Beal Conjecture & Equivalent Beal Conjecture Proved

"5% of the people think; 10% of the people think that they think; and the other 85%would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

The author proves both the original Beal conjecture and the equivalent Beal conjecture. The original Beal conjecture states that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor. The equivalent Beal conjecture states that if A, B, C, x, y, z are positive integers and A, B, and C are coprime (have no common prime factors), and $x, y, z > 2$, then the equation $A^x + B^y = C^z$ has no solutions. The principles applied in both proofs are based on the same properties of the factored Beal equation. However the proof of the equivalent conjecture is by contradiction. The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation.

Options

Option 1
Beal Conjecture Proved
(Original conjecture)

Preliminaries
Introduction

The following is from the first page of the author's high school practical physics note book: Science is the systematic observation of what happens in nature and the building up of body of laws and theories to describe the natural world. Scientific knowledge is being extended and applied to everyday life. The basis of this growing knowledge is experimental work. To prove Beal conjecture, one will be guided by the observational properties of the factored Beal equation.

Observation 1: $\boxed{2^3 + 2^3 = 2^4}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$2^3 + 2^3 = 2^4$$

$$2^3 + 2^3 = 2^3 \bullet 2$$

$$\underbrace{2^3}_{k}(\underbrace{1+1}_{L}) = \underbrace{2^3}_{M} \bullet \underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side..

Note above that the greatest common power of the prime factors on the left of the equation is the same as a power of the prime factor on the right side of the equation.

Note also, the following

The ratio $\dfrac{K}{M} = \dfrac{2^3}{2^3} = 1$.

If $\dfrac{K}{M} = 1$, then $K = M$

Similarly, $\dfrac{P}{L} = \dfrac{2}{1+1} = 1$.

If $\dfrac{P}{L} = 1$, then $P = L$

Corresponding relationship formula

Let r , s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^x = F^z t^z$$

$$r = s = t = 2$$

$$x = 3.y = 3, z = 4$$

$$(D = 1, E = 1, F = 1)$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$

$$K = M, \; P = L$$

Observation 2: $\boxed{7^6 + 7^7 = 98^3}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$7^6 + 7^7 = 98^3$$
$$7^6 + 7^6 \bullet 7 = (49 \bullet 2)^3$$
$$7^6 + 7^6 \bullet 7 = 7^6 \bullet 2^3$$
$$7^6(1+7) = 7^6 \bullet 2^3$$
$$\underbrace{7^6}_{K}(\underbrace{1+7}_{L}) = \underbrace{7^6}_{M} \bullet \underbrace{2^3}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{7^6}{7^6} = 1$.

Similarly, $\frac{P}{L} = \frac{2^3}{1+7} = 1$.

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^x = F^z t^z$$
$$r = s = t = 7$$
$$x = 6, y = 7, z = 3$$
$$(D = 1, E = 1, F = 14)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M,\ P = L$$

Observation 3: $\boxed{3^3 + 6^3 = 3^5}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$3^3 + 6^3 = 3^5$$
$$3^3 + (3 \bullet 2)^3 = 3^5$$
$$3^3 + 3^3 \bullet 2^3 = 3^5$$
$$3^3(1 + 2^3) = 3^3 \bullet 3^2$$
$$\underbrace{3^3}_{K}(\underbrace{1+8}_{L}) = \underbrace{3^3}_{M} \bullet \underbrace{3^2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{3^3}{3^3} = 1$.

Similarly, $\frac{P}{L} = \frac{3^2}{1+8} = 1$.

