Chen’s Formulas of the Fine-structure Constant (viXra:2002.0203vH)

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper gives two series of formulas of the fine-structure constant \( \alpha \) which are reasonable, precise, smart and elegant. It also demonstrates there are two values of \( \alpha \), i.e., \( \alpha_1=1/137.035999037435 \) and \( \alpha_2=1/137.035999111818 \), which are consistent with but much more accurate than those experiment measured values. The formulas consist of \( 2\pi \)-e formulas and some factors related to nucleon numbers of nuclides. A brief explanation of the fine-structure constant shows \( 1/\alpha \approx 137.036 \) is the equal ratio factor between 112 and 168 (more precisely 168-1/3). Based on these, all 119th to 170th ideal extended elements were predicted, the speed of light in atomic units was mathematically calculated by \( c_{au}=1/(\alpha_1 \alpha_2)^{1/2}=137.035999074627 \), Schrödinger equation of hydrogen atom was simplified and correlated with \( \alpha_1/\alpha_2 \), classical electron radius was hypothetically calculated to be 2.81794032658(43) fm and proton charge radius was hypothetically calculated to be 0.83302720999(13) fm. In the end, it was found that the approximate rational numbers of \( 2\pi \) marvelously related to nuclides, a mathematic shell model of nuclides was established and a picture of elements and ideal extended elements was depicted.

Keywords: formulas; the fine-structure constant; the ideal extended elements; the speed of light; Schrödinger equation of hydrogen atom; the proton charge radius; \( 2\pi \).

1. Introduction

The fine-structure constant (Sommerfeld’s constant) is a critical dimensionless constant in physics, it is a century mystery of physics, it has been one of the biggest enigmas in physics since it was introduced by Arnold Sommerfeld in 1916. Its definition, some interpretations and the latest measured values are as follows\(^1\): 

\[
\alpha = \frac{\lambda_c}{2\pi a_0}, \quad \alpha = \frac{2\pi r_e}{\lambda_0}, \quad \frac{a_0}{r_e} = \frac{1}{\alpha^2}; \quad \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{v_e}{c}, \quad \frac{c}{v_e} = \frac{1}{\alpha}
\]

in atomic units, the speed of light \( c_{au} = \frac{1}{\alpha} \)

the 2014 CODADA recomended value: \( \alpha = 1/137.035999139(31) \)
the 2018 CODADA recomended value: \( \alpha = 1/137.035999084(21) \)

Science 13 April 2018 reported value: \( \alpha = 1/137.035999046(27) \)
The ratio of Bohr radius of hydrogen atom $a_0$ to the classical electron radius $r_e$ is $1/\alpha^2$. The ratio of the speed of light $c$ to the line velocity of ground state electron in hydrogen atom $v_e$ is $1/\alpha$, this means in atomic units $c=1/\alpha$ and $E=mc^2=m/\alpha^2$ or $\alpha^2=m/E$. In quantum electrodynamics it substantially characterizes the strength of electromagnetic interaction between elementary charged particles such as electron and proton, so it is the coupling constant of electric charges. It is one of the 25 fundamental constants (could not be calculated theoretically, could only be determined by experiments) in Standard Model of physics and should be the most important one. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. However, to our knowledge, up to now (except this work), no one knows how it comes from, no one could give reasonable explanations to it or formulas of it since it was introduced.

In 2016 Paul Davis gave the following comment\(^3\): “Physicists have long wondered where this number, $1/137.035999$, comes from. Is there a deep reason why $\alpha$ has to be precisely this number for the world to function as it does? There is a long history of attempts to derive $\alpha$ from physical theory or to concoct a mathematical formula that has this value. For a brief time in the 1920s, when it looked as if $\alpha$ might be exactly $1/137$, astronomer Arthur Eddington searched for a theory that would throw up the numbers naturally, but his ideas ultimately led nowhere. Then in 1969 a young Swiss mathematician, Armand Wyler, pointed out that $(9/16\pi^3)(\pi/5!)^{1/4}$ comes close to $1/137.036$, which matched the value of $\alpha$ to the precision known at the time. However, his formula was not accompanied by any credible theory and was regarded as little more than a numerical curiosity. Several other attempts at $\alpha$ numerology have been made since, none of which have gained traction in the physics community.”

As for the fascination of the fine-structure constant, in the middle of 1980s, Richard Feynman stated\(^4\): “It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the hand of God wrote that number, and we don’t know how He pushed his pencil.”

This paper shows how God pushed his pencil to write the fine-structure constant and how God used it to coordinate elements.
2. 2π-e formula(s)

2π-e formula, its related formulas and their preliminary applications were deduced independently by us from April to December of 2013.

Fig. 1. Diagram of y=1/x.

Euler-Mascheroni constant γ : \[ \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{\infty} = \ln \infty + \gamma \]

As for y=1/x (Fig. 1), \( \gamma_y = \sum_{n=1}^{\infty} \delta_{n+1} + \sum_{n=1}^{\infty} \delta_{n-1} = \frac{1}{2} + \gamma_1 = 0.5 + 0.077215 \cdots = 0.577215 \cdots \)

\[ \gamma_y = \sum_{n=1}^{\infty} \delta_{n+1} = \lim_{N \to \infty} \left( \sum_{n=1}^{N} \frac{1}{n} - \int_{1}^{N+1} \frac{1}{x} \, dx \right) - \frac{1}{2} \]

Generally \( \gamma_x = \lim_{N \to \infty} \left( \sum_{n=1}^{N} \frac{1}{n} - \int_{1}^{N+1} \frac{1}{x} \, dx \right) - \frac{1}{2} \), \( s \in \mathbb{N} \)

As for y = \log(x) (Fig. 2), \( \delta_{n+1} = \int_{n}^{n+1} \ln x \, dx - \frac{1}{2} \ln \frac{n+1}{n} - \ln n = (x \ln x - x) \bigg|_{n}^{n+1} - \frac{1}{2} \ln(n+1) - 1 \)

\( (n+1) \ln(n+1) - n \ln n - \frac{1}{2} \ln(n+1) - \frac{1}{2} \ln n = (n+\frac{1}{2}) \ln(1+\frac{1}{n}) - 1 \)

\[ \gamma_{e,N} = \sum_{n=1}^{N} \delta_{n+1} = \sum_{n=1}^{N} [(n+\frac{1}{2}) \ln(1+\frac{1}{n}) - 1] = \sum_{n=1}^{N} \frac{\left(1 + \frac{1}{n}\right)^{\left(n+\frac{1}{2}\right)}}{e^n} = \ln \prod_{n=1}^{N} \left(1 + \frac{1}{n}\right)^{\left(n+\frac{1}{2}\right)} \]

\[ \gamma_e = \gamma_{e,N} = \sum_{n=1}^{N} \delta_{n+1} = \lim_{N \to \infty} \left( \sum_{n=1}^{N} \ln(n) - \frac{\log(N+1)}{2} \right) \]

\[ \ln N! = \sum_{n=1}^{N} \ln n = \int_{1}^{N+1} \ln x \, dx - \sum_{n=1}^{N} \delta_{n+1} - \sum_{n=1}^{N} \delta_{n-1} = (x \ln x - x) \bigg|_{1}^{N+1} - \ln e^{N+1} - \ln e^{-\gamma_e} - \sum_{n=1}^{N} \ln(n) - \ln N \]

\( = (N+1) \ln \left( \frac{N+1}{e} \right) + \frac{1}{2} \ln(N+1) = \ln \left[ \frac{e^{\gamma_e} \frac{N+1}{e^{N+1}}}{\sqrt{N+1}} \right] \]

\( N! = \frac{e^{\gamma_e} \frac{N+1}{e^{(N+1)}}}{\sqrt{N+1}}, \text{ compared to Stirling formula : } N! \sim \sqrt{2\pi N} \left( \frac{N}{e} \right)^N \)

\( (N+1)! = (N+1)N! \sim \sqrt{2\pi(N+1)} \left( \frac{N+1}{e^{(N+1)}} \right)^{N+1}, \text{ } N! \sim \sqrt{\frac{2\pi}{N+1}} \left( \frac{N+1}{e} \right)^{N+1} \)

Compared to previous formula, gives \( \sqrt{2\pi} \sim e^{\gamma} \) or \( 2\pi = (\frac{e^{\gamma}}{e^{2\gamma}})^2 \)

2π - e formula(s): \( 2\pi = (\frac{e^{\gamma}}{e^{2\gamma}})^2 = e^2 e^{\frac{e^2}{2}} e^{\frac{e^3}{3}} e^{\frac{e^4}{4}} \cdots, (2\pi)_k = (\frac{e^{\gamma}}{e^{2\gamma}})^2 = e^2 e^{\frac{e^2}{2}} e^{\frac{e^3}{3}} \cdots e^{\frac{e^k}{k}} \)

\( \gamma_e = 0.0810614668 \cdots, e^{\gamma} = 1.0844375 \cdots \)
2π-e formula is an expanding form of Stirling formula. To our knowledge, it was first deduced by us. If it was new, it could be named Chen’s 2π-e formula.

3. Some Formulas Related to 2π-e Formula

The following formulas which correlate each other and has similar form could be called Chen’s natural group formulas, and the form is called natural group.

\[
1 = 4\gamma_1 + \frac{4\gamma_2}{1(1 + 1)} + \frac{4\gamma_3}{2(2 + 1)} + \frac{4\gamma_4}{3(3 + 1)} + \cdots
\]

\[
= \beta_1 + \frac{\sum_{n=1}^N B_{2n}(\pi / 2)^{2n}}{(2n)!} - \frac{\sum_{n=1}^N B_{2n}(3\pi / 2)^{2n}}{(2n)!}
\]

\[
N \sim \frac{3}{2} \beta + \frac{\sum_{n=1}^N B_{2n}(2\pi)^{2n}}{2(2n)!}
\]

\[
e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots
\]

\[
2\pi = \left(\frac{e}{e^\gamma}\right)^2 = e^2 + e^2 + e^2 + \cdots
\]

\[
B, B_{2n} \text{: the Bernoulli numbers such as } -\frac{1}{2}, -\frac{1}{6}, -\frac{1}{30}, -\frac{1}{42}, -\frac{1}{30}, \cdots
\]

\[
\gamma_e = \lim_{N \to \infty}\left(\sum_{n=1}^N \log(n) - \frac{\log(n + 1)}{n}\right) = 0.0810614668\cdots
\]

\[
\gamma_s = \lim_{N \to \infty}\left(\sum_{n=1}^N \frac{1}{2} - \frac{N}{x^n}\right) = \frac{1}{2}, \quad s \in \mathbb{N}
\]

\[
\gamma_1 = 0.077215\cdots, \quad \gamma_2 = 0.144934\cdots, \quad \gamma_3 = 0.24899\cdots, \quad \gamma_4 = 0.36122\cdots, \quad \gamma_5 = 0.433349\cdots, \quad \gamma_6 = 0.5
\]

\[
\gamma_e, \gamma_1, \gamma_2, \gamma_3, \cdots \text{are called Chen’s natural group constants (analogue to Bernoulli numbers)}
\]

The following are some other formulas related to 2π-e Formula.

\[
\sqrt{2\pi} = e^{\gamma_e}, \quad e = \sqrt{2\pi e^e} = \sqrt{2\pi (1 + \sum_{n=1}^N \frac{\gamma_e^n}{n!})}
\]

\[
\gamma_e = \sum_{n=1}^\infty (n + 1) \log(1 + \frac{1}{n}) - 1 = \sum_{n=1}^\infty \frac{(2n - 1)B_{2n} \pi^{2n} - 2(2n)!}{2(2n + 1)!} = \frac{1}{4} \sum_{s=1}^\infty \gamma_e^n s(s + 1)
\]

\[
\gamma_s = \sum_{n=1}^\infty (n + 1) \ln(1 + \frac{1}{n}) - \int_1^\infty \frac{x}{x^n} \ln(1 + \frac{1}{x}) dx
\]

\[
\gamma_e = \frac{1}{2} \lim_{N \to \infty} \sum_{n=1}^N (2n - 1)B_{2n} \pi^{2n} - \ln N
\]

\[
\frac{\pi}{2} = \left(\frac{e}{e^\gamma}\right)^2, \quad e = \sqrt{\frac{\pi}{2} e^e} = \sqrt{\frac{\pi}{2} (1 + \sum_{n=1}^N \gamma_e^n n!)}; \quad \frac{\pi}{2} = \left(\frac{e^{\gamma_e}}{e^e}\right)^2, \quad \gamma = \frac{\ln \pi}{2} + 2\gamma_e
\]

\[
\frac{\pi}{2} = \sum_{n=1}^\infty \frac{B_{2n} \pi^{2n}}{2n(2n)!} + \sum_{n=1}^\infty \zeta(2n) - 1 = \frac{3}{4} \zeta(2) - \sum_{k=1}^\infty \frac{1}{k^{2n}}
\]

\[
\sum_{n=1}^\infty \frac{B_{2n} \pi^{2n}}{2n(2n)!} = \sum_{n=1}^\infty \frac{B_{2n} (2n^2 + 1) \pi^{2n}}{2n(2n + 1)!}
\]
4. Some Applications of $2\pi$-e Formula and its Related Formulas

(1). $2\pi$-e formula is basically an algebraic expanding of Stirling formula, but it is more meaningful, it exhibits the relationship between $2\pi$ and $e$. In $2\pi$-e formula, $\gamma$ is a real constant with geometric definition like Euler-Mascheroni constant $\gamma$. With $2\pi$-e formula and its related formulas, $2\pi$ can be calculated from $e$ and vice versa. So it is the real $2\pi$-e relationship formula.

$$2\pi = \left(\frac{e}{e^n}\right)^2 = e^2 \frac{e^2}{1} \frac{e^2}{(\frac{1}{2})^3} \frac{e^2}{(\frac{3}{2})^3} \frac{e^2}{(\frac{5}{2})^3} \cdots$$

$$e = \sqrt{2\pi e^x} = \sqrt{2\pi} \left(1 + \sum_{n=1}^{\infty} \frac{\gamma_n}{n!}\right), \quad \gamma_n = \frac{(2^{2n-1} - 1) B_{2n}}{2(2n+1)!}$$

(2). $2\pi$-e formula demonstrates $2\pi$ is a natural constant rather than $\pi$. $\pi/2$ is somewhat fundamental but not as complete as $2\pi$. $\pi$ is neither fundamental nor complete. In 2001 mathematician Bob Palais said “$\pi$ is wrong5. $2\pi$-e formula and the Taylor expansion of $e$ have similar form (natural group form), this should give a conclusive proof that $2\pi$ is a real natural constant and $\pi$ is not.

$$2\pi = \left(\frac{e}{e^n}\right)^2 = e^2 \frac{e^2}{1} \frac{e^2}{(\frac{1}{2})^3} \frac{e^2}{(\frac{3}{2})^3} \frac{e^2}{(\frac{5}{2})^3} \cdots \Rightarrow 2\pi \text{ or } \sqrt{2\pi} \text{ is a natural constant}$$

$$\frac{\pi}{2} = \left(\frac{e}{e^n}\right)^2 \Rightarrow \pi \text{ or } \sqrt{\pi} \text{ is almost a natural constant}$$

$$\pi = \left(\frac{e}{e^n}\right)^2 \Rightarrow \pi \text{ or } \sqrt{\pi} \text{ is not a natural constant}$$

Table 1 lists some points of view of Piist who support $\pi$ is a natural constant, Tauist who support $2\pi$ is a natural constant and this work which supports the later.

