The electron model and Nature’s constants

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Summary

This paper recaps our electron model – including our explanation of the anomaly – and offers some reflections on Nature’s fundamental constants. We will also present a theoretical explanation of the radius of the Zitterbewegung charge – aka the classical electron radius – using an electromagnetic mass calculation. While, in the previous version of the paper, we limited ourselves to a classical (non-mainstream) explanation of Schwinger’s $\alpha/2\pi$ factor, we also offer some reflections on a possible explanation of the higher-order factors in the anomaly of the magnetic moment of an electron.

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Introduction

The idea of a force combines (1) the idea of a charge – a force acts on a charge, right? – and (2) the idea of inertia—resistance to a change of the state of motion. Logically, this leads one to conclude that a charge should have some mass. Why? Because any force on a zero-mass charge would give it infinite momentum. A brief look at the (relativistically correct) force law makes this rather obvious:

\[ \mathbf{F} = m_v \cdot \mathbf{a} = \frac{d(m_v \cdot \mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt} \]

\[ m_v = \gamma \cdot m_0 = \frac{1}{\sqrt{1 - v^2/c^2}} \cdot m_0 \]

The velocity \( \mathbf{v} \) in the *inertial* reference frame (i.e. the reference frame of the object we would be looking at) is equal to zero: the Lorentz factor is, therefore, equal to \( \gamma = 1 \), and \( m_v = m_0 \). Hence, if we have a finite force \( \mathbf{F} \) acting on a zero-mass object, its acceleration \( \mathbf{a} \) has to be infinite so as to yield a finite \( 0 \cdot \infty \) product.

It is, therefore, quite nice that our ring current model of an electron yields a non-zero rest mass for the pointlike charge inside of the electron. Let us quickly recap the basics of it.

The anomaly of the electron’s radius

Most ring current or *Zitterbewegung* models of an electron assume the pointlike \( zbw \) charge is whizzing around the center of the \( zbw \) oscillation *at the speed of light*. We think that assumption is a mathematical idealization. This is why the anomalous magnetic moment is *not* an anomaly: the assumption that the elementary charge has no dimension or structure whatsoever is bound to result in an ‘anomaly’ between our measurements and these ‘good theories’ we have about the structure of electrons, photons and protons.\(^1\)

Let us do some calculations. Because \( \hbar \) and \( c \) have precisely *defined* values since the 2019 revision of SI units, we can calculate the Compton radius from the mass—not approximately, but *exactly*.\(^2\) The CODATA value for the electron mass is equal to:

\[ m_{\text{CODATA}} = 9.1093837015(28) \times 10^{-31} \text{ kg} \]

Based on this, we can calculate a theoretical electron radius based on a ring current model of the electron.\(^3\) Interpreting \( c \) as the tangential velocity of the \( zbw \) charge – and also using the Planck-Einstein

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\(^1\) Mathematical idealizations are just what they are: we need the math and the mathematical ideas that come with it (including the ideas of nothingness and infinity) to describe reality – math was Wittgenstein’s ladder to understanding – but Planck’s quantum of action, and the finite speed of light, effectively tell us our mathematical ideas are what they are: idealized notions we use to describe a reality which is, in the end, quite finite. Something that has no dimension whatsoever probably exists in our mind only. As for the notion of a ‘good theory’, we refer to Dirac’s remarks on gauge and renormalization theory.

\(^2\) Note that the radius is inversely proportional to the mass. The Compton radius of a muon-electron or a proton, for example, is much smaller than the Compton radius of an electron. As for the term ‘good theories’, this is, obviously, a bit of a cynical reference to Dirac’s 1975 comments on renormalization theories: “This so-called ‘good theory’ involves neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it is small – not neglecting it just because it is infinitely great and you do not want it!”