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^x = F^z t^z$$
$$r = s = t = 3$$
$$x = 3, y = 3, z = 5$$
$$(D = 1, E = 2, F = 1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M,\ P = L$$

Observation 4: $\boxed{2^9 + 8^3 = 4^5}$

Identify the greatest common factor.of all three terms of the equation and factor it out on the left side.

$$2^9 + 8^3 = 4^5$$
$$2^9 + ([2^3])^3 = ([2^2])^5$$
$$2^9 + 2^9 = 2^{10}$$
$$2^9(1+1) = 2^9 \bullet 2$$
$$\underbrace{2^9}_{k}(\underbrace{1+1}_{L}) = \underbrace{2^9}_{M} \bullet \underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{2^9}{2^9} = 1.$

Similarly, $\frac{P}{L} = \frac{2}{1+1} = 1.$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^x = F^z t^z$$
$$r = s = t = 2$$
$$x = 9, y = 3, z = 5$$
$$(D = 1, E = 4, F = 2)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \ P = \ L$$

Observation 5: $\boxed{34^5 + 51^4 = 85^4}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$34^5 + 51^4 = 85^4$$
$$(17 \bullet 2)^5 + (17 \bullet 3)^4 = (17 \bullet 5)^4$$
$$17^5 \bullet 2^5 + 17^4 \bullet 3^4 = 17^4 \bullet 5^4$$
$$17^4(17 \bullet 2^5 + 3^4) = 17^4 \bullet 5^4$$
$$\underbrace{17^4}_{k}(\underbrace{17 \bullet 2^5 + 3^4}_{L}) = \underbrace{17^4}_{M} \bullet \underbrace{5^4}_{P}$$

(Note: $17 \bullet 2^5 + 3^4 = 17 \bullet 32 + 81 = 625;$ $5^4 = 625$)

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{17^4}{17^4} = 1.$

Similarly, $\frac{P}{L} = \frac{5^4}{17 \bullet 2^5 + 3^4} = 1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^x = F^z t^z$$
$$r = s = t = 17$$
$$x = 5, y = 4, z = 4$$
$$(D = 2, E = 3, F = 5)$$
$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$$
$$K = M, \ P = \ L$$

Note above that one factored out s^y .

One will apply the switch from r^x to s^y in the conjecture proof.

Observation 6: $\boxed{3^9 + 54^3 = 3^{11}}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$3^9 + 54^3 = 3^{11}$$
$$3^9 + (3^3 \bullet 2)^3 = 3^{11}$$
$$3^9 + 3^9 \bullet 2^3 = 3^{11}$$
$$3^9(1 + 2^3) = 3^9 \bullet 3^2$$
$$\underbrace{3^9}_{k}(\underbrace{1 + 2^3}_{L}) = \underbrace{3^9}_{M} \bullet \underbrace{3^2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side

Note the following

The ratio $\frac{K}{M} = \frac{3^9}{3^9} = 1$.

Similarly, $\frac{P}{L} = \frac{3^2}{1 + 2^3} = 1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^x = F^z t^z$$
$$r = s = t = 3$$
$$x = 9, y = 3, z = 11$$
$$(D = 1, E = 18, F = 1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \ P = L$$

Observation 7: $\boxed{33^5 + 66^5 = 33^6}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$33^5 + 66^5 = 33^6$$
$$(11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5 = (11 \bullet 3)^6$$
$$11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5 = 11^6 \bullet 3^6$$
$$11^5(3^5 + 2^5 \bullet 3^5) = 11^5 \bullet 11 \bullet 3^6$$
$$\underbrace{11^5}_{k}(\underbrace{3^5 + 2^5 \bullet 3^5}_{L}) = \underbrace{11^5}_{M} \bullet \underbrace{11 \bullet 3^6}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{11^5}{11^5} = 1$.

Similarly, $\frac{P}{L} = \frac{11 \bullet 3^6}{3^5 + 2^5 \bullet 3^5} = 1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.**

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^x = F^z t^z$$
$$r = s = t = 11$$
$$x = 5, y = 5, z = 6$$
$$D = 3, E = 6, F = 3$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \ P = L$$

Summary of Observations 1-7

The most important and useful observation in the above examples is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation. This observation will be useful in proving Beal conjecture.