<table>
<thead>
<tr>
<th></th>
<th>Piist</th>
<th>Tauist</th>
<th>This work</th>
</tr>
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<td>$\pi d$</td>
<td>$2\pi R$</td>
<td>$2\pi R$</td>
</tr>
<tr>
<td>Area of a circle</td>
<td>$\pi R^2$</td>
<td>(1/2)(2\pi R)R</td>
<td></td>
</tr>
<tr>
<td>Volume of sphere</td>
<td>$(4/3) \pi R^3$</td>
<td>(2/3)(2\pi R)</td>
<td>(2\pi R^2/3)2R</td>
</tr>
<tr>
<td>Volume of n-dimension sphere</td>
<td>$\frac{\pi}{\Gamma(n/2+1)} R^n$</td>
<td>$(2\pi)^{n^2} \frac{R^n}{2^n \Gamma(n/2+1)}$</td>
<td>$2\pi R^n \frac{V_{n-2}}{n}$</td>
</tr>
<tr>
<td>Euler’s identity</td>
<td>$e^{x}+1=0$</td>
<td>$e^{2n}=1$</td>
<td>$e^{2n}=1$</td>
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<tr>
<td>Gauss integral</td>
<td>$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$</td>
<td>$\int_{-\infty}^{\infty} e^{-x^2} dx = \frac{e^{-1}}{\sqrt{\pi}}$</td>
<td></td>
</tr>
</tbody>
</table>
(3). As $2\pi$ is a square number, the frequent appearing of its square root in some important equations such as Gaussian distribution (normal distribution) and Maxwell-Boltzmann distribution becomes reasonable and understandable. And the distributions can be transformed as follows.

Standard Normal Distribution: $f(x, 0, 1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = e^{\frac{x^2}{2} + \frac{1}{2} - 1}$

Maxwell – Boltzmann Distribution: $f(v) = \frac{2}{\sqrt{2\pi}} v^2 \left( \frac{m}{kT} \right)^{\frac{3}{2}} \exp\left( -\frac{mv^2}{2kT} \right) = 2\left( \frac{m}{kT} \right)^{\frac{3}{2}} v^2 e^{\frac{mv^2}{2kT}}$  

(4). Euler’s identity (Euler’s equation) $e^{i\pi} + 1 = 0$ is called God formula and the most beautiful formula in mathematics. However, as $2\pi$ is the real natural constant and $\pi$ is not, $e^{2\pi i} = 1$ should be more beautiful.

(5). $\gamma = \ln(2\pi) + \gamma_c$ may help to prove $\gamma$ is an irrational number or even a transcendental number.


(7). The mathematic expression of chirality is $\pm 2\pi$. This concept is helpful for us to establish “Chirality and Poetry Model of Atomic Nuclei” (2017/12-2018/3).

(8). Based on the above theories, Chen’s formulas of the fine-structure constant were deduced (2018/4-6), then modified and extended (2018/7-2020/6).

5. Original Inspiration for Formulas of the Fine-structure Constant

1. According to $\alpha = \frac{e^2}{4\pi\hbar c} = \frac{\lambda_e}{2\pi a_0} = \frac{2\pi e}{\lambda_e} \approx \frac{1}{137.036}$, the formulas of $\alpha$ should relate to $2\pi$.

2. $\frac{137.036}{2\pi} = 21.81, 137.036 \times 2\pi = 6.28318$

3. According to $2\pi - e$ formula: $2\pi = (\frac{e}{e^2})^2 = e^2 e^2 e^2 e^2 e^2 e^2 e^2 e^2 e^2 \ldots$

$2\pi$ is a square number, suppose $21.81 = x^2, \ x = 4.670 \approx 14/3$

so: $\frac{1}{\alpha} \approx (\frac{14}{3})^2 2\pi$ or $\alpha \approx (\frac{3}{14})^2 \frac{1}{2\pi}$ (Discover: about 2 am on 2018/4/12)

4. Apply with $2\pi - e$ formula (in the afternoon of 2018/4/12, a meeting in the morning)

$\alpha = (\frac{3}{14})^2 \frac{1}{(2\pi)_{112}} = (\frac{3}{14})^2 \frac{1}{e^2 e^2 e^2 \ldots e^2} = 137.035781520$, closest to the real value.

As 112 is one of the most important stable numbers and the 112th element $^{288}Cn$ is the natural end of elements according to our Chen's Chirality and Poetry Model of Atomic Nuclei.

So: Eureka! Subsequently transformed to: $\alpha = \frac{6^2}{7(2\pi)_{112}} \approx 137.035781520$,

Finally modified to: $\alpha = \frac{6^2}{7(2\pi)_{112}} \times \frac{1}{112} = 137.035999037435$
6. Logical Deduction of Chen's Formulas of the Fine-structure Constant

Physicist Richard Feynman noticed a hydrogen-like atom with Z protons and only one electron, according to Bohr model, the line velocity of the nth rank electron \( v_{e/Z/n} \) satisfies:
\[
\frac{v_{e/Z/n}}{c} = \frac{Ze^2}{n^4\pi e_0 hc} Z \alpha, \quad \text{as} \quad v_{e/Z/n} \leq c, \quad \alpha = \frac{v_{e/Z/n}}{c} \approx \frac{1}{Z_{\text{max-ideal}}} = \frac{1}{Fy} = \frac{1}{137}
\]
The 137th hydrogen-like element Fy (Feynmanium) is an ideal (imaginative) element, in reality, the above formula should be modified to:
\[
\alpha = f(Z_{\text{real}}) \frac{1}{Z_{\text{max-real}}}
\]
According to Chen's Chirality and P-Theory Model of Atomic Nuclei, 
\[
Z_{\text{max-real}} = 112 \approx 2 \cdot 56, \quad \text{so} \quad \alpha = f(Z_{\text{real}}) \frac{1}{Z_{\text{max-real}}} = f(Z_{\text{real}}) \frac{1}{112}
\]
Compared to \( \alpha = \frac{\lambda}{2\pi a_0} \), the formula should have a \( 2\pi \) factor:
\[
\alpha = f(Z_{\text{real}}) \frac{1}{Z_{\text{max-real}}} = \frac{n}{m(2\pi)} \frac{1}{Z_{\text{max-real}}} = \frac{6^2}{7 \cdot (2\pi)_{112}} = 1/136.8
\]
Apply with \( 2\pi \cdot e \) formula:
\[
2\pi e^2 = \left( \frac{2}{1} \right)^3 \left( \frac{3}{2} \right)^5 \left( \frac{5}{3} \right)^7 \ldots
\]
The formula is transformed to:
\[
\alpha = \frac{n}{m(2\pi)} \frac{1}{Z_{\text{max-real}}} = \frac{6^2}{7 \cdot (2\pi)_{112}} = \frac{6^2}{7 \cdot e^2 \cdot e^2 \cdot e^2 \ldots} = \frac{1}{112} = 1/137.035782
\]
Above deduction on 2018/4/12, only \( (2\pi)_{112} \) gives the closest value to \( \alpha \), this coincidence of one part per infinity proves the formula itself is correct.

Added an calibration factor \( (\delta=1/75^2) \) on 2018/4/20, the accurate formula is:
\[
\alpha_i = \frac{\lambda}{2\pi a_0} = \frac{6^2}{7 \cdot e^2 \cdot e^2 \cdot e^2 \ldots} = \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435
\]

By the same procedure but compared to \( \alpha = \frac{2\pi r_c}{\lambda_c} \), the other formula is:
\[
\alpha_2 = \frac{2\pi r_c}{\lambda_c} = \frac{13 \cdot e^2}{10^2} \frac{279}{278} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = 1/137.035999111818
\]
Discover: 2018/4/24; Revise: 2018/9/18-20 (280 \( \rightarrow \) 278, \( -\frac{1}{39^2} + \frac{1}{780^2} \rightarrow -\frac{1}{3 \cdot 29 \cdot 64} \))

Another amazing coincidence is \( 6^2 \) and \( 10^2 \) are square numbers in accordance with \( 2\pi = (e/\epsilon_c)^2 \)

This also demonstrates that \( \alpha \) has two values with two kinds of formulas.
\[
\text{As } f(Z_{\text{real}}) = \frac{n}{m(2\pi)_k} \text{ or } f(Z_{\text{real}}) = \frac{m(2\pi)_k}{n}, \text{ m n k } \delta \text{ should relate to nucleon numbers of nuclides.}
\]
7. The Two Most Important Formulas

The above two formulas for $\alpha_1$ and $\alpha_2$ were our first gained formulas and are the most important formulas among their serial formulas which will be given followed in this paper. Calculation to give the values of $\alpha_1$ and $\alpha_2$ is shown in Fig. 3 and Table 2.

Fig. 3. Calculation diagram of $\alpha_1$ and $\alpha_2$ (2018/4-6).

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In these two formulas (deduced from the modification of $Z_{\text{max}}$), there are some factors that are essentially related to nucleon numbers of some nuclides especially some important stable numbers (stipulated by Chen’s Chirality and Poetry Model of Atomic Nuclei\(^7\)) such as 28, 42, 56, 83, 84, 112, 126, 166, 167, 168 et al. And these numbers correlate with each other. This kind of relationship is shown in the follows.

A brief illustration of the relationships between the fine-structure constant and nuclides:

Above nuclides indicate that 136 – 138, which can be called the fine-structure constant numbers, definitely relate to 112 and 166 – 168 (double of 56 and 83 – 84, the most stable numbers in nuclides).

Relations to nuclides (7(2\pi)\[\alpha\][\pi] \approx 44; \[\text{nucleon proton } \pi \text{ number]}

\[\begin{align*}
\alpha_1 &= \frac{6^2 \pi}{7^2} \left(\frac{1}{112 + \frac{1}{278}}\right) = \frac{6^2}{7^2} \left(\frac{1}{278}\right) \left(\frac{113}{112}\right) \left(\frac{1}{112 + \frac{1}{278}}\right)
= \frac{1}{1 + 1.3759999037435}
\end{align*}\]

Relations to nuclides (13(2\pi)\[\alpha\][\pi] \approx 82; \[\text{nucleon proton } \pi \text{ number]}

\[\begin{align*}
\alpha_1 &= \frac{13 \cdot (2\pi)}{278} \left(\frac{1}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}}\right)
= \frac{13 \cdot (2\pi)}{278} \left(\frac{1}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}}\right)
= \frac{1}{1 + 1.3759999111818}
\end{align*}\]

The value of the front part of each above formula is almost equal to 1/(3/2)\[1/2\] (because 112 is the element natural proton end and 168 is the element natural neutron end as shown in \[\text{112Cn}_{168+3}\]), so the formulas can be transformed to the follows.

\[\begin{align*}
\alpha_1 &= \alpha_1 \cdot (3/2)^\[2\]
= \left(\frac{1}{2} - \frac{1}{3 \cdot 112 + 1}\right) \left(\frac{1}{2 \cdot 3 \cdot 5 \cdot 13 \cdot 23 \cdot 30 \cdot 64}\right) \left(\frac{1}{112} + \frac{1}{75}\right)
= 1/1 + 1.3759999037435
\end{align*}\]

Relations to nuclides:

\[\begin{align*}
\alpha_2 &= \alpha_2 \cdot (3/2)^\[2\]
= \left(\frac{1}{2} - \frac{1}{3 \cdot 112 + 1}\right) \left(\frac{1}{2 \cdot 7 \cdot 11 \cdot 19 \cdot 29 \cdot 36 \cdot 75}\right) \left(\frac{1}{112} - \frac{1}{3 \cdot 29 \cdot 64}\right)
= 1/1 + 1.3759999111818
\end{align*}\]
8. The Integrated Fine-structure Constant

Multiplication of $\alpha_1$ and $\alpha_2$ should almost divide out the $2\pi$ factors and give $3/2$ and $112 \times 112$ factors, this means $\alpha_1\alpha_2$ is almost equal to $112 \times 168$, so we define $\alpha_c = (\alpha_1\alpha_2)^{1/2}$ as the integrated fine-structure constant or Chen’s fine-structure constant.

$$\frac{1}{\alpha_c^2} = \frac{1}{\alpha_1\alpha_2} \frac{2\pi \alpha_0}{2\pi r_0} = \frac{\alpha_0}{r_0} = \left(\frac{\varepsilon_0}{\varepsilon}\right)^2$$

$$= 112 \times (168 - \frac{1}{3} \cdot \frac{12 \cdot 47}{6 - 29 \cdot 23 \cdot 30 - 79 / 47})$$

$$= 136(138 + \frac{1}{2} \cdot \frac{10 \cdot 29}{12 - 53(6 - 51 - 1) - 27 / 47})$$

$$= 137(137 + \frac{1}{13} \cdot \frac{7 \cdot 29}{32 - 33 - 39 + 16 / 49})$$

$$= 112 \cdot 167,664,378,788,408 = 1877,886,504,2381$$

$$\alpha_c = \alpha_1\alpha_2 = \sqrt{\frac{6^2}{7 \cdot (2\pi)_{112}^{3/2}}} = \frac{1}{75^2} \left[1 + \frac{1}{112} - \frac{1}{3 \cdot 29 - 64}\right]$$

$$= \frac{13 \cdot 3^2}{7 \cdot 5^2} \left[\frac{2 \cdot 3 - 19}{113} \cdot \frac{e^3}{114} \cdot \frac{2 \cdot 139}{115} \cdot \frac{9 \cdot 31}{2 \cdot 139} \cdot \frac{1}{557} \cdot \frac{1}{112} - \frac{1}{30^3 \cdot 5} - \frac{1}{60^3 \cdot 15} - \frac{1}{120^3 \cdot 15 \cdot 29}\right]$$

$$= 1/1877,886,504,2381$$

9. A Brief Explanation of the Fine-structure Constant

According to Chen’s Chirality and Poetry Model of Atomic Nuclei $^7$, the ratio of neutron number $N$ to proton number $Z$ in nuclides increases from 1/1 to 3/2 (eventually slightly above 3/2) along with the increasing of atomic number, for example, from $^{14}$Si to $^{56}$Fe, $^{56}$Cu to $^{112}$Cn. In this process, $(3/2)^{1/2}$ will act as a transition foothold. As for nuclide $^{112}$Cn with $Z=112$, $N=168+5$ and $112/168=3/2$, 137 is just right their $(3/2)^{1/2}$ times intermediate stage. This should be why 137 exists and what’s the real meaning of 137.
The relationships between the fine-structure constant and elements are mainly reflected by correlation of nucleon numbers of 56, 68-69, 82, 82-84, 112, 136-138, and 166-168 which are derived from the three key numbers 112, 137 and 168, and by correlation of nucleon numbers of the other factors in Chen’s formulas of the fine-structure constant. The former type is illustrated as follows.