\(^3\) NIST gives CODATA values for the Compton wavelength of an electron. It also gives a measure of the electron’s classical electron radius, which is the Compton radius divided by the fine-structure constant. We will leave it to the reader to verify those values against our calculations and reflect about those results.
and mass-energy equivalence relation – we get the following theoretical value for the ring current radius of an electron:

\[ a = \frac{c}{\omega} = \frac{ch}{E} = \frac{ch}{mc^2} = \frac{\hbar}{mc} = \frac{\lambda_c}{2\pi} \approx 0.38616 \text{ pm} \]

This is the Compton radius, and we interpret it as the effective radius for inelastic (Compton) scattering of photons. The Compton radius is to be distinguished from the radius of the pointlike zbw charge inside, which we will (later) calculate as \( r_e = \alpha r_c = \alpha \hbar / m_e c \).

We can also calculate the Compton radius from the CODATA value for the magnetic moment:\(^4\)

\[ \mu_{\text{CODATA}} = 9.2847647043(28) \times 10^{-24} \text{ J} \cdot \text{T}^{-1} \]

Indeed, the magnetic moment is the product of the current and the area of the loop, and the current is the product of the elementary charge and the frequency. The frequency is, of course, the velocity of the charge divided by the circumference of the loop. Because we assume the velocity of our charge is equal to \( c \), we get the following radius value:

\[ \mu = I \pi a^2 = q_e f \pi a^2 = q_e \frac{c}{2\pi a} \pi a^2 = q_e c \frac{a}{2} \iff a = \frac{2\mu}{q_e c} \approx 0.38666 \text{ pm} \]

We should note that we get a value that is slightly different from the theoretical \( a = c / \omega = \hbar / mc \) radius: we have an anomaly. We can confirm this anomaly by re-doing this calculation using the Planck-Einstein relation to calculate the frequency:

\[ \mu = I \pi a^2 = q_e f \pi a^2 = q_e \omega \frac{a^2}{2} \iff \mu = \frac{2\mu}{q_e \omega} = \frac{2\mu \hbar}{q_e E} = \frac{2\mu \hbar}{q_e mc^2} \approx 0.38638 \text{ pm} \]

We again get a slightly different value. These approximate 0.38666 and 0.38638 pm values we get out of our radius calculation using the CODATA value for the magnetic moment are slightly larger than the theoretical \( a = \hbar / mc \) value we get based on the mass or the Compton wavelength, which is 0.38616 fm—more or less.\(^5\) So, yes, we do have an anomaly.

Hence, we will want to think of the radius based on the mass or the Compton wavelength as some kind of theoretical radius and so we will put it in the denominator. We can write it like we want, with or without some subscript: \( a = a_{\text{CODATA}} = a_m = a_\lambda = a_C \). In contrast, we will write the radius based on our calculation using the magnetic moment as \( a_\mu \). We can then write the anomaly as:\(^6\)

\[ \frac{a_\mu - a}{a} \approx 0.00115965 \iff \frac{a_\mu}{a} = 1.00115965 \ldots \]

You will immediately recognize the anomaly. It is, effectively, equal to about 99.85% of Schwinger’s factor: \( \alpha / 2\pi = 0.00116141 \ldots \)

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4 We should put a minus sign as per the convention but, because we are interested in magnitudes here, we will omit it. It will, hopefully, confuse the reader less, rather than more.

5 We encourage the reader to re-do the calculations so as to arrive at more precise results.

6 We used the first of the two radii one can calculate from the magnetic moment. The reader can re-do the calculations using the second of the two anomalous radii.
The anomaly of the electron’s magnetic moment

Let us, for good order, also recalculate the anomaly of the magnetic moment. We will follow a slightly different presentation than the usual one but you will see the logic is not very different. We first calculate a new theoretical value for the magnetic moment using the Compton radius, which we will denote as $\mu_a$. When writing it all out, we get this:

$$\mu_a = \pi a^2 = q_e \frac{c}{2 \pi a} \pi a^2 = q_e \frac{c^2}{2} a = \frac{q_e c}{2m} \hbar \approx 9.27401 ... \times 10^{-24} \text{ J} \cdot \text{T}^{-1}$$

We can now calculate the anomaly — against the CODATA value — once more$^7$:

$$\frac{\mu_a - \mu}{\mu} = 0.00115965 ...$$

We get the same anomaly—not approximately but exactly. That is what we would expect: in the zbw or ring current model, the anomaly is not only related to the actual magnetic moment but to the actual radius as well. This should not surprise us: the magnetic moment is, of course, proportional to the radius of the loop.$^8$ Hence, if the actual magnetic moment differs from the theoretical one, then the actual radius must also differ from the theoretical one.