1. $2^3+2^3=2^4$ $\underbrace{2^3}_{K}(\underbrace{1+1}_{L})=\underbrace{2^3}_{M}\bullet\underbrace{2}_{P}$	**2.** $3^3+6^3=3^5$ $\underbrace{3^3}_{K}(\underbrace{1+8}_{L})=\underbrace{3^3}_{M}\bullet\underbrace{3^2}_{P}$	**3.** $7^6+7^7=98^3$ $\underbrace{7^6}_{K}(\underbrace{1+7}_{L})=\underbrace{7^6}_{M}\bullet\underbrace{2^3}_{P}$
4. $2^9+8^3=4^5$ $\underbrace{2^9}_{K}(\underbrace{1+1}_{L})=\underbrace{2^9}_{M}\bullet\underbrace{2}_{P}$	**5.** $34^5+51^4=85^4$ $\underbrace{17^4}_{K}(\underbrace{17\bullet 2^5+3^4}_{L})=\underbrace{17^4}_{M}\bullet\underbrace{5^4}_{P}$	**6.** $3^9+54^3=3^{11}$ $\underbrace{3^9}_{K}(\underbrace{1+2^3}_{L})=\underbrace{3^9}_{M}\bullet\underbrace{3^2}_{P}$
7. $33^5+66^5=33^6$ $\underbrace{11^5}_{K}(\underbrace{3^5+2^5\bullet 3^5}_{L})=\underbrace{11^5}_{M}\bullet\underbrace{11\bullet 3^6}_{P}$	**Corresponding relationship formulas** $\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}\ \ K=M,\ \ P=\ L$ **or** $\underbrace{s^y}_{K}[\underbrace{E^y+D^xr^x\bullet s^{-y}}_{L}]=\underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P};K=M,\ \ P=\ L$	

Properties of the Factored Beal Equation

Let r, s and t be prime factors of A, B and C .respectively, such that $A=Dr$ **,** $B=Es$ **,** $C=Ft$ **.** where D, E and F are positive integers; and the equation becomes $(Dr)^x+(Es)^y=(Ft)^z$.

Step 1: Factor out r^x on the left side of the equation and on the right side of the equation, replace t^z by $t^x\bullet t^{z-x}$ (Note $t^x\bullet t^{z-x}=t^z$)

$$(Dr)^x+(Es)^y=(Ft)^z$$

$$D^xr^x+E^ys^x=F^zt^z$$

$$\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P};\ K=M,\ \ P=\ L$$

For the factorization $\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$ with respect to r^x, $r^x=t^x$ $(K=M)$

Step 2: Factor out s^y on the left side of the equation and on the right side of the equation, replace t^z by $t^y\bullet t^{z-y}$ (Note $t^y\bullet t^{z-y}=t^z$)

$$(Es)^y+(Dr)^x+=(Ft)^z$$

$$E^ys^y+D^xr^x=F^zt^z$$

$$\underbrace{s^y}_{K}[\underbrace{E^y+D^xr^x\bullet s^{-y}}_{L}]=\underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}\ \ ;K=M,\ \ P=\ L$$

For the factorization $\underbrace{s^y}_{K}[\underbrace{E^y+D^xr^x\bullet s^{-y}}_{L}]=\underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$ with respect to s^y, $s^y=t^y$ $(K=M)$

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Proof of Beal Conjecture

(Original Conjecture)

Given: $A^x + B^y = C^z$, A, B, C, x, y, z are positive integers and $x, y, z > 2$.

Required: To prove that A, B and C have a common prime factor.

Plan: Let r, s and t be prime factors of A, B and C .respectively, such that $A = Dr$ **,** $B = Es$ **,**

$C = Ft$ **.** where D, E and F are positive integers, The proof would be complete after showing that $r = s = t$

The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation. Two main steps are involved in the proof. In the first step, one will determine how r *and* t are related, and in the second step, one will determine how s *and* t are related.