Several clusters of ideal extended elements (ie) Fy and Ch are hence predicted.

10. Comparison to Experiment Determined Values

The above two calculated values of the fine-structure constant, i.e., \( \alpha_1 = \frac{1}{137.035999037435} \) and \( \alpha_2 = \frac{1}{137.035999111818} \) are consistent with those experiment measured values\(^2\), but much more accurate with several more digits.

The above theoretical analysis and formulas also demonstrate there are two different values of the fine-structure constant, i.e. \( \alpha_1 \) and \( \alpha_2 \). Accordingly, we have found that up to now the experiment determinations of \( \alpha \) have almost proved this because the \( \alpha \) ranges measured by two different but accurate methods couldn’t overlap each other\(^2\). It seems that the time comes to a critical point to prove there are two values of the fine-structure constant theoretically and experimentally.

11. Theoretical Calculation of the Speed of Light

In atomic units, the line velocity of the ground state electron in hydrogen atom can be assigned as the natural unit of speed \( \left( \frac{v_e}{\alpha} = 1 \right) \), then the speed of light becomes the reciprocal of the fine-structure constant, i.e., \( c_{au} = \frac{1}{\alpha} = 137.035999 \). However, we have demonstrated that there are two values of \( \alpha \), but the speed of light shouldn’t have two values, so by referring to Maxwell’s formula of calculating the speed of light

\[
\frac{112}{1/\alpha_2} \approx \frac{1}{137.036} \quad \text{or} \quad \frac{112}{1/\alpha_1} \approx \frac{1}{168 - 1/3}
\]

\[
137.036^2 \approx 112 \cdot (168 - 1/3)
\]

\[
112 \cdot \left( \frac{3}{2} - \frac{1}{336 + 1} \right)^2 \approx 137.036, \quad 137.036 \cdot \left( \frac{3}{2} - \frac{1}{336 + 1} \right)^2 \approx 168 - 1/3
\]

\[
\frac{112}{1/\alpha_2} \approx \frac{1}{137.036} \quad \text{or} \quad \frac{112}{1/\alpha_1} \approx \frac{1}{168 - 1/3}
\]

\[
137^2 \approx 112 \cdot 168, \quad 112 \left( \frac{3}{2} \right)^2 \approx 137, \quad 137 \left( \frac{3}{2} \right)^2 \approx 168
\]
electromagnetic wave or light, it should be reasonable to suppose the speed of light to be the integrated fine-structure constant, i.e., \( c_{au} = 1/\alpha_c = 1/(\alpha_1 \alpha_2)^{1/2} = 137.035999074627 \).

It means we’ve theoretically/mathematically calculated the speed of light, the formula is intrinsically consistent with Maxwell’s formula, and the value is much accurate.

In atomic units \( (e = m_e = \hbar = 1 \) and \( \epsilon_0 = \frac{1}{4\pi} \), \( v_{class} = \alpha c_{au} = \frac{e^2}{4\pi\alpha_0 \hbar} = 1 \), so \( c_{au} = \frac{1}{\alpha_c} \).

There are two \( \alpha \) (\( \alpha_1 \) and \( \alpha_2 \)), but there shouldn't be two \( c \) or \( c_{au} \), so it should be: \( c_{au} = \frac{1}{\alpha c_{au}} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} \) (\( au \: atomic \: units \))

Compared to Maxwell Formula \( c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \), \( c_{au} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} \) should be reasonable.

\[ c_{au} = \frac{1}{\mu_{au} \epsilon_{au}} = \alpha c_{au}, \mu_{au} = 4\pi\alpha_1 \alpha_2 \] (2019/11/30)

So the theoretical formula of the speed of light in atomic units is as follows:

\[ c_{au} = \frac{1}{\alpha_1} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \frac{1}{\sqrt{\frac{6^2}{7\pi(2\pi)^{1/2}} \left[ \frac{1}{12^2} + \frac{1}{10^2} + \frac{1}{112 - \frac{1}{29 \cdot 64}} \right]}} \]

\[ = \frac{5}{3} \sqrt{\frac{(2\pi)^{1/2}}{13(2\pi)^{1/2}} \left[ \frac{1}{12^2} - \frac{1}{30^2 \cdot 5} - \frac{1}{60^2 \cdot 15} - \frac{1}{120^2 \cdot 15 \cdot 29} \right]} \]

\[ = \frac{5}{3} \sqrt{\frac{11}{12^2}} \cdot \frac{2^2}{23} \cdot \frac{23}{11} \cdot \frac{23}{2^2} \cdot \frac{23}{11} \cdot \frac{23}{2^2} \cdot \frac{23}{11} \cdot \frac{23}{2^2} \cdot \frac{23}{11} \]

\[ = \sqrt{\frac{1}{12^2} - \frac{1}{3112 + 1} \cdot \frac{1}{12^2 - \frac{1}{3112 + 1}} \cdot \frac{1}{2^2 \cdot \frac{3}{2}} \cdot 14 - 53 \cdot 193 - 312 \cdot \frac{1}{2} \cdot 2 \cdot 29} \]

\[ = \sqrt{137.03599903743 \times 137.03599911818} = 137.035999074627 \]


Discover: 2019/12/16; Revise and Supplement: 2020/1/5 – 8, 2/24, 3/28-29

12. The Special 29 and 75 Factors

In the above formulas some factors especially 29 and 75 appear several times. This feature should be analyzed and explained. Accompanying N/Z ratio from 1/1 to slightly above 3/2 along with the increasing of atomic number, 29Cu_{34,36} is the critical point of N/Z ratio approaching \( (3/2)^{1/2} \) and 75Re_{110,112} is the critical point of N/Z ratio approaching 3/2 (Table 3, Fig. 4 and Fig. 5), so 29 and 75 are important factors and hence frequently appear in the formulas.

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Z: atomic number, N: average neutron number or neutron number of the most stable isotope.

1. N/Z from 1/1 (C) to slightly above 3/2 (such as 112Cn which is the natural end of elements demonstrated by Chen’s Chirality and Poetry Model of Atomic Nuclei²).

2. For 29Cu, N/Z ratio 1.19 is near to (3/2)½=1.22, slightly less is because of stability effect.

3. For 32Re, N/Z ratio 1.48 is near to 3/2=1.50, slightly less is because of stability effect.

4. From C to 112Cn, the middle of N/Z 1.5 range is at (76.5-5)/(112-5)=0.668≈2/3 position.

Fig. 4 and Fig. 5 shows that stability effect of nucleon number 64 makes the neutron numbers of 29Cu’s isotopes are relatively less (34 and 36) than normal so that its N/Z ratio is a little less than (3/2)½ which is otherwise it should be. Also the
stability effect of nucleon numbers 110 and 112 make the neutron numbers of \( ^{75}\text{Re} \)’s nuclides are relatively less (110 and 112) than normal so that its N/Z ratio is a little less than 3/2 which otherwise it should be.

**Fig. 4. Complete Graph of N/Z Ratios of Elements** (2019/4/23-24).

![Fig. 4. Complete Graph of N/Z Ratios of Elements](image)

**Fig. 5. Partially Amplified Graph of N/Z Ratios of Elements** (2019/4/24).

![Fig. 5. Partially Amplified Graph of N/Z Ratios of Elements](image)
The general trend of $N/Z$ ratio of elements is from $1/1$ ($^6C_6$) to slightly above $3/2$ ($^{112}Cn_{173}$) definitely. However, the increasing process is not smooth, the $N/Z$ ratio rising fluctuates consecutively. According to Chen’s Chirality and Poetry Model of Atomic Nuclei, there are some stable numbers (magic numbers) which can bring about this kind of fluctuation (Table 4 and Fig. 6).

### Table 4. Effect of Stable Numbers on $N/Z$ ratio’s fluctuation (2019/4/22).

<table>
<thead>
<tr>
<th>Element</th>
<th>Z</th>
<th>N(Average)</th>
<th>$Z(3/2)^{1/2}$</th>
<th>$N-Z(3/2)^{1/2}$</th>
<th>Stable Number</th>
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<tbody>
<tr>
<td>K</td>
<td>19</td>
<td>20.13</td>
<td>23.27</td>
<td>-3.17</td>
<td>20</td>
</tr>
<tr>
<td>Ca</td>
<td>20</td>
<td>20.12</td>
<td>24.49</td>
<td>-4.41</td>
<td>20+20</td>
</tr>
<tr>
<td>Sc</td>
<td>21</td>
<td>24</td>
<td>25.72</td>
<td>-1.74</td>
<td></td>
</tr>
<tr>
<td>Ti</td>
<td>22</td>
<td>25.92</td>
<td>26.94</td>
<td>-1.07</td>
<td>22+26=48</td>
</tr>
<tr>
<td>V</td>
<td>23</td>
<td>28.00</td>
<td>28.17</td>
<td>-0.23</td>
<td>28</td>
</tr>
<tr>
<td>Cr</td>
<td>24</td>
<td>28.06</td>
<td>29.39</td>
<td>-1.39</td>
<td>28</td>
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<tr>
<td>Mn</td>
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<td>30</td>
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<td></td>
</tr>
<tr>
<td>Fe</td>
<td>26</td>
<td>29.91</td>
<td>31.84</td>
<td>-1.99</td>
<td>26+30=56</td>
</tr>
<tr>
<td>Co</td>
<td>27</td>
<td>32.00</td>
<td>33.07</td>
<td>-1.14</td>
<td></td>
</tr>
<tr>
<td>Ni</td>
<td>28</td>
<td>30.76</td>
<td>34.29</td>
<td>-3.60</td>
<td>28+30=58, 28+32=60</td>
</tr>
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<td><strong>Cu</strong></td>
<td><strong>29</strong></td>
<td><strong>34.62</strong></td>
<td><strong>35.52</strong></td>
<td><strong>-0.97</strong></td>
<td><strong>64</strong></td>
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<tr>
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<td>30</td>
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<td>36.74</td>
<td>-1.36</td>
<td>30+34=64, 30+36=66</td>
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<tr>
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<td>31</td>
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<td>37.97</td>
<td>0.75</td>
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<tr>
<td>Ge</td>
<td>32</td>
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<td>39.19</td>
<td>1.44</td>
<td>32+40=72</td>
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<tr>
<td>As</td>
<td>33</td>
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<td>47.89</td>
<td>44.09</td>
<td>3.71</td>
<td>36+48=84</td>
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</tbody>
</table>

**Fig. 6. Effect of Stable Numbers on $N/Z$ ratio’s fluctuation (2019/4/22-23)**

If there were no stability effect, the $N/Z$ ratio of Cu should be at the red spot.
13. \( \alpha_1/\alpha_2 \) in Schrödinger Equation of Hydrogen Atom

Stationary Schrödinger Equation \(-\frac{\hbar^2}{2m} \nabla^2 \psi + U \psi = E \psi\), applied to hydrogen atom:

\[
\nabla^2 \psi + \frac{2m}{\hbar^2} \left( E + \frac{e^2}{4\pi \varepsilon_0 r} \right) \psi = 0, \quad E = -\frac{m e^4}{2n^2(4\pi \varepsilon_0)^2 \hbar^2}, \text{ do substitution and simplification:}
\]

\[
\frac{2m}{\hbar^2} \left( \frac{m e^4}{2n^2(4\pi \varepsilon_0)^2 \hbar^2} - \frac{e^2}{4\pi \varepsilon_0 r} \right) \psi = \nabla^2 \psi, \quad \left[ \frac{1}{n^2} \left( \frac{m e^2}{4\pi \varepsilon_0 \hbar} \right)^2 - \frac{2}{r} \frac{m e^2}{4\pi \varepsilon_0 \hbar^2} \right] \psi = \nabla^2 \psi,
\]

As \( \sqrt{\alpha_1 \alpha_2} = \frac{v}{c} = \frac{e^2}{4\pi \varepsilon_0 \hbar c}, \lambda_e = \frac{h}{m_c} \) and \( \alpha_i = \frac{\lambda_e}{2\pi a_0} \):

\[
\left[ \frac{1}{n^2} \left( \frac{\alpha_i}{\alpha_2} \right)^2 - \frac{2}{r} \frac{\alpha_i}{\lambda_e} \right] \psi = \nabla^2 \psi,
\]

\[
\left[ \frac{1}{n^2} \alpha_i \left( \frac{\alpha_i}{\alpha_2} \right)^2 - \frac{2}{r} \frac{\alpha_i}{\lambda_e} \right] \psi = \nabla^2 \psi
\]

As \( \alpha_i / \alpha_2 \approx 1 \), simplified to: \( \left[ \frac{1}{n^2} \alpha_i^2 - \frac{2}{\alpha_0} \right] \psi = \nabla^2 \psi \) (factor 2 seems not beautiful)

In atomic units (\( au: e = m_e = \hbar = 1 \) and \( \varepsilon_0 = \frac{1}{4\pi} \)),

\[
a_{\text{au}} = \frac{4\pi \varepsilon_0 \hbar^2}{m_e e^2} = 1, \quad \nu = \frac{e^2}{4\pi \varepsilon_0 \hbar} = 1, \quad e_{\text{au}} = \frac{e}{\alpha_e} = \frac{1}{\alpha_e} = \frac{1}{\sqrt{\alpha_i \alpha_2}}
\]

\[
\left[ \frac{1}{n^2} \left( \frac{\alpha_i}{\alpha_2} \right) - \frac{2}{r_{\text{au}}} \frac{\alpha_i}{\alpha_2} \right] \psi = \nabla^2 \psi, \quad \text{or} \quad \left( \frac{e_{\text{au}}^2}{\alpha_i^2 n^2} - \frac{2 e_{\text{au}}}{\alpha_i} \right) \psi = \nabla^2 \psi
\]

the above equation could be called Schrödinger-Chen equation of hydrogen atom, the later form of the equation shows factor 2 is still reasonable and beautiful.