At this point, the reader may wonder how we get a theoretical value for the magnetic moment. We get it from the same ring current model. We can just equate the two formulas we presented for the magnetic moment:

$$a = \sqrt{\frac{2\mu \hbar}{q_e mc^2}} \implies \sqrt{\frac{2\mu \hbar q_e^2 c^2}{4\mu^2 q_e mc^2}} = \sqrt{\frac{\hbar q_e}{2\mu m}} = 1 \implies \mu = \frac{q_e}{2m} \hbar$$

The mass of the zbw charge

Our assumption is that the anomaly is not an anomaly at all. We get it because of our mathematical idealizations. We think the assumption that the electron is just a pointlike or dimensionless charge is non-sensical: when thinking of what might be going on at the smallest scale of Nature, we should abandon these mathematical idealizations: an object that has no physical dimension whatsoever does — quite simply — not exist.

Likewise, we should not assume that the pointlike zbw charge is whizzing around at exactly the speed of light. It can be very near $c$, but not quite equal to $c$. Hence, its theoretical rest mass will also be very close to zero, but not exactly zero. As a result, we will have some real radius $r$ that is probably not quite equal to the Compton radius $a = \hbar/mc$ as well.

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$^7$ You should watch out with the minus signs here – and you may want to think why you put what in the denominator – but it all works out!

$^8$ We have a squared radius in the numerator of the formula for the magnetic moment, and a non-squared radius factor in the denominator.
Let us write it all out. What should we put where? The greater value – based on the greater radius – should be in the denominator, so we write:

\[
\frac{\mu_r}{\mu_a} = \frac{\frac{q_e}{2m} \frac{\hbar}{2\pi}}{v \cdot r} = \frac{\hbar}{m \cdot v \cdot r} = \frac{c \cdot a}{v \cdot r}
\]

Note that, from the \( v = r \cdot \omega \) and \( c = a \cdot \omega \) relations\(^9\), we then get the following result:

\[
\frac{\mu_r}{\mu_a} = \frac{c \cdot a}{v \cdot r} = \frac{\omega \cdot a^2}{\omega \cdot r^2} = \frac{a^2}{r^2} \Leftrightarrow \frac{c}{v} = \frac{a}{r}
\]

This helps us to with interpretation of our results: because \( v \) must be smaller than \( c \), the identity shows the real radius \( r \) must also be slightly smaller than \( a = \hbar/mc \). If there would be no anomaly – in other words: if our mathematical idealization would match reality – then the formulas just becomes unity (everything is equal to 1). However, we know the anomaly exists, and it is very nearly equal to \( 1 + \alpha/2\pi \).

For all practical purposes – we think a 99.85% explanation is pretty good – we will just equate it and re-write the expression above as:

\[
1 + \frac{\alpha}{2\pi} = \frac{2\pi + \alpha}{2\pi} \Leftrightarrow v \cdot r = \frac{2\pi \cdot c \cdot a}{2\pi + \alpha} = \frac{2\pi \cdot c \cdot \frac{\hbar}{mc}}{2\pi + \alpha} = \frac{\hbar}{m(2\pi + \alpha)}
\]

\[
\Leftrightarrow L = m \cdot v \cdot r = \frac{\hbar}{2\pi + \alpha}
\]

So now we need to answer the question: what is the real velocity \( v \) and what is the real radius \( r \) of our zbw charge? We will come to that. We first ask the reader to note something quite essential here:

**Mainstream quantum mechanics assumes angular momentum must come in units of \( \hbar \), and mainstream physicists think that is a direct implication of – or even an equivalent to – the Planck-Einstein law: \( E = h \cdot f = \hbar \cdot \omega \). The calculation above brings some nuance to this statement: angular momentum does not come in exact units of \( \hbar \). There is an anomaly, and we think the anomaly is part and parcel of Nature.**