Proof:

Step 1: One will factor out r^x

$(Dr)^x + (Es)^y = (Ft)^z$

$D^x r^x + E^y s^x = F^z t^z$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$

$K = M$, $P = L$ (Properties of factored Beal equation)

From above, $r^x = t^x$;

If $r^x = t^x$, then $r = t$. $(\log r^x = \log t^x;\ x\log r = x\log t;\ \log r = \log t;\ r = t)$

Step 2: One will factor out s^y

$(Es)^y + (Dr)^x + = (Ft)^z$

$E^y s^y + D^x r^x = F^z t^z$

$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$$

$K = M$, $P = L$ (Properties of factored Beal equation)

From above, $s^y = t^y$;

If $s^y = t^y$, then $s = t$. $(\log s^y = \log t^y;\ y\log s = y\log t;\ \log s = \log t;\ s = t)$

It has been shown in Step 1 that $r = t$, and in Step 2 that, $s = t$; therefore, $r = s = t$.

Since $A = Dr$ **,** $B = Es$ **,** $C = Ft$ and $r = s = t$, A, B and C have a common prime factor, and the proof is complete.

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Option 2

Proof of Equivalent Beal Conjecture

The equivalent Beal conjecture states that if A, B, C, x, y, z are positive integers and A, B, and C are coprime (have no common prime factors), and $x, y, z > 2$, then the equation $A^x + B^y = C^z$ has no solutions. Here, one will assume at the beginning of the proof that A, B, and C do not have any common prime factors. The proof would be constructed by contradiction.

Given: A, B, C, x, y, z are positive integers and A, B, and C are coprime, with $x, y, z > 2$**.**

Required: To prove that the equation $A^x + B^y = C^z$ has no solutions.

Plan: Let r, s and t be prime factors of A, B and C .respectively, such that $A = Dr$, $B = Es$, $C = Ft$ where D, E and F are positive integers, and $\boxed{r \neq s \neq t}$**,** The proof would be complete after showing that $r = s = t$, which would be a contradiction to the assumption that $r \neq s \neq t$.
The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation. Two main steps are involved in the proof. In the first step, one will determine how r *and* t are related, and in the second step, one will determine how s *and* t are related. The steps here are the same as those in original conjecture proof, except that the end of step 2 would conclude by contradiction.

Proof:

Step 1: One will factor out r^x

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^x = F^z t^z$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$

$K = M$, $P = L$ (Properties of factored Beal equation)

From above, $r^x = t^x$;

If $r^x = t^x$, then $r = t$. ($\log r^x = \log t^x$; $x\log r = x\log t$; $\log r = \log t$; $r = t$)

Step 2: One will factor out s^y

$$(Es)^y + (Dr)^x + = (Ft)^z$$

$$E^y s^y + D^x r^x = F^z t^z$$

$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$$

$K = M$, $P = L$ (Properties of factored Beal equation)

From above, $s^y = t^y$;

If $s^y = t^y$,then $s = t$. ($\log s^y = \log t^y$; $y\log s = y\log t$; $\log s = \log t$; $s = t$)

Since it has been shown in Step 1 that $r = t$, and in Step 2 that, $s = t$; $r = s = t$**.**
This result. $\boxed{r = s = t,}$ is a contradiction to $\boxed{r \neq s \neq t}$. of the hypothesis, and therefore, the equation $A^x + B^y = C^z$ (= $(Dr)^x + (Es)^y = (Ft)^z$) is **not** true and has no solutions.
The proof is complete.

Discussion

It is interesting that to prove the equivalent Beal conjecture from the original conjecture proof, all one had to do was to assume at the beginning of the proof that A, B and C do not have any common prime factors and then produce a contradictory result that A, B and C have a common prime factor. The principles upon which relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation are sound, since they are based on numerical sample problems research. The results in this paper may encourage those who have already proved the equivalent conjecture to prove the original conjecture, which stresses positively on the common prime factor, and such a proof may make Honorable Beal feel better about his conjecture.

Conclusion

Both the original Beal conjecture and the equivalent Beal conjecture have been proved in this paper. The principles applied in both proofs are based on the properties of the factored Beal equation. However, the proof of the equivalent conjecture is by contradiction. The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation.

PS: Other proofs of Beal Conjecture by the author are at viXra:2001.0694; viXra:1702.0331; viXra:1609.0383; viXra:1609.0157;

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