As \( \alpha_i / \alpha_2 \approx 1 \), simplified to: \( \left[ \frac{1}{n^2} - \frac{2}{r_{\text{au}}} \right] \psi = \nabla^2 \psi \)

Discover: 2018/4-6; Revise: 2019/12/13 (add \( au \) form)

\[
\alpha_i/\alpha_2 = \frac{137.035999111818}{137.035999037435} = 1.0000000005428 = 1 + \frac{23\cdot 59}{25 \cdot 10^{11}} = (1 + \frac{23\cdot 59}{50 \cdot 10^{11}})^2
\]

\[
\frac{\sqrt{\alpha_i/\alpha_2}}{1 + \frac{23\cdot 59}{50 \cdot 10^{11}}} = 1.0000000002714
\]

Relations to nuclides: \( ^{118}_{50}\text{Sn}, ^{114}_{50}\text{Te}, ^{109}_{44}\text{Re}, ^{105}_{50}\text{Pd}, ^{103}_{50}\text{Ru}, ^{137}_{56}\text{Ba}, ^{131}_{50}\text{Sn}, ^{137}_{56}\text{Ba} \)

2019/8/29

Solution of Schrödinger equation of hydrogen atom gives some quantum numbers such as \( n, l \) and \( m_\ell \) which determine the electron shell structure and the chemical
properties of atoms. That means Schrödinger equation of hydrogen atom is the base of chemical periodicity of elements. On the other hand, from above analysis, we have already demonstrated the formulas of the fine-structure constant $\alpha$ are derived from Chen’s Chirality and Poetry Model of Atomic Nuclei and hence mainly connected to the stability of atomic nuclei. So, a question is whether and how $\alpha$ is connected to Schrödinger Equation of hydrogen atom. This question should reveal the connection of the theory of electron shell of atoms and the theory of nuclei of elements. The above deduction provides the answer. The fine-structure constant $\alpha$ relates to Schrödinger Equation of hydrogen atom in $\alpha_1/\alpha_2$ way which is subtle and negligible but could show the equation is really reasonable and beautiful.

14. The Two Kinds of General Formulas of the Fine-structure Constant

Based on the above two formulas of $\alpha_1$ and $\alpha_2$, it should be reasonable to assume there are two kinds of serial formulas of $\alpha_1$ and $\alpha_2$ which are listed in follows. Among these formulas, the above two first discovered formulas are the most fundamental and important. Some formulas both with a big m and an extra large k should be more important referring to the trend of the approximate values of $\alpha$.

Approximate formulas:

$$\alpha_{1-m} = \frac{n}{m \cdot (2 \pi) k} \frac{1}{112} = \frac{1}{m \cdot e^2 \frac{e^2}{\left(2 \right)^3} \frac{e^2}{\left(3 \right)^2} \cdots \frac{e^2}{k^2 \cdot \left(k+1\right)^{2k+1}}} \approx \frac{1}{137.036}$$

$$\alpha_{2-m} = \frac{n}{m \cdot (2 \pi) k} \frac{1}{112} = \frac{1}{n \frac{1}{k^2 \cdot \left(k+1\right)^{2k+1}}} \approx \frac{1}{137.036}$$

Accurate Formulas:

$$\alpha_{1-m} = \frac{n}{m \cdot (2 \pi) k} \frac{1}{112 + \delta_1} = \frac{1}{m \cdot e^2 \frac{e^2}{\left(2 \right)^3} \frac{e^2}{\left(3 \right)^2} \cdots \frac{e^2}{k^2 \cdot \left(k+1\right)^{2k+1}}} \frac{1}{112 + \delta_1}$$

$$= \frac{1}{137.0359999037435}$$

$$\alpha_{2-m} = \frac{n}{m \cdot (2 \pi) k} \frac{1}{112 - \delta_2} = \frac{1}{n \frac{1}{k^2 \cdot \left(k+1\right)^{2k+1}}} \frac{1}{112 - \delta_2}$$

$$= \frac{1}{137.035999111818}$$

Discover: 2019/6/27; Revise: 2019/7/2-3
### Table 5. Parameters and Results of Approximate Formulas of $\alpha_1$ (2019/7/2).

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>k</th>
<th>$\alpha_{1-m'}$</th>
<th>m</th>
<th>n</th>
<th>k</th>
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**Fig. 7. Results of Approximate Formulas of $\alpha_1$ (2019/7/2).**

1. $\alpha_{1-7'}$ is the first most accurate and reasonable formula, so assume $\alpha_1=\alpha_{1-7}$.
2. The m and k are bigger, the more accurate the $\alpha_{1-m'}$ is. So there should be a big m=M make $\alpha_{1-M'}$ is much close to the real $\alpha_1$. Here M is assumed to be 133 or 170.
\[ \alpha_{1,1} = \frac{6}{1 - e^2} = \frac{1}{137.035999037435} \]

\[ \alpha_{1,2} = \frac{11}{2} \frac{2}{2} \frac{2}{2} = \frac{1}{137.035999037435} \]

\[ \alpha_{1,3} = \frac{2^2 \cdot 2^2 \cdot 2^2}{2^5} = \frac{1}{137.035999037435} \]

\[ \alpha_{1,4} = \frac{26}{67} \frac{1}{14 - 9.71 + \frac{1}{8}} = \frac{1}{137.035999037435} \]

\[ \alpha_{1,5} = \frac{31}{31} \frac{28}{3} \frac{28}{27} = \frac{1}{137.035999037435} \]

\[ \alpha_{1,6} = \frac{62}{62} \frac{2}{2} \frac{2}{2} = \frac{1}{137.035999037435} \]

\[ \alpha_{1,7} = \frac{1}{7} \frac{2}{2} \frac{2}{2} = \frac{1}{137.035999037435} \]

\[ \alpha_{1,8} = \frac{1}{8} \frac{2}{2} \frac{2}{2} = \frac{1}{137.035999037435} \]
\[ \alpha_{1-9} = \frac{47}{3^2 \cdot e^2 \cdot e^2 \cdots e^2} \cdot \frac{2}{\left( \frac{3^2}{2} \right)^5} \cdot \frac{1}{112 + \frac{1}{4 \cdot 17 - \frac{1}{2(8 \cdot 9 \cdot 37 - 1) + \frac{3 \cdot 17}{157}}} = 1/137.035999037436 \]

\[ \alpha_{1-11} = \frac{3 \cdot 19}{11 \cdot e^2 \cdot e^2 \cdots e^2} \cdot \frac{2}{\left( \frac{3^2}{2} \right)^5} \cdot \frac{1}{112 + \frac{1}{35 - \frac{88 \cdot 41 - 5 \cdot 53}{2 \cdot 22 - 13}}} = 1/137.035999037435 \]

\[ \alpha_{1-13} = \frac{13 \cdot e^2 \cdot e^2 \cdots e^2}{1 \cdot \left( \frac{3^2}{2} \right)^5} \cdot \frac{47}{112 + \frac{1}{137 - \frac{6 \cdot (2 \cdot 27 - 59) + 1}{950}}} = 1/137.035999037435 \]

\[ \alpha_{1-16} = \frac{4 \cdot e^2 \cdot e^2 \cdots e^2}{3^2 \cdot \left( \frac{3^2}{2} \right)^5} \cdot \frac{17}{112 + \frac{28}{-18 \cdot (141 + 1) + 173}} = 2 \cdot (2 \cdot 75 - 1) \]

\[ \alpha_{1-17} = \frac{2 \cdot 22}{17 \cdot e^2 \cdot e^2 \cdots e^2} \cdot \frac{21}{112 + \frac{1}{137 - \frac{2 \cdot 19 \cdot 23 \cdot 59 - 30}{100}}} = 1/137.035999037435 \]

\[ \alpha_{1-19} = \frac{2 \cdot 7^2}{19 \cdot e^2 \cdot e^2 \cdots e^2} \cdot \frac{38}{112 + \frac{1}{137 - \frac{2 \cdot (8 \cdot 54 - 1) + 54}{19^2}}} = 1/137.035999037440 \]

\[ \alpha_{1-20} = \frac{2 \cdot 5 \cdot e^2 \cdot e^2 \cdots e^2}{1 \cdot \left( \frac{3^2}{2} \right)^5} \cdot \frac{59}{112 + \frac{1}{32 \cdot 45 - 79 + \frac{22}{3 \cdot 17}}} = 1/137.035999037435 \]
\[ \alpha_{1,22} = \frac{22 \cdot e^2}{2} \frac{e^2}{2} \cdots \frac{e^2}{2} \frac{27 \cdot 29}{2 \cdot 17 \cdot 23} \frac{1}{5(2157-1)} \frac{1}{112} + \frac{1}{2 \cdot [2 \cdot 3 \cdot 17 \cdot (10 \cdot 19 + 1) + 1]} + \frac{29}{49} = 1/137.035999037435 \]

\[ \alpha_{1,23} = \frac{23 \cdot e^2}{2} \frac{e^2}{2} \cdots \frac{e^2}{2} \frac{1}{35} = 1/137.035999037435 \]

\[ \alpha_{1,25} = \frac{5^2 \cdot e^2}{2} \frac{e^2}{2} \cdots \frac{e^2}{2} \frac{4 \cdot 13 \cdot 43}{2 \cdot 17 \cdot 1} + \frac{1}{123(16-17-1) + 1/25} = 1/137.035999037435 \]

\[ \alpha_{1,27} = \frac{27 \cdot e^2}{2} \frac{e^2}{2} \cdots \frac{e^2}{2} \frac{1}{114 + 1/19 + 18/23 + 1/23} = 1/137.035999037435 \]

\[ \alpha_{1,29} = \frac{29 \cdot e^2}{2} \frac{e^2}{2} \cdots \frac{e^2}{2} \frac{1}{112} + \frac{1}{6 \cdot 8 \cdot (12 \cdot 26 - 1) + 11/18} = 1/137.035999037434 \]

\[ \alpha_{1,31} = \frac{31 \cdot e^2}{2} \frac{e^2}{2} \cdots \frac{e^2}{2} \frac{1}{112} + \frac{1}{12 \cdot 11 \cdot 7 \cdot 4.49 + 5.41} = 1/137.035999037434 \]

\[ \alpha_{1,32} = \frac{1}{15} = 1/137.035999037435 \]

\[ \alpha_{1,33} = \frac{23 \cdot e^2}{2} \frac{e^2}{2} \cdots \frac{e^2}{2} \frac{1}{112} + \frac{1}{25 \cdot 29 - 5 \cdot 83 + 19 \cdot 23} = 1/137.035999037435 \]
$$\alpha_{1,33} = \frac{1}{137.035999037435}$$

$$\alpha_{1,34} = \frac{1}{137.035999037435}$$

$$\alpha_{1,36} = \frac{1}{137.035999037435}$$

$$\alpha_{1,43} = \frac{1}{137.035999037435}$$

$$\alpha_{1,50} = \frac{1}{137.035999037435}$$
\[ \alpha_{1-96} = \frac{17 \cdot 29 \left( \frac{3}{2} \right)^3 \left( \frac{17}{1} \right)^2}{2 \cdot 7 \cdot 5 \cdot 43 \cdot 1} = 1 + \frac{163 \left( 8 \cdot 21 \cdot 37 + 1 \right)}{50 \cdot 10^{11}} \]

\[ = \frac{1}{137.035999037435} \]

\[ \alpha_{1-103} = \frac{103 \cdot e^2 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2}{6 \cdot (12 \cdot (8 \cdot (67 + 1) + 1) + \frac{3}{4})} = 1 + \frac{1}{137.035999037435} \]

\[ \alpha_{1-138} = \frac{683}{133 \cdot (2\pi)} = \frac{1}{112 + \frac{14651}{50 \cdot 10^{11}}} = \frac{1}{7 \cdot 19 \cdot e^2 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2} = \frac{1}{112 + \frac{59 \cdot 210}{(13 \cdot (8 \cdot 7 \cdot 17 + 1))^{71(12 \cdot 29 + 1)}}} \]

\[ = \frac{1}{137.035999037435} \]

\[ \alpha_{1-140} = \frac{6 \cdot 20 \cdot 1 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2}{140 \cdot e^2 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2} = \frac{1}{4 \cdot 9 \cdot (2 \cdot 3 \cdot 29 \cdot 47 + 1) + \frac{29}{54}} \]

\[ = \frac{1}{137.035999037435} \]

\[ \alpha_{1-155} = \frac{5 \cdot 31 \cdot e^2 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2}{100 \cdot e^2 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2} = \frac{1}{19 \cdot 210 \cdot e^2 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2} = \frac{1}{112 + \frac{5 \cdot 17 \cdot 31 \cdot (2 \cdot 13 \cdot 17 + 1) - \frac{15}{43}}{19777} \cdot 10^{11}} \]

\[ = \frac{1}{137.035999037435} \]

\[ \alpha_{1-170} = \frac{873}{170 \cdot e^2 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2} = \frac{1}{3 \cdot 97 \cdot 8 \times 10^{11}} = \frac{1}{137.035999037435} \]

\[ \alpha_{1-178} = \frac{170 \cdot e^2 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2}{170 \cdot e^2 \left( \frac{3}{2} \right)^3 \left( \frac{1}{2} \right)^2} = \frac{1}{43 \cdot 97 \cdot 8 \times 10^{11}} = \frac{1}{137.035999037435} \]
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**Table 6. Parameters and Results of Approximate Formulas of \( \alpha_2 \) (2019/7/3).**

**Fig. 8. Results of Approximate Formulas of \( \alpha_2 \) (2019/7/3).**

3. \( \alpha_{2:13} \) is the first most accurate and reasonable formula, so assume \( \alpha_2 = \alpha_{2:13} \).