In contrast, we believe the Planck-Einstein relation to be true—not approximately but exactly. Hence, we believe that the frequency \( f \) or \( \omega \) of the Zitterbewegung oscillation is, effectively equal to \( f = E/\hbar \) or \( \omega = E/\hbar \), precisely. If we believe that to be true, then the following relations explain the anomaly\(^10\):

\[^{9}\text{The reader may wonder why we take } \omega \text{ to be a constant. The answer is: we take the energy (or mass) of an electron as a given, and we take the Planck-Einstein relation (} \omega = E/\hbar \text{) as a given too! The geometry of the situation gives us everything here!}\]

\[^{10}\text{We are just using the tangential velocity formula here to do the substitution that is being done: } c = a \cdot \omega \text{ and } v = r \cdot \omega \text{ and – yes – we assume stable particles respect the Planck-Einstein relation, which we believe to be true—as opposed to the quantum-mechanical theorem in regard to angular momentum which, as mentioned, we believe to be very nearly true.}\]
\[ \frac{\mu_r}{\mu_o} = 1 + \frac{\alpha}{2\pi} = \frac{c \cdot a}{v \cdot r} = \frac{\omega \cdot a^2}{\omega \cdot r^2} = \frac{a^2}{r^2} \Rightarrow r = \frac{a}{\sqrt{1 + \frac{\alpha}{2\pi}}} \approx 0.99942 \cdot \frac{h}{mc} \]

We can plug this into the \( \beta = \frac{v}{c} = \frac{r}{a} \) relation to get the relative velocity:

\[ \beta = \frac{v}{c} = \frac{r}{a} = \frac{a}{a \cdot \sqrt{1 + \frac{\alpha}{2\pi}}} = \frac{1}{\sqrt{1 + \frac{\alpha}{2\pi}}} \approx 0.99942 \]

We can now calculate the real rest mass of the pointlike \( zbw \) charge:

\[ m_0 = \sqrt{1 - \beta^2} \cdot m_{\gamma} = \sqrt{1 - \beta^2} \cdot \frac{m_e}{2} = \sqrt{1 - \frac{1}{1 + \frac{\alpha}{2\pi}}} \cdot \frac{m_e}{2} = \frac{\alpha}{\sqrt{2\pi + \alpha}} \cdot \frac{m_e}{2} \approx 0.017 \cdot m_e \approx 0.034 \cdot m_{\gamma} \]

Hence, we arrive at the conclusion that the rest mass of the pointlike \( Zitterbewegung \) charge is equal to about 1.7% of the rest mass of the electron \( (m_e) \), or 3.4% of its relativistic mass \( (m_{\gamma}) \). Is this a credible result? We think so, but we will let the reader re-do the calculations.

**The fundamental units of physics**

These results are all wonderful—too wonderful for most, I guess.\(^{11}\) We may relate them to a more philosophical question: what is *fundamental* in Nature? In other words, what are *first principles*, and what can be *derived* from them?

Planck’s constant \( (h) \) models a fundamental *cycle* in Nature, and we consider the absolute speed of light \( (c) \) to be another fundamental *fact*. From these two, we get the idea of a force. Indeed, the physical dimension of Planck’s constant is a force over some distance during some time \( (F \cdot \Delta s \cdot \Delta t) \). Hence, combining \( h \) and \( c \), we could define a natural unit for the force, based on whatever natural unit we would want to choose for distance and time—say, the second for time and the light-second for distance, although smaller units would be much more convenient at the sub-atomic scale.\(^{12}\)

As mentioned, the idea of a force combines two ideas: it acts on a charge, but the charge must have some *inertia* to a change in its state of motion. Otherwise, we get nonsense as we, hopefully, managed to demonstrate in our introduction to this paper.

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\(^{11}\) We sent this to two very distinguished physicists whose name we won’t reveal out of respect. We will just mention that both have done highly relevant work in the area of measuring the (electric) charge radius of the proton. To their credit, both bothered to react. One reacted by saying he finds this ‘interesting’. The other reacted in the same manner but added some of the calculations look like ‘numerology’. We, obviously, do not take the last comment very seriously. We believe our calculations are very *real*: no quantum-mechanical hocus-pocus here!

