43, 645–646 (1972).

15 Muller, M. E. A note on a method for generating points uniformly on n-dimensional spheres. *Comm. Assoc. Comput. Mach.* **2**, 19–20 (1959).

16 Kasper, J. C., Maruca, B. A., Stevens, M. L. & Zaslavsky, A. Sensitive Test for Ion-Cyclotron Resonant Heating in the Solar Wind. *Phys. Rev. Lett.* **110**, 091102-1–091102-5 (2013).