4. The m and k are bigger, the more accurate the \( \alpha_{2-m'} \) is. So there should be a big m=M make \( \alpha_{2-M} \) is much close to the real \( \alpha_2 \). Here M is assumed to be 253 or 269.
\[ \alpha_{2-1} = \frac{e^2}{1} \left( \frac{e^2}{2} \right)^3 \left( \frac{e^2}{3} \right)^5 \left( \frac{e^2}{4} \right)^7 = \frac{1}{112 - 1 + \frac{1}{3} + \frac{1}{16} + 41 \cdot (12 - 13 + 1) + \frac{13}{41}} = \frac{1}{1/137.035999111816} \]

\[ \alpha_{2-2} = \frac{2^2 \cdot e^2}{e^2} \left( \frac{2^2 \cdot e^2}{e^2} \right)^3 \left( \frac{2^2 \cdot e^2}{e^2} \right)^5 \left( \frac{2^2 \cdot e^2}{e^2} \right)^7 = \frac{1}{112 - 1 + \frac{1}{3} + \frac{1}{16} + 41 \cdot (12 - 13 + 1) + \frac{13}{41}} = \frac{1}{1/137.035999111816} \]

\[ \alpha_{2-3} = \frac{2^3 \cdot e^2}{e^2} \left( \frac{2^3 \cdot e^2}{e^2} \right)^3 \left( \frac{2^3 \cdot e^2}{e^2} \right)^5 \left( \frac{2^3 \cdot e^2}{e^2} \right)^7 = \frac{1}{112 - 1 + \frac{1}{3} + \frac{1}{16} + 41 \cdot (12 - 13 + 1) + \frac{13}{41}} = \frac{1}{1/137.035999111816} \]

\[ \alpha_{2-4} = \frac{2^4 \cdot e^2}{e^2} \left( \frac{2^4 \cdot e^2}{e^2} \right)^3 \left( \frac{2^4 \cdot e^2}{e^2} \right)^5 \left( \frac{2^4 \cdot e^2}{e^2} \right)^7 = \frac{1}{112 - 1 + \frac{1}{3} + \frac{1}{16} + 41 \cdot (12 - 13 + 1) + \frac{13}{41}} = \frac{1}{1/137.035999111816} \]
\[
\alpha_{24} = 19 e^2 \frac{e^2}{\left(\frac{2}{3}\right)^2} \frac{e^2}{\left(\frac{2}{3}\right)^4} \cdots \frac{e^2}{\left(\frac{2}{3}\right)^{15}} = \frac{1}{137.035999111818}
\]

\[
\alpha_{223} = 23 \left(\frac{e^2}{\left(\frac{2}{3}\right)^2} \frac{e^2}{\left(\frac{2}{3}\right)^4} \cdots \frac{e^2}{\left(\frac{2}{3}\right)^{15}} \right) = \frac{1}{137.035999111818}
\]

\[
\alpha_{224} = 5 \cdot 37 \left(\frac{e^2}{\left(\frac{2}{3}\right)^2} \frac{e^2}{\left(\frac{2}{3}\right)^4} \cdots \frac{e^2}{\left(\frac{2}{3}\right)^{15}} \right) = \frac{1}{137.035999111818}
\]

\[
\alpha_{225} = \frac{193}{2 \cdot (4 \cdot 23 - 1)} + \frac{1}{32 \cdot 10} = \frac{1}{137.035999111818}
\]

\[
\alpha_{227} = 4 \cdot 13 \left(\frac{e^2}{\left(\frac{2}{3}\right)^2} \frac{e^2}{\left(\frac{2}{3}\right)^4} \cdots \frac{e^2}{\left(\frac{2}{3}\right)^{15}} \right) = \frac{1}{137.035999111818}
\]

\[
\alpha_{229} = 223 \left(\frac{e^2}{\left(\frac{2}{3}\right)^2} \frac{e^2}{\left(\frac{2}{3}\right)^4} \cdots \frac{e^2}{\left(\frac{2}{3}\right)^{15}} \right) = \frac{1}{137.035999111818}
\]
\[ \alpha_{2-31} = \frac{31 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2}{(\frac{1}{2})^3 \cdot (\frac{1}{2})^3} \cdot \frac{5\gamma}{58} = 1/137.035999111819 \]

\[ \alpha_{2-32} = \frac{2 \cdot 4^2 \cdot e^2 \cdot e^2 \cdot e^2}{(\frac{1}{2})^3 \cdot (\frac{1}{2})^3} \cdot \frac{42}{41} = 1/137.035999111818 \]

\[ \alpha_{2-33} = \frac{33 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2}{(\frac{1}{2})^3 \cdot (\frac{1}{2})^3} \cdot \frac{139}{138} = 1/137.035999111818 \]

\[ \alpha_{2-36} = \frac{6^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2}{(\frac{1}{2})^3 \cdot (\frac{1}{2})^3} \cdot \frac{191}{190} = 1/137.035999111818 \]

\[ \alpha_{2-37} = \frac{37 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2}{(\frac{1}{2})^3 \cdot (\frac{1}{2})^3} \cdot \frac{2 \cdot 43}{5 \cdot 17} = 1/137.035999111818 \]
\[ \alpha_{-125} = \left( \frac{5 \cdot 5^2 \cdot \epsilon^2 \langle \frac{3}{2} \rangle^3 \langle \frac{3}{2} \rangle^3 \cdots \langle \frac{3}{2} \rangle^3}{31^2} \right) \frac{4294}{4293} \times 1 \]

\[ \alpha_{-253} = \left( \frac{5 \cdot 5^2 \cdot \epsilon^2 \langle \frac{3}{2} \rangle^3 \langle \frac{3}{2} \rangle^3 \cdots \langle \frac{3}{2} \rangle^3}{31^2} \right) \frac{28187}{28186} \times 1 \]

\[ \alpha_{-269} = \left( \frac{5 \cdot 5^2 \cdot (2\Pi) \langle \frac{3}{2} \rangle^3 \langle \frac{3}{2} \rangle^3 \cdots \langle \frac{3}{2} \rangle^3}{41655} \right) \frac{41654}{310309} \times 1 \]

\[ \alpha_{-279} = \left( \frac{5 \cdot 5^2 \cdot (2\Pi) \langle \frac{3}{2} \rangle^3 \langle \frac{3}{2} \rangle^3 \cdots \langle \frac{3}{2} \rangle^3}{41655} \right) \frac{41654}{310309} \times 1 \]

\[ \alpha_{-289} = \left( \frac{5 \cdot 5^2 \cdot (2\Pi) \langle \frac{3}{2} \rangle^3 \langle \frac{3}{2} \rangle^3 \cdots \langle \frac{3}{2} \rangle^3}{41655} \right) \frac{41654}{310309} \times 1 \]

In above formulas, there are many amazing coincidences. As 136=8×17 and 138=6×23, 17 and 23 both appear in \( \alpha_{1-1} \), \( \alpha_{1-17} \), \( \alpha_{1-22} \), \( \alpha_{1-23} \), \( \alpha_{1-25} \), \( \alpha_{1-59} \), \( \alpha_{1-103} \), \( \alpha_{1-133} \), \( \alpha_{2-17} \) and \( \alpha_{2-23} \), frequently appears in \( \alpha \) and 23 frequently appears in \( \alpha \), 157 and 257 in \( \alpha_{1-50} \) should relate to \( ^{100}\text{Fm}^{*} \), \( ^{173} \text{in} \alpha_{1-16} \) should relate to \( ^{112}\text{Cn}^{*} \), and so on. As the factors in formulas of \( \alpha \) are reasonably assumed to relate to nuclides, some ideal extended elements such as \( ^{156,137,138}\text{Fy}^{208,209,210} \) and \( ^{169}\text{Ch}^{257} \) are predicted.

**15. Radius of Electron and Proton**

The classical electron radius \( r_p \) has been calculated very accurately. However, the proton charge radius \( r_p \) hasn’t yet been determined precisely. Recent two experiments
measured \( r_p \) and had given the best results up to now which was \( r_p = 0.833(19) \) fm\(^9\) and \( r_p = 0.831(19) \) fm\(^{10}\), and hence CODADA revised its recommended data of \( r_p \) to 0.8414(19) fm. Here we give our calculation results of \( r_e \) and \( r_p \). And it seems there is \( a_p \) similar to \( \alpha \). \( a_p \) could be called “the second fine-structure constant”.

Ratio of Bohr radius of hydrogen atom to classical electron radius:

\[
a_p = \frac{r_e}{\alpha} = \frac{1}{\alpha \alpha_s} = \frac{1}{112 \times (168 - \frac{1}{3} + \frac{1}{2 \cdot 3 \cdot 47} - \frac{1}{2 \cdot 2 \cdot 29 \cdot 53 \cdot 59} - 79 / 47)} = 18788.865042381
\]

\[
r_e = a_s^2 a_p = r_e a_s = \frac{5.29177210903(80) \times 10^{-11} \text{m}}{18788.865042381} = 2.81794032658(43) \text{ fm}
\]

Comparable to CODATA recommended value \( r_e = 2.8179403262(13) \text{ fm} \) but more precise.

Ratio of Bohr radius of hydrogen atom to the proton charge radius should have the similar form, and is assumed to have the following hypothetical formulas:

\[
a_p \approx \frac{r_e}{a_p} \approx c p p \approx 252^2 \cdot \frac{1}{\alpha \alpha_s} \approx \frac{1}{247} \cdot \frac{1}{30 \cdot (2 \cdot 100 - 1) + \frac{8}{45}} = 252.040872632515^2
\]

\[
r_e = a_s^2 a_p \approx \frac{5.29177210903(80) \times 10^{-11} \text{m}}{63524.60147736} = 0.833027202999(13) \text{ fm}
\]

\( \alpha_p \approx \frac{r_e}{a_p} \approx 252.040872632515^2 \) could be called the second fine-structure constant.

2019/12/19-23

\[
\begin{align*}
\alpha_{p1} & = \frac{5 \cdot 3.3^3}{8 \cdot (2 \pi)^3} \times 1 \times 225 + \frac{1}{4 \cdot 112} = 1 = 1/252.040872632515 \\
\alpha_{p2} & = \frac{23 \cdot 2^2}{2 \cdot 9^2} \times 1 \times 225 = \frac{3 \cdot 16 \cdot 29}{71} = 1/252.040872632514
\end{align*}
\]
\[ \alpha'_{P2} = \frac{22 \cdot (-2\pi)_{164}}{5 \cdot 31} = \frac{1}{225 - \frac{1}{7 \cdot 137}} = \frac{1}{125 \cdot 203 - \frac{1}{8 + 17}} = 1/252.00487623512 \]

\[ \alpha'_{P2} = \frac{21 \cdot (-2\pi)_{126}}{2 \cdot 37} = \frac{1}{45 - \frac{1}{16 - 29 + \frac{1}{20 - 13^2 + 179 + \frac{1}{8/17}}} = 1/252.00487623512 \]

\[ 2\pi = 6.2831853 \cdots \approx \frac{4 \cdot 157}{100} = \frac{157}{25} = 6.28 \equiv \frac{3 \cdot 7 \cdot 44 + 68}{100^2} = \frac{68.323}{10^2} \]

\[ 2\pi \approx \frac{4 \cdot 157}{100} = \frac{157}{25} = 6.28 \quad \frac{55\text{ Mn}}{100} \quad \frac{100\text{ Ru}}{66} \quad \text{Gd} \quad \frac{118\text{ Sn}}{139} \quad \frac{118\text{ Ta}}{120} \quad \frac{118\text{ Mo}}{120} \quad \frac{118\text{ Cd}}{120} \]

\[ 2\pi \approx \frac{16 \cdot 3 \cdot 7 \cdot 11 \cdot 17}{98 \cdot 44 \cdot 56} \approx \frac{3 \cdot 112 - 11 \cdot 17}{186 \cdot 4} \approx \frac{2 \cdot 168 \cdot 11 \cdot 17}{131 \cdot 136} \]

\[ 2\pi \approx \frac{4 \cdot 4}{7} = \frac{6.2857 \cdots}{8.888} = \frac{18 \cdot 30}{4 \cdot 44} \approx \frac{487 \cdot 30}{64 \cdot 44} = 12.18 \]

\[ 2\pi \approx \frac{201}{32} = 6.2812 \cdots \approx \frac{18 \cdot 12}{4 \cdot 44} \approx \frac{487 \cdot 12}{64 \cdot 44} = 32.17 \]

\[ 2\pi \approx \frac{245}{39} = 6.2820 \cdots \approx \frac{3 \cdot 17 \cdot 2}{4 \cdot 46} = 1.39 \]

\[ 2\pi \approx \frac{289}{46} = \frac{6.2862 \cdots}{2 \cdot 23} = \frac{4 \cdot 23}{5 \cdot 46} = 0.93 \]

\[ 2\pi \approx \frac{333}{53} = 6.2830 \cdots \approx \frac{9 \cdot 37}{4 \cdot 53} = \frac{37 \cdot 53}{4 \cdot 37} = 3.1 \]

\[ 2\pi \approx \frac{377}{60} = 6.2833 \cdots \approx \frac{4 \cdot 35}{3 \cdot 60} = \frac{24 \cdot 35}{3 \cdot 60} = 1.3 \]

\[ 2\pi \approx \frac{465}{74} = 6.2837 \cdots \approx \frac{3 \cdot 31}{4 \cdot 74} = 0.9 \]

\[ 2\pi \approx \frac{509}{81} = 6.2839 \cdots \approx \frac{3 \cdot 59}{4 \cdot 81} = 0.9 \]

\[ 2\pi \approx \frac{622}{99} = 6.2838 \cdots \approx \frac{3 \cdot 62}{4 \cdot 99} = 0.9 \]

\[ 2\pi \approx \frac{2 \cdot 3 \cdot 5 \cdot 17 - 1}{9^2} = 6.2838 \cdots \approx \frac{9 \cdot 31}{4 \cdot 81} = 1.3 \]

16. Direct Relationships between 2\pi and Nuclides

In Chen's formulas of the fine-structure constant, there are 2\pi-e formulas, in which k gets certain numbers and relate to nucleon numbers of some nuclides. So in the end of this paper we feel curious about whether 2\pi directly relate to nuclides.
The approximate rational numbers of $2\pi$ (could be called $2\pi$ formulas) relate to nuclides marvelously. This means $2\pi$ (along with $2\pi$-e formula) plays important roles in atomic nuclei, and acts as a rational number rather than an irrational number in the world of atomic nuclei.