\(^{12}\) The only requirement for a natural distance and time unit is that the speed of light as expressed in these units should equal unity: \( c = 1 \). Hence, our choice for such units will involve some idea of *scale*. In mathematical terms, these units would all be equivalent because they differ by a proportionality constant only. There is a natural constant relating various scales: the fine-structure constant. We will come back to this.
The first idea – a force acting on a charge – may be used to define a natural unit for the charge which, in this case, is the electric charge.\(^\text{13}\)

The second idea would define a natural mass unit. For the first, it is quite obvious that the charge of an electron – which is nothing but the \(\text{zbw}\) charge inside – would be the right choice. For the second, we may briefly wonder whether we shouldn’t consider the mass of the \(\text{zbw}\) charge, but the sensible answer here is obvious: we can only measure the mass of the electron and, hence, the mass of the electron should probably be our natural mass unit as well.

Apart from \(\hbar\) and \(c\), we also have the fine-structure constant \(\alpha\). How can we define it? The answer is: we cannot. We can only \textit{measure} it from the anomalies and from the finer structure of the hydrogen spectrum. Both are related, even if these interrelations are, obviously, \textit{not} self-evident.

Of course, while we can (probably) not \textit{define} its numerical value, we may try to explain what it \textit{is}. We have done so using our ring current model – not only for the electron itself but also for electron orbitals. These analyses lead us to characterize the fine-structure constant as a scaling constant but, as evident from our previous papers\(^\text{14}\), it scales various physical dimensions—not only the dimensions in space!

\textbf{What about the Uncertainty Principle?}

Is there any room for the Uncertainty Principle in our analysis? There is. We like to think of Planck’s constant as a vector. Indeed, the force in the \(F \cdot \Delta s \cdot \Delta t\) must have some direction. This direction may wander around. This is equivalent to saying that the plane of the ring current evolves in time which, of course, also means that the direction of the magnetic moment is changing all of the time. When a magnetic field is being applied, the electron snaps into place, so to speak.\(^\text{15}\)

However, we have no idea of how \textit{exactly} the angle of the plane of the \textit{Zitterbewegung} or rotary motion of the charge could change: we cannot think of some obvious \textit{clue} here. If we could, we would not hesitate to further develop this paper. However, it is, for us, not a priority to develop some answer to this question. Why not? Because it doesn’t matter: we do not need to \textit{explain} anything here. About half of the electrons that are entering a Stern-Gerlach apparatus will have their spin \textit{up}, more or less, and the other half will have it \textit{down}. The magnetic field then, somehow, \textit{snaps} them into place. Of course, the reader will object to such reasoning: there should be some inertia here too, isn’t it?

We cannot say much to that, except the obvious: apparently, there is \textit{no} inertia here. Why? We don’t know. All we can say is that it is a direct consequence of the Planck-Einstein relation. The question is related to the next: what keeps the current going, and what keeps the charge in its orbit?

\(^{13}\) We wrote about the idea of a strong charge in our previous papers as part of our calculations of the electromagnetic radius of a proton \((4\hbar/mc \approx 0.841 \text{ fm})\). Indeed, something must explain the extraordinarily small radius and, likewise, the extraordinarily large mass of a proton (the radius is inversely proportional to the mass in the ring current model). We may, therefore, want to think of a fundamental oscillation of some other charge – a \textit{strong} charge – to explain the extra mass. This idea would lead to a distinction between the idea of an electromagnetic mass and the idea of a strong mass. However, we are very reluctant to engage in such theory because we would like to think of other theoretical models here. We may, for example, want to think that the electromagnetic oscillation might have different \textit{modes} or higher harmonics. We are inspired here by the fact that the ring current model is easily applicable to the heavier variant of the electron—the \textit{muon}.

\(^{14}\) See our paper on the meaning of the fine-structure constant (https://vixra.org/abs/1812.0273).

\(^{15}\) We obviously also think of the Larmor precession as an actual \textit{or real} precessional motion of the \textit{zbw} charge.
What keeps the current going?