17. Correlations among $\alpha$, $2\pi$ and nuclides

Some Chen’s formulas of the fine-structure constant and $2\pi$ formulas correlate with each others with the same factors and all together relate to the same nuclides. For example, $\alpha_{1.50}$ and $2\pi \approx 4 \times 157/100$ have the same 157 and 100 factors, $\alpha_{1.50}$ and $2\pi \approx 3 \times 7 \times 44 \times 68/100^2$ have the same 100, 7, 11 and 16 factors, and they relate to the same corresponding nuclides. They also have common factors with $\alpha_{1,7}$ and $\alpha_{2,13}$ which should relate to $2\pi \approx 5 \times 7^2/3/13$ and $2\pi \approx 13 \times 29/4/3/5$.

$$\alpha_{1,9} = \frac{47}{3^2 \cdot e^2 \cdot e^2 \cdot \ldots \cdot e^2} \frac{1}{112 + \frac{1}{4 \cdot 17}} = \frac{1}{2(8 \cdot 9 \cdot 37 - 1) + \frac{3 \cdot 17}{157}}$$

$$\alpha_{1,50} = \frac{2.257}{100 \cdot e^2 \cdot e^2 \cdot \ldots \cdot e^2} \frac{1}{112 + \frac{1}{29 \cdot 61 + \frac{157}{16.11}}}$$

$$2\pi \approx \frac{4 \cdot 157}{100} = \frac{157}{25} = 6.28$$

$$\alpha_{1,7} = \frac{6^2}{7 \cdot e^2 \cdot e^2 \cdot \ldots \cdot e^2} \frac{1}{112 + \frac{1}{75^2}}$$

$$\alpha_{2,13} = \frac{13 \cdot e^2 \cdot e^2 \cdot \ldots \cdot e^2}{1 \cdot \left(\frac{279}{278}\right)^{157}} \frac{1}{10^2} \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}}$$

$$2\pi \approx \frac{245}{39} = \frac{5 \cdot 7^2}{3 \cdot 13} = 6.2820\ldots$$

$$2\pi \approx \frac{377}{60} = \frac{13 \cdot 29}{4 \cdot 3.5} = 6.2833\ldots$$

$\alpha_{1,22}$ relates to $2\pi \approx 2 \times 22/7$, $2\pi \approx 17^2/7/23$ and $2\pi \approx 2 \times 355/113$ as follows. And $2\pi \approx 17^2/7/23$ also relates to $\alpha_{1,1}$, $\alpha_{1,17}$, $\alpha_{1,22}$, $\alpha_{1,23}$, $\alpha_{1,25}$, $\alpha_{1,59}$, $\alpha_{1,103}$, $\alpha_{1,133}$, $\alpha_{2,17}$ and $\alpha_{2,23}$, in which both 17 and 23 factors appear.

$$\alpha_{1,22} = \frac{113}{22 \cdot e^2 \cdot e^2 \cdot \ldots \cdot e^2} \frac{1}{112 + \frac{1}{\left(\frac{2779}{278}\right)^{157}}} \frac{1}{2(2 \cdot 3 \cdot 17 \cdot (10 \cdot 19 + 1) + 1) + \frac{29}{49}}$$

$$2\pi \approx \frac{2 \cdot 22}{7} = 6.2857\ldots$$
\( \alpha_{1-13} \) and \( \alpha_{1-43} \) relate to \( 2\pi \approx 3 \times 67/32, 2\pi \approx 5 \times 7^2/39, 2\pi \approx 17^2/46 \) and others as follows.

\[
\begin{align*}
\alpha_{1-13} &= \frac{e^2}{13} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \ldots \left( \frac{3}{2} \right)^{\frac{3}{2}} \frac{e^2}{112 + \frac{1}{137} \left( 6(2.27 \cdot 59 + 1) + \frac{9}{50} \right)} \\
\alpha_{1-43} &= \frac{e^2}{13-17} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \ldots \left( \frac{3}{2} \right)^{\frac{3}{2}} \frac{e^2}{112 + \frac{1}{8 \cdot (12 \cdot 83 + 1) + \frac{4}{3 \cdot 13}}} \\
\end{align*}
\]

\( 2\pi \approx \frac{3 \cdot 67}{32}, 2\pi \approx \frac{5 \cdot 7^2}{3 \cdot 13}, 2\pi \approx \frac{289}{46} = \frac{17^2}{2 \cdot 23}, 2\pi \approx \frac{13 \cdot 39}{60}, 2\pi \approx \frac{30 \cdot 31}{4 \cdot 37} \)

\( \alpha_{1-11}, \alpha_{1-36}, \alpha_{2-24}, \alpha_{2-23}, \alpha_{2-37} \) and \( \alpha_{2-125} \) relate to \( 2\pi \approx 9 \times 37/53, 2\pi \approx 15 \times 31/2/37 \) and \( 2\pi \approx (30 \times 17-1)/81 \) as follows.

\[
\begin{align*}
\alpha_{2-11} &= \frac{e^2}{11} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \ldots \left( \frac{3}{2} \right)^{\frac{3}{2}} \frac{e^2}{57} \left( \frac{19}{18} \right)^{37} \frac{1}{112 + \frac{1}{35} \left( 88 \cdot 41 \right) - \frac{5 \cdot 53}{22 \cdot 13}} \\
\alpha_{2-36} &= \frac{e^2}{6} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \ldots \left( \frac{3}{2} \right)^{\frac{3}{2}} \frac{e^2}{63} \left( \frac{2}{31} \right)^{125} \frac{1}{112 + \frac{1}{5 \cdot (31 \cdot 42 - 1) + \frac{3 \cdot 31}{14 \cdot 13}}} \\
\alpha_{2-24} &= \frac{e^2}{5 \cdot 37} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \frac{e^2}{112 - \frac{1}{257} + \frac{1}{10 \cdot (12 \cdot 13 \cdot 83 + 1) + \frac{23}{81}}} \\
\alpha_{2-23} &= \frac{e^2}{3 \cdot 59} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \frac{e^2}{2 \cdot (40 \cdot 23 - 1) + \frac{9}{32 \cdot 10}} \\
\alpha_{2-37} &= \frac{e^2}{3.5 \cdot 19} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \frac{e^2}{2 \cdot 81 \cdot 7 \cdot 23} \frac{1}{112 - \frac{1}{4 \cdot 3 \cdot 5 \cdot 13} + \frac{1}{5 \cdot 37 \cdot 1 \cdot 49}} \\
\alpha_{2-125} &= \frac{e^2}{5 \cdot 5 \cdot 23} \left( \frac{3}{2} \right)^{\frac{3}{2}} \left( \frac{3}{2} \right)^{\frac{3}{2}} \ldots \left( \frac{3}{2} \right)^{\frac{3}{2}} \frac{e^2}{81 \cdot 53 \cdot 31 \cdot (12 \cdot 23 - 1)} = \frac{1}{112 - \frac{1}{101 \cdot (20 \cdot (12 \cdot 89 + 1) + 1)}} \\
2\pi \approx \frac{9 \cdot 37}{53} = 6.2830 \ldots, 2\pi = \frac{15 \cdot 31}{2 \cdot 37} = 6.2837 \ldots, 2\pi \approx \frac{30 \cdot 17 - 1}{81} = 6.2839 \ldots
\end{align*}
\]
18. Chen’s Mathematic Shell Model of Nuclides

In overall, there are multi-correlations among $\alpha$, $2\pi$ and nuclides. It seems there should be a mathematical shell model of nuclides, in which the core is $2\pi$ formulas and the middle layer is $2\pi$-e formulas and the outer layer is Chen’s formulas of $\alpha$ (Fig. 9, $\phi$ is explained in Section 21). The nucleon numbers, stability and abundance of nuclides are regulated by these formulas, especially by their integer factors.

![Chen's Mathematic Shell Model of Nuclides](image)

Dr. Gang Chen (2020/1/12-13, 3/1)

19. Ideal Extended Elements

In the deduction of Chen’s formulas of the fine-structure constant, it was reasonably assumed the factors in them related to nucleon numbers of nuclides, and it seems this assumption is quite correct. So by somewhat correlation and decoding methodology, all 119th to 170th ideal extended elements were predicted (Table 7). In addition, nuclides can even relate to naked $2\pi$’s approximate rational numbers ($2\pi$ formulas). Some typical examples of correlations of ideal extended elements with formulas of $\alpha$ and $2\pi$ are listed as follows.

Example 1: Correlations of 100, 121,125,126,157, 257, 169, et al.

$$
\alpha_{1-9} = \frac{47}{3^3 \cdot e^1 \cdot e^3 \cdot e^7 \cdots e^{19}} \left( \frac{\pi}{9} \right)^{112+1} \frac{1}{4 \cdot 17} \frac{1}{2(8 \cdot 9 \cdot 37 - 1) + \frac{3 \cdot 17}{157}}
$$

$$
\alpha_{1-50} = \frac{2 \cdot 257}{100 \cdot e^1 \cdot e^3 \cdot e^7 \cdots e^{181}} \left( \frac{\pi}{181} \right)^{112+1} \frac{1}{29 \cdot 61 - \frac{157}{16 \cdot 11}}
$$

$$
\alpha_{2-24} = \frac{5 \cdot 37}{2^2 \cdot 6 \cdot e^1 \cdot e^3 \cdot e^7 \cdots e^{128}} \left( \frac{\pi}{62} \right)^{112+1} \frac{1}{257 \cdot 5 \cdot \frac{1}{10 \cdot (12 \cdot 13 \cdot 83 + 1) + \frac{23}{81}}}
$$

$$
2\pi \approx \frac{4 \cdot 157}{100} \quad 2\pi \approx \frac{16 \cdot 3 \cdot 7 \cdot 11 \cdot 17}{100} \quad 2\pi \approx \frac{4 \cdot 11}{7} \quad 2\pi \approx \frac{17 \cdot 2}{23} \quad 2\pi \approx \frac{3 \cdot 31}{4 \cdot 37} \quad 2\pi \approx \frac{4 \cdot 5 \cdot 71}{2 \cdot 113}
$$

$^{106}Ru, ^{168}Te, ^{169}Te, ^{180}Pt, ^{257}Mo, ^{120}Sn, ^{32}Ch^{\alpha}, ^{244}Cm^{\alpha}, ^{125}Sb^{\alpha}, ^{127}Sb^{\alpha}, ^{169}Ch^{\alpha}, ^{404}Ch^{\alpha}$
Example 2: Correlations of 83, 126, 84, 125, 209, 112, 173, 285, 115 and 137

\[ \alpha_{1-16} = \frac{83}{4^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2} \]

\[ \alpha_{1-25} = \frac{1}{3 \cdot 43} \]

\[ \alpha_{1-32} = \frac{1}{15 \cdot 11} \]

\[ \alpha_{2-30} = \frac{1}{77} \]

\[ \alpha_{2-32} = \frac{1}{13 \cdot 19} \]

\[ \alpha_{2-31} = \frac{1}{2^2 \cdot 22} \]

\[ \alpha_{2-32} = \frac{1}{139} \]

\[ \alpha_{2-31} = \frac{1}{139} \]

\[ \alpha_{2-32} = \frac{1}{139} \]

\[ \frac{2 \pi}{10} \approx \frac{16 \cdot 3 \cdot 7 - 11 - 17}{100} = \frac{411}{7} \approx \frac{17^2}{2 \cdot 23} \]

**Table 7.** Correlations of Ideal Extended Elements (IEE) with Formulas of \( \alpha \) and \( 2\pi. \)

<table>
<thead>
<tr>
<th>IEE</th>
<th>Page</th>
<th>( \alpha )</th>
<th>( 2\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>113NH</td>
<td>10 19 21 28 29 31</td>
<td>( \alpha^2 \cdot \alpha_{1-5,7} \cdot \alpha_{2-2,22,23,31,37,38,253} )</td>
<td>( 2\pi \approx 4 \times 355/226 )</td>
</tr>
<tr>
<td>114Fl</td>
<td>19 23 28 31</td>
<td>( \alpha_{1-11,133,155} \cdot \alpha_{2-37,38} )</td>
<td>( 2\pi \approx 17/46 )</td>
</tr>
<tr>
<td>115Mc</td>
<td>20 21 25 31</td>
<td>( \alpha_{1-1,16,23} \cdot \alpha_{2-5} )</td>
<td>( 2\pi \approx 17/46 )</td>
</tr>
<tr>
<td>116Lv177, 117Ts177</td>
<td>10 20 22 27 31</td>
<td>( \alpha_{1,13,59} \cdot \alpha_{2-3,23} / \alpha^2 )</td>
<td>( 2\pi \approx 622/99 )</td>
</tr>
<tr>
<td>118Og</td>
<td>20 22 23 27</td>
<td>( \alpha_{1-17,20,50,59,133} \cdot \alpha_{2-19,269} )</td>
<td>( 2\pi \approx 44/7 )</td>
</tr>
<tr>
<td>119-122Ch179-182</td>
<td>21-23 28 31 37 39 44</td>
<td>( \alpha_{1-23,29,50,170} \cdot \alpha_{3,37} \cdot \alpha_{p/2} \cdot \alpha_{c_{uu}} )</td>
<td>( 2\pi \approx 44/7 ) et al.</td>
</tr>
<tr>
<td>123Ch183/185</td>
<td>19 20 21 25 28</td>
<td>( \alpha_{1-4,11,16,17,31} \cdot \alpha_{2-1,4,5,9,32} )</td>
<td>( 2\pi \approx 333/53 \approx 465/74 ) et al.</td>
</tr>
</tbody>
</table>
Chen’s Picture of Elements and Ideal Extended Elements

20. Chen’s Picture of Elements and Ideal Extended Elements

Chen’s Picture of Elements and Ideal Extended Elements
Dr. Gang Chen (2018/1-3; 2020/2/2-5, 17, 19, 22-26)
The relationships between elements and ideal extended elements (the frontier of elements) and an overall picture of them were depicted as above.

21. Some Supplements

Supplement 1:

Supplement 2:

Supplement 3:

Table 8. Relationships of factors in $\alpha_1$-7 and $\alpha_2$-13 with primordial nuclides (2020/2/16-17).

<table>
<thead>
<tr>
<th>Nuclides</th>
<th>$^{3}$Li$_{4}$</th>
<th>$^{29}$Cu$_{34}$</th>
<th>$^{31}$Ga$_{40}$</th>
<th>$^{64}$Gd$_{92}$</th>
<th>$^{79}$Re$_{112}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PN before</td>
<td>5</td>
<td>70</td>
<td>78</td>
<td>209</td>
<td>252</td>
</tr>
<tr>
<td>PN all</td>
<td>285</td>
<td>285</td>
<td>285</td>
<td>285</td>
<td>285</td>
</tr>
<tr>
<td>Ratios</td>
<td>1/57</td>
<td>14/57</td>
<td>26/95</td>
<td>11/15</td>
<td>84/95</td>
</tr>
</tbody>
</table>

1. 3, 29, 31, 64, 75 and 112 are factors in $\alpha_1$ and $\alpha_2$.
2. PN: primordial nuclides; PN all: usually regarded as 286.
3. Nucleon number 285 of $^{112}$Cn$_{173}$ would relate to PN all, or PN all should be 285 rather than 286, and $^{235}$U should not be a primordial nuclide.
4. $^{235}$U$_{141}$ should not be a primordial nuclide, its relative stability (but not much stable) should come from relative stable nucleon numbers 92=96-4 and 143=11×13, so number of PN would become 285 from 286.
Supplement 4: Correlations of factors in $\alpha$ ($\alpha_{1-7}$, $\alpha_{2-13}$ and $\alpha_{4-50}$) and nuclides

In this scheme there are several important clues based on factors in the formulas of $\alpha_{1-7}$, $\alpha_{2-13}$ and $\alpha_{4-50}$ such as 6 (36, 48, 138, 144, et al), 7 (56, 84, 112, 126, 166-168, 210, 252), 10 (30, 50, 70, 100, 120, 130, 170, 200, 210, 220, 250, 310, 330, 400, 420), 11 (44, 88, 121, 134, 176, 187, 209, 220, 330, 363), 13 (26, 143, 169, 221, 364), 29 (87,145, 348), 25 (75, 100, 125, 200, 250, 400), 31 (93 124 186 310, 372), 61 (122, 244), 64 (136 et al), 137 (68, 69, 136,138), 139, 157 (314), 257, et al. And these clues correlate each others. These relationships are strong proofs that Chen’s formulas of the fine-structure constant are correct, otherwise so many coincidences couldn’t be explained.

In addition, numbers 7, 13 and 50 in $\alpha_{1-7}$, $\alpha_{2-13}$ and $\alpha_{4-50}$ may have the following relationships: (13+7)(13-7) =50+70=120 and $\alpha_{50}$Sn. And Sn is special, it has the most stable nuclides (up to 10) among which $\alpha_{50}$Sn has the most relative abundance.

Supplement 5: Other two formulas of the fine-structure constant

$$\alpha_{1/9=11} = \frac{1}{11 + \frac{1}{84} - \frac{1}{11 \cdot 17 \cdot 53 + \frac{2}{191} + \frac{1}{157}}} = 1/137.035999037435$$

$$\alpha_{2/20=25} = \frac{1}{25 - \frac{1}{14 + \frac{1}{251} - \frac{8}{(12 \cdot 43 \cdot 227 + 1) \cdot \frac{1}{37}}} = 1/137.03599911818$$
Supplement 6: Other formulas of the speed of light $c_{au}$

$$c_{au} = \frac{1}{\alpha} = \frac{1}{\sqrt{\alpha_{\text{H}} \alpha_{\text{D}}}}$$

$$= \frac{(11 + \frac{1}{84} - \frac{1}{11 \cdot 17 \cdot 53 + \frac{2 \cdot 191}{3 \cdot 157}}) \cdot (112 + \frac{1}{75}) \cdot 25 \cdot (112 - \frac{1}{3 \cdot 29 \cdot 64})}{9 \cdot (20 + \frac{1}{2} - \frac{1}{14} + \frac{1}{251})} - \frac{1}{8 \cdot (12 \cdot 43 \cdot 227 + 1)} - \frac{8}{37}$$

$$= \sqrt{137.035999074627 \times 137.035999111818} = 137.035999074627$$

$c_{au}$

$$= \frac{1}{\alpha} = \frac{1}{\sqrt{\alpha_{\text{H}} \alpha_{\text{D}}}}$$

$$= \frac{(11 + \frac{1}{84} - \frac{1}{11 \cdot 17 \cdot 53 + \frac{2 \cdot 191}{3 \cdot 157}}) \cdot (112 + \frac{1}{75}) \cdot 25 \cdot (112 - \frac{1}{3 \cdot 29 \cdot 64})}{9 \cdot (20 + \frac{1}{2} - \frac{1}{14} + \frac{1}{251})} - \frac{1}{8 \cdot (12 \cdot 43 \cdot 227 + 1)} - \frac{8}{37}$$

$$= \sqrt{137.035999074627 \times 137.035999111818} = 137.035999074627$$

$c_{au}$

$$= \frac{1}{\alpha} = \frac{1}{\sqrt{\alpha_{\text{H}} \alpha_{\text{D}}}}$$

$$= \frac{(11 + \frac{1}{84} - \frac{1}{11 \cdot 17 \cdot 53 + \frac{2 \cdot 191}{3 \cdot 157}}) \cdot (112 + \frac{1}{75}) \cdot 25 \cdot (112 - \frac{1}{3 \cdot 29 \cdot 64})}{9 \cdot (20 + \frac{1}{2} - \frac{1}{14} + \frac{1}{251})} - \frac{1}{8 \cdot (12 \cdot 43 \cdot 227 + 1)} - \frac{8}{37}$$

$$= \sqrt{137.035999074627 \times 137.035999111818} = 137.035999074627$$
Supplement 7: Comparison of formulas of $1$, $N$, $e$, $2\pi$, $\pi/2$, $\phi$, $\alpha$, $\alpha_c$, $c_{au}$ and $\alpha_{pic}$

$$1 = 4\gamma' + \frac{4\gamma_1}{1(1+1)} + \frac{4\gamma_2}{2(2+1)} + \frac{4\gamma_3}{3(3+1)} + \cdots$$

$$N \sim -\frac{3}{2}B + \sum_{n=1}^{\infty} \frac{B_{2n}(2\pi)^{2n}}{2(2n)!}$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

$$2\pi = \left(\frac{e}{e'}\right)^2 = e^2 \left(\frac{2}{1}\right)^3 \left(\frac{3}{2}\right)^4 \left(\frac{4}{3}\right)^5 \cdots$$

$$\phi = \frac{\sqrt{5} - 1}{2} = 0.618\ldots$$

$$\alpha_c = \frac{6^2}{7 \cdot e^2 \left(\frac{2}{1}\right)^3 \left(\frac{3}{2}\right)^4 \cdots \left(\frac{113}{112}\right)}$$

$$\alpha_{pic} = \frac{\sqrt{\alpha_c \alpha_z}}{\alpha_e} = 1/137.0359999118181$$

$$c_{au} = \frac{1}{\alpha_e} = \frac{1}{\sqrt{\alpha_c \alpha_z}} = \frac{5}{3} \frac{7 \cdot (2\pi)^{112} (112^2 - \frac{1}{30^2 \cdot 5} + \frac{1}{60^2 \cdot 15} - \frac{1}{120^2 \cdot 15 \cdot 29})}{\sqrt{13 \cdot (2\pi)^{278} (112^2 - \frac{1}{30^2 \cdot 5} + \frac{1}{60^2 \cdot 15} - \frac{1}{120^2 \cdot 15 \cdot 29})}}$$

$$\alpha_{pic} = \frac{1}{\alpha_{c_{pic}}} = \frac{1}{\alpha_{c} \alpha_{c_{au}}}$$

$$\approx 25.112 \left(\frac{1}{4} \frac{1}{46} + \frac{1}{55 \cdot 100} + \frac{1}{9 \cdot 25 \cdot 13 \cdot (20 \cdot 293 + 1) - \frac{4}{23}}\right) = 25.040872632515^2 = 63524.60147736$$

(Supposed)
The relations of the above formulas are sophisticated. In general, some formulas such as 1, N, e and 2\(\pi\) have similar form (called the natural group form), some formulas such as \(\phi\), \(\alpha\), \(\alpha_c\) and \(c_{\text{au}}\) can be divided into rational parts and irrational parts for each which may imply they have the same reasonability. In addition, 2\(\pi\), \(\pi/2\), \(\phi\), \(\alpha\), \(\alpha_c\) and \(c_{\text{au}}\) are all proportional constants, so they should have some similar or the same regularities.

**Supplement 8:** Comparison of pictures of elements and \(\phi\)

With the hints of the above formulas, it is not strange that the gold section (\(\phi\approx0.618\)) appears in the elements, it should appear in some places with some forms.

**Fig. 11**

Imagine a one-dimension creature lives in the line 0-1, he is familiar with 0-0.618 line space and can reach 0.618-1 line space, if he is enough smart, he may feel there should be an ideal extended line space from 0 to -1.618, but he couldn't reach all or can only get the margin of it. The same situation is suitable for us, we live in the space of elements, we mainly utilize the stable elements and can use some radioactive elements before the 112th element Cn, moreover, there should be a space for ideal extended elements from the 119th to the 170th, a few of which we can synthesize, many of which we can't, but this space should exist. This situation is also suitable for our lives in the earth, the solar system and the universe, or even in the mater, dark matter and dark energy, except that the proportion ratios should be different.

**Supplement 9:** Primordial nuclides and Fibonacci sequences

2\(\pi\) connects to nuclides and 2\(\pi\) also connects to Gold Section \(\phi\) as described by
Ramanujan’s formulas, so $\phi$ should connect to nuclides. And Fibonacci sequences are the integer presentations of $\phi$, so Fibonacci sequences should connect to nuclides.

These connections are described as follows.

Fibonacci Sequence $p_1$: 1 8 9 17 26 43 69 112

1 16, 18 9 19 35, 37 119
69 100 112 137

42

Fibonacci Sequence $p_2$: 1 10 11 21 32 53 85 138

138 52 60 85 121 209 338 547

Note: 69 100, relay of the numbers 56 and 100.

Fibonacci Sequence $n_1$: 0 8 8 16 24 40 64 104 168

0 4 8 12 20 32 56 100

Fibonacci Sequence $n_2$: 0 4 8 16 24 40 64 104 168

1 9 17 25 42 67 109 176 285

As stated in Section 4, the mathematic expression of chirality is $\pm 2\pi$. There are 10 fingers in a pair of human hands, and there are 14 finger segments in a single hand. It was assumed by us that the numbers 10/20 and 28/56 stand for a pair of “hands” in the world of nuclides. So nuclides $^{10}Ne$ and $^{20}Ca$ stand for two pairs of “hands” emerging gradually, and the numbers of primordial nuclides just before them are the numbers of Fibonacci Sequence $e$. This means chirality or $\pm 2\pi$ is the inner essence and $\phi$ or Fibonacci sequence is the outer expression of nuclides.
Primordial Nuclides

Primordial nuclides (PN) before Ba take about 0.618 part of all (176/285)
Numbers of PN before C, 8B, 10Ne, 14Si, 20Ca, 28Ni, 40Zr, 56Ba and 112Cn are
9 8 17 25 42 67 109 176 and 285 which is Fibonacci Sequence e

Primordial Nuclides and Fibonacci Sequence e
Dr. Gang Chen (2018/1-3, 2020/2/29-3/1)

Fig. 12

The Integrated Picture of Nuclides and Fibonacci Sequences
The Nuclide Picture was taken from Wikipedia
Dr. Gang Chen (2018/1-3; 2020/3/1-3, 4/24)

Fig. 13
Why are there two pairs of "hands" in the world of nuclides? This should be
because a pair of "hands" takes the right "hand" as priority and the other pair
takes left "hand" as priority. So, (5, 10, 14, 20, 28, 40, 56, 112) could be defined to be Numbers of
Chirality they are connected to Fibonacci Sequence in nuclides (Fig. 13).