It is an obvious question: what keeps the ring current going? It is related to the other obvious questions: what keeps the zbw charge in its orbit, and why does the energy not radiate away?

Here also, we can only provide exploratory or speculative answers. Most current ring or Zitterbewegung theorists – think of David Hestenes and others – think the ring current generates the magnetic field that keeps it going. As such, they compare it to a superconducting ring of current.

We like this comparison and then we do not. We like it because a superconducting ring of current also keeps going without radiating any energy away. However, we also note superconduction is being explained in a very different way in mainstream mechanics: the explanation involves Bose condensation and (Cooper) pairs of electrons. We are quite mystified by that. At the same time, we did seem to be able to offer common-sense explanations for quite a few quantum-mechanical mysteries now (the physical meaning of the wavefunction, the wavelike behavior and interference of electrons and photons, the anomalous magnetic moment, the proton radius, etcetera). Hence, we may be able to explain superconductivity in some easier way one day too!

However, we have a second objection: it would seem a superconducting ring can have any radius. In contrast, the electron has only one specific Compton radius, and there is nothing that keeps the charge in its orbit. I think of that puzzle as a real ‘fine-tuning problem’. So far, we can only make sense of it by assuming our two-dimensional oscillator model\(^\text{16}\) is, somehow, more fundamental than what I’ll refer to as Hestenes’ ‘superconduction’ model. We get our ‘perpetuum mobile’ – so to speak – directly from (i) accepting Einstein’s mass-energy equivalence relation \(E = m \cdot c^2\) for what it is, (ii) interpreting \(c\) as the tangential velocity of the zbw charge\(^\text{17}\) \((c = a \cdot \omega)\), and (iii) the Planck-Einstein relation \(E = \hbar \cdot \omega\):

\[
a = \frac{c}{\omega} = \frac{c \hbar}{m c^2} = \frac{\hbar}{m c} = \frac{\lambda c}{2\pi} \approx 0.386159268 \text{ pm}
\]

We admit it is still mysterious, but it is the best we’ve got. All the rest – most of the Standard Model, that is\(^\text{18}\) – looks even more mysterious to us. It looks like a remake of the intellectual battle between Ptolemaic and Copernican models: both yield results, but one is significantly simpler than the other. History will decide which model wins. Until that day, we should just try to heed Wittgenstein’s advice:

“\textit{Wovon man nicht sprechen kann, darüber muß man schweigen.}”

An explanation for the classical electron radius

We think of the classical electron radius as the radius of the zbw charge inside of the electron. The CODATA value of the classical electron radius is this:

\[
r_{\text{CODATA}} = 2.8179403262(13) \times 10^{-15} \text{ m}
\]


\(^\text{17}\) We also referred to the zbw charge as a naked charge: it has no properties except its charge. It has, therefore, zero rest mass and that is why it moves around at lightspeed: the slightest force on it will cause an infinite acceleration.

\(^\text{18}\) We think of the Higgs field here, for example. In our model, a charge comes with a (tiny) mass. No need for hocus-pocus!
This value corresponds, more or less\(^\text{19}\), to the theoretical \(r_e = \alpha r_C = \frac{\alpha \hbar}{m_e c}\) value when applying the \(\alpha = \frac{q_e^2}{4\pi \varepsilon_0 \hbar c}\) CODATA definition\(^\text{20}\):

\[
r_e = \alpha \frac{\hbar}{mc} = \frac{q_e^2}{4\pi \varepsilon_0 \hbar c} \cdot \frac{c \hbar}{mc^2} = \frac{q_e^2}{4\pi \varepsilon_0 mc^2} = 2.81794032666895 \ldots \text{fm}
\]

Do we think this might be the real radius of the \(\text{zbw}\) charge at the core of the electron? We do. Richard Feynman gets the following interesting formula when calculating the electromagnetic mass or energy of a sphere of charge with radius \(\alpha = \frac{q_e^2}{4\pi \varepsilon_0 \hbar c}\):

\[
U = \frac{1}{2} \frac{e^2}{a} = \frac{1}{2} \frac{q_e^2}{4\pi \varepsilon_0} \frac{1}{r_e} = \frac{1}{2} \frac{q_e^2}{4\pi \varepsilon_0} \frac{mc}{\alpha \hbar} = \frac{1}{2} \frac{q_e^2}{4\pi \varepsilon_0} \frac{4\pi \varepsilon_0 \hbar mc^2}{q_e^2 \hbar} = \frac{1}{2} mc^2
\]

In fact, Feynman does not write it like this, but we inserted and used the \(\alpha = \frac{q_e^2}{4\pi \varepsilon_0 \hbar c}\) and \(a = r_e = \frac{\alpha \hbar}{mc}\) identities above. The point is: we get only half of the (rest) energy or (rest) mass of the electron out of this assembly. Feynman was puzzled by that \(\frac{1}{2}\) factor: where is the other half? He should not have been puzzled by it: he is assembling the \(\text{zbw}\) charge here—\textit{not} the electron as a whole. Hence, the missing mass is in the \textit{Zitterbewegung} or orbital/circular motion of the \(\text{zbw}\) charge.

We can now \textit{derive} the classical electron radius from the formula above:

\[
U = \frac{1}{2} \frac{e^2}{2 r_e} = \frac{1}{2} \frac{q_e^2}{4\pi \varepsilon_0} \frac{1}{r_e} \iff r_e = \frac{1}{2} \frac{q_e^2}{4\pi \varepsilon_0} U = \alpha \frac{\hbar c}{2 m_e c^2} = \alpha \frac{\hbar}{m_e c} = \alpha r_C
\]

This is a nice result. Mystery solved?

Maybe. Maybe not. We did gloss over some rather important details here. Feynman was assembling a thin spherical shell of charge here—as opposed to a uniformly charged \textit{sphere} of charge, in which case the coefficient becomes \(3/5\) instead of \(1/2\).\(^\text{21}\) So is our \(\text{zbw}\) charge a thin spherical shell of charge or a uniformly charged \textit{sphere} of charge? Our honest answer is: we don’t know. The formulas suggest the former—and that makes sense, instinctively: negative charges repel each other, so they are always on the outside of a conductor.

However, perhaps we should not push our classical ideas too far here. There are a few other—more important—things that don’t make sense here.

First, one should note that Feynman did not include the energy we associated with the spin of the \(\text{zbw}\) charge in this energy calculation. He only calculated \textit{potential} energy when assembling the elementary charge by bringing infinitesimally small charges together. This undermines the logic of the derivation above.

\(^\text{19}\) The reader should note the final digits of the two values are different.

\(^\text{20}\) The use of point estimates yields a slightly different value but it is well within the standard error. Hence, we consider the results to be equivalent. NIST confirms our intuition here: the relative uncertainty on the Compton wavelength and the classical electron radius is of the same order: 3 and \(4.5 \times 10^{-10}\) respectively. A 50% or 1/2 factor—one more! We suspect it’s the \(\frac{1}{2}\) factor in the effective mass.

\(^\text{21}\) See: \url{https://www.feynmanlectures.caltech.edu/II_28.html}. The basic idea is to ‘assemble’ the elementary charge by bringing infinitesimally small charge \textit{fractions} together.

\(^\text{22}\) See: \url{https://www.feynmanlectures.caltech.edu/II_08.html}. 
More importantly, the $m_e/2$ mass of our $zbw$ charge is relativistic mass in our model: the pointlike $zbw$ charge only acquires its mass $m_e/2$ mass because it is zittering around at (almost) the speed of light. In other words, the ring current model tells us most of the energy is kinetic. To be precise, if our calculations are correct, then about 96.6% of the mass (or energy) of the $zbw$ charge is kinetic.

So what can we say? Not all that much, for the time being. We don’t think we managed to fully solve all of the quantum-mechanical mysteries. However, we do think that we have a perfectly consistent realist interpretation of quantum mechanics here. To be precise, we think we have a theory here which explains all of the mysterious intrinsic properties of an electron (its mass, its radius for elastic as well as inelastic scattering, and its magnetic moment) using common-sense physics.