Supplement 10: Other formulas of the speed of light c
Supplement 11: Construct formulas of the fine-structure constant with Wallis formula of $\pi/2$ instead of $2\pi$-e formula

Wallis Formula of $\pi/2$:

Traditional format: \[
\frac{\pi}{2} = \frac{2 \cdot 4 \cdot 6 \cdot 8}{1 \cdot 3 \cdot 5 \cdot 7} \cdot \frac{10 \cdot 2}{12 \cdot 14} \cdots = \prod_{n=1}^{\infty} \left( \frac{2n}{2n-1} \right).
\]

Natural group format: \[
\left( \frac{\pi}{2} \right) = \frac{2 \cdot 4 \cdot 6 \cdot 8}{3 \cdot 5 \cdot 7 \cdot 9} \cdot \frac{10 \cdot 2}{12 \cdot 14} \cdots = \prod_{n=1}^{\infty} \left( \frac{2n}{2n+1} \right).
\]

Comparable and similar to $2\pi$ - $e$ formula: \[
(2\pi)_k = e^{2\frac{k}{2}} \left( \frac{3}{2} \right)^{3/2} \left( \frac{4}{3} \right)^{4/3} \cdots \left( \frac{k+2}{k} \right)^{k+1}.
\]

So, Wallis Formula of $\pi/2$ could be used to construct formulas of $\alpha$.

Note: There should be $(2\pi)_k \sim 4(\frac{\pi}{2})^{k/2}$, or $(2\pi)_k$ and $(\frac{\pi}{2})^{3k/2}$ keep the same accuracy.

\[
\alpha_{1-7-\text{Wallis}} = \frac{9}{113} + \frac{1}{2 \cdot 19 \cdot 49 - 1} = 1/137.035999037435
\]

\[
\alpha_{1-22-\text{Wallis}} = \frac{4 \cdot 2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 2}{3 \cdot 5 \cdot 7 \cdot 9} \cdot \frac{12 \cdot 14 \cdot 16 \cdot 18}{19 \cdot 21 \cdot 23 \cdot 25} = 1/137.035999037435
\]

\[
\alpha_{1-29-\text{Wallis}} = \frac{4 \cdot 2 \cdot 4 \cdot 6 \cdot 8 \cdot 9 \cdot 11 \cdot 13 \cdot 15 \cdot 17 \cdot 19}{3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 15 \cdot 17 \cdot 19} = 1/137.035999037435
\]
\[ \alpha_{1-155-\text{Wallis}} = \frac{199}{5 \cdot 31 \cdot \left( \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \cdot \ldots}{3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot \ldots} \cdot 11964 \cdot 2 \cdot 31 \cdot 193 \right)}{11965 \cdot 12 \cdot (12 - 83 + 1) + 1} + \frac{1}{8 \cdot 71 \cdot (10 - 13 - 29 - 1)} = 1/137.035999037435 \]

\[ \alpha_{1-170-\text{Wallis}} = \frac{9 \cdot 97}{4 \cdot 10 - 17 \cdot \left( \frac{2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \cdot \ldots}{3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot \ldots} \cdot 103324 \cdot 6 \cdot 17 \cdot (4 \cdot 11 - 23 + 1) \right)}{103325 \cdot 2 \cdot 26 \cdot (4 \cdot 7 - 71 - 1) + 1} + \frac{1}{7 \cdot 19 \cdot 83 \cdot 25 \cdot 10^{11}} = 1/137.035999037435 \]

\[ \beta_{\text{Li}_4 \text{Be}_5} = \frac{10^{12}}{27} \text{B} \]
### Supplement 12: Comparison of two kinds of formulas of the speed of light

\[
c_{\text{uo}} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \cdot \alpha_{123}} - \frac{5/3}{\frac{7}{13} (2\pi)^{12}} (112^2 - \frac{1}{30^2} \cdot 5 \cdot \frac{1}{60^2} \cdot 15 \cdot \frac{1}{120^2} \cdot 15 \cdot 29)}
\]

\[= \frac{1}{137.035999074627} \quad \text{(Refer to Page 12)}
\]

\[
c_{\text{uo}} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \cdot \alpha_{123}} - \frac{5/3}{\frac{7}{13} (2\pi)^{12}} (112^2 - \frac{1}{9} \cdot 5 \cdot 10^9 + \frac{1}{4})} = \frac{1}{137.035999074627}
\]

\[
c_{\text{uo}} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-7} \cdot \alpha_{12-13} \cdot \alpha_{12-3}}} = \frac{5/3}{\frac{7}{13} (2\pi)^{12}} (112^2 + \frac{1}{15} \cdot 4 \cdot 7.1 + \frac{1}{6} \cdot (48 \cdot (12 - 29 - 1))}
\]

\[= \frac{1}{137.035999074627}
\]
\[ c_m = \frac{1}{\alpha_r} = \frac{1}{\alpha_{r_{1-29}}} = \frac{223 (2\pi)^{107}}{149 (2\pi)^{53.81}} (112^2 + \frac{1}{6.23} - \frac{1}{5.67 - 100 + \frac{17}{25}}) = 137.035999074627 \]

\[ c_m = \frac{1}{\alpha_r} = \frac{1}{\alpha_{r_{1-29-Wallis}}} = \frac{223 (2\pi)^{107}}{149 (2\pi)^{53.81}} (112^2 + \frac{1}{6.23} - \frac{1}{5.67 - 100 + \frac{17}{25}}) = 137.035999074627 \]

\[ c_m = \frac{1}{\alpha_r} = \frac{1}{\alpha_{r_{1-36}}} = \frac{277 (2\pi)^{107}}{5.37 (2\pi)^{53.81}} (112^2 + \frac{1}{6.41 - 47} - \frac{3.29}{139}) = 137.035999074627 \]

\[ c_m = \frac{1}{\alpha_r} = \frac{1}{\alpha_{r_{1-36-Wallis}}} = \frac{277 (2\pi)^{107}}{5.37 (2\pi)^{53.81}} (112^2 + \frac{1}{6.41 - 47} - \frac{3.29}{139}) = 137.035999074627 \]

\[ c_m = \frac{1}{\alpha_r} = \frac{1}{\alpha_{r_{1-36-Wallis}}} = \frac{277 (2\pi)^{107}}{5.37 (2\pi)^{53.81}} (112^2 + \frac{1}{6.41 - 47} - \frac{3.29}{139}) = 137.035999074627 \]
Leibniz Formula of $\pi/4$ could be used to construct formulas of $\alpha$.

Note: There should be $(2\pi)_k \sim 8(\pi/4)^{1/2}$.

That means $(2\pi)_k$ and $(\pi/4)^{1/2}$ become convergent at the same speed.

The larger the $k$ is, the better is the accuracy and the less is the error.
\[ \alpha_{4-7-GL} = \frac{36}{8 \cdot 7 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 6 \cdot 36 + 1})} + \frac{1}{112} + \frac{1}{64 \cdot 13} - \frac{1}{128 \cdot 3 \cdot (4 \cdot 3 \cdot 17 \cdot 23 - 1)} = 1/137.035999037435 \]

\[ \alpha_{4-22-GL} = \frac{113}{8 \cdot 22 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 5 \cdot 13 \cdot 23 + 1})} + \frac{1}{112} + \frac{1}{5 \cdot 17 \cdot (2 \cdot 3 \cdot 7 \cdot 13 - 1) + \frac{3}{44}} = 1/137.035999037435 \]

\[ \alpha_{4-29-GL} = \frac{149}{8 \cdot 29 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot (4 \cdot 7 \cdot 11 - 1) + 1})} + \frac{1}{112} + \frac{6 \cdot 25 \cdot 79}{28 \cdot 5 \cdot 23} = 1/137.035999037435 \]

\[ \alpha_{4-36-GL} = \frac{5 \cdot 37}{8 \cdot 36 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 4 \cdot 113 + 1})} + \frac{1}{112} + \frac{3 \cdot 29 \cdot 47}{2 \cdot 47 - 4 \cdot 83 - 1} = 1/137.035999037435 \]

\[ \alpha_{4-43-GL} = \frac{13 \cdot 17}{8 \cdot 43 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 128 \cdot 3 + 1})} + \frac{1}{112} + \frac{1}{4 \cdot (64 \cdot 9 + 1) - \frac{136}{137}} = \frac{1}{137} \]

or

\[ \alpha_{4-43-GL} = \frac{13 \cdot 17}{8 \cdot 43 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 128 \cdot 3 + 1})} + \frac{1}{112} + \frac{1}{4 \cdot (64 \cdot 9 + 1) - \frac{12 \cdot 11 \cdot 13}{25 \cdot 10 \downarrow}} = 1/137.035999037437 \text{ or } 1/137.035999037435 \]

\[ \alpha_{4-50-GL} = \frac{257}{8 \cdot 50 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot 173 + 1})} + \frac{1}{112} + \frac{1}{16 \cdot 11 \cdot 13 - \frac{28}{6 \cdot 31}} = 1/137.035999037436 \]
\[ \alpha_{-59-Gl} = \frac{3 \cdot 101}{8 \cdot 59 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 6 \cdot 29^2 + 1}) 112 + \frac{1}{27 \cdot (8 \cdot 9 \cdot (8 \cdot 3 \cdot 13 - 1) - 1) + \frac{10}{23}} = \alpha_{-59-Gl} \]

\[ \alpha_{-81-Gl} = \frac{4 \cdot 13}{81 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 6 \cdot 7 \cdot 73 + 1}) 112 + \frac{1}{5 \cdot 23 \cdot (36 \cdot 47 + 1) + \frac{19}{25}} = \alpha_{-81-Gl} \]

\[ \alpha_{-96-Gl} = \frac{7 \cdot 29}{8 \cdot 96 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 10 \cdot (30 \cdot 37 - 1) + 1}) 112 + \frac{1}{9 \cdot 103 \cdot (4 \cdot 17 \cdot 41 + 1)} = \alpha_{-96-Gl} \]

\[ \alpha_{-103-Gl} = \frac{23^2}{8 \cdot 103 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 18 \cdot 139 + 1}) 112 + \frac{2 \cdot (10 \cdot 23 \cdot 41 + 1)}{25 \cdot 10^{10}} = \alpha_{-103-Gl} \]

\[ \alpha_{-133-Gl} = \frac{36 \cdot 19 - 1}{8 \cdot 7 \cdot 19 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 16 \cdot 3 \cdot 17 \cdot 29 + 1}) 112 + \frac{3 \cdot 107 \cdot 151}{4 \cdot 10^{11}} = \alpha_{-133-Gl} \]

\[ \alpha_{-140-Gl} = \frac{36 \cdot 20 - 1}{8 \cdot 7 \cdot 20 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 22 \cdot 167 + 1}) 112 + \frac{1}{128 \cdot 19 \cdot 83 + \frac{2}{17}} = \alpha_{-140-Gl} \]

\[ \alpha_{-155-Gl} = \frac{199}{10 \cdot 31 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 6 \cdot 7 \cdot 17 + 1}) 112 + \frac{9 \cdot 7 \cdot 29 \cdot 107}{25 \cdot 10^{11} or 113 \cdot 173} = \alpha_{-155-Gl} \]
\[
\alpha_{\text{1-170-Gl}} = \frac{9.97 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 6 \cdot 19 \cdot (2 \cdot 17^2 - 1) + 1})}{112 + \frac{7 \cdot 23 \cdot 41}{25 \cdot 10^{11}}}
\]

\[
= 1 / 137.035999037435
\]

\[
\alpha_{\text{2-10-Gl}} = \frac{8 \cdot 10 \cdot (1 - \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 18 \cdot 11 + 1})}{7 \cdot 11}
\]

\[
= 1 / 137.035999111818
\]

\[
\alpha_{\text{2-13-Gl}} = \frac{8 \cdot 13 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 28 \cdot 19 + 1})}{100}
\]

\[
= 1 / 137.035999111818
\]

\[
\alpha_{\text{2-23-Gl}} = \frac{8 \cdot 23 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 4 \cdot 7 \cdot 11 + 1})}{3 \cdot 59}
\]

\[
= 1 / 137.035999111818
\]

\[
\alpha_{\text{2-29-Gl}} = \frac{8 \cdot 29 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 4 \cdot (24 \cdot 13 + 1) + 1})}{223}
\]

\[
= 1 / 137.035999111818
\]

\[
\alpha_{\text{2-33-Gl}} = \frac{4 \cdot 33 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 24 \cdot 11 + 1})}{127}
\]

\[
= 1 / 137.035999111818
\]
\[
\alpha_{2-36-GL} = \frac{8 \cdot 36 \cdot \left(1 - \frac{1}{3} \cdot \frac{1}{5} + \frac{1}{7} - \cdots + \frac{1}{2 \cdot 2 \cdot 181 + 1}\right)}{2 \cdot 138 + 1} - \frac{1}{112 - \frac{1}{16 \cdot (2 \cdot 7 \cdot 11 \cdot 17 - 1) + \frac{6}{7}}}
\]

Supplement 14: Construct formulas of the speed of light with \(\alpha_{1-GL}\) and \(\alpha_{2-GL}\).
Supplement 15: DNA-Protein model of formulas of α and nuclides

DNA: α_{i} = α_{i-7} = 62 \cdot \frac{e^{2} \cdot e^{2} \cdot \ldots \cdot e^{2}}{\left(\frac{\pi}{2}\right)^{4} \cdot \left(\frac{\pi}{2}\right)^{6}} = \frac{1}{137.03599907435}

Gene Factors: 3 4 5 6 7 9 12 18 25 36 44 (7 \cdot 2 \cdot \pi \approx 44) 75 112 113 225

Direct Derived Factors: 30 35 42 88 150 226;


Protein: 112.3 C_{6} 6.7 12.3 C_{6} 3 4 5 6 7 9 \ldots 4 71 312,314 2 173 366,372 173 113 171 125,126 187,188 137 209 144,147 222 12 225 1 125 }

et al.
DNA: $\alpha_{2.13} = \frac{13 \cdot e^2}{(\frac{e^2}{1})^{\frac{1}{3}}} \frac{e^2}{(\frac{e^2}{2})^{\frac{1}{5}}} \frac{e^2}{(\frac{e^2}{2})^{\frac{1}{11}}} = 1/137.035999111818$

Gene Factors: 2 3 5 9 10 13 29 31 64 82 (13 2 82) 100 112 139


Emerging Factors: 19 38 57 71 114 145 187 208 209 210 et al.

Protein: $\alpha_{2.45} = \frac{5 \cdot e^2}{\left(\frac{e^2}{1}\right)^{\frac{1}{3}}} \frac{e^2}{\left(\frac{e^2}{2}\right)^{\frac{1}{5}}} \frac