We, therefore, hope that we have managed to convince the reader that the assumption that the electron is just a dimensionless charge is non-sensical. When thinking of what might be going on at the smallest scale of Nature, we should abandon our mathematical idealizations: an object that has no physical dimension whatsoever does – quite simply – not exist. Pointlike and zero-dimension are not the same: the pointlike $zbw$ charge has some (tiny) dimension.

### The higher-order factors in the explanation of the anomaly

We know that Schwinger’s $\alpha/2\pi$ factor explains most but not all of the anomalous magnetic moment. Schwinger’s factor is a very good first-order factor: it explains about 99.85% explanation of the measured anomaly, but so that’s not all of it. We write:

$$\frac{\mu_a - \mu}{\mu} = \frac{a_\mu - a}{a} = \frac{\alpha}{2\pi} + \ldots$$

So how can we explain the $n^{th}$-order factors ($n > 1$) that follow? We have not any detailed calculations here, but we think we have an logical explanation. As mentioned earlier, the $\mu = 1\pi r^2 = q_e f \pi r^2 = q_e \frac{v}{2\pi} \pi r^2 = \frac{1}{2} q_e r v$ tells us that the moment is proportional to the radius of the loop, and the factor of proportionality is $q_e v/2$. Hence, electric charge that is closer to the theoretical $a = h/mc$ radius will make a proportionally larger contribution to the magnetic moment. Hence, Feynman’s conceptualization of the elementary charge – which is the *Zitterbewegung* charge in our model – as an assembly of infinitesimally small charges is useful here, once again.

Let us illustrate this point by thinking about the *physicality* of what we are modeling here. We can rewrite the equation above as follows$^{23}$:

$$\frac{a_\mu - a}{a} = \frac{\alpha}{2\pi} + \ldots \iff a_\mu - a = (\alpha + \ldots) \cdot \frac{a}{2\pi}$$

This is a very interesting equation. A priori, one might have expected that the difference between the $a = h/mc$ Compton radius and the actual radius $r$ would be of the order of $\alpha a$. Why? Because $\alpha a$ is the classical electron radius, which explains elastic scattering. We, therefore, think it is, in effect, the actual radius of the $zbw$ charge inside of the electron. But we have a $1/2\pi$ factor here, and it is rather obvious that we cannot explain it away. This $1/2\pi$ factor is equal to about 0.16.

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$^{23}$ We re-write the $n^{th}$-order factors ($n > 1$) here: we simply multiply them by $2\pi$ as we bring the $1/2\pi$ factor out of the brackets.
It makes us think of the concept of the *effective center of charge*, which we used in an earlier attempt to provide a classical explanation for the anomalous magnetic moment. As we accepted the idea of an effective radius of the \( zbw \) charge, we think the concept of an effective center of charge still makes sense. However, it obviously needs further tuning.

We will be honest and admit we had hoped there would be some recursive logic in our electron model—and it is there! We calculated a so-called *real radius of the Zitterbewegung* based on the *definition* of the fine-structure constant, but that calculation is based on the idea of the \( zbw \) charge being pointlike. But then we say that the \( zbw \) charge is *not* pointlike — or not dimension-less. We say it has a radius itself: the classical electron radius—rather obvious, but we do need to make the point here.

It is, therefore, rather obvious — to us: we hope the reader will see the point too — that some small corrections to the calculations will need to be made. We think these small corrections must, somehow, explain the \( n^{\text{th}} \)-order factors \((n > 1)\) in the mainstream explanation of the anomalous magnetic moment.

Jean Louis Van Belle, 21 February 2020

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24 We refer to our very first *Classical Calculations of the Anomalous Magnetic Moment* ([https://vixra.org/abs/1906.0007](https://vixra.org/abs/1906.0007)), which we now think of as being useful but too simple. We think of it as being too simple because we were wedded to the idea of the \( zbw \) charge moving at lightspeed. This model makes much more sense, but it implies we have an actual radius that is actually *larger* than the theoretical \( a = \frac{\hbar}{mc} \) radius—rather than smaller, as we assumed in the mentioned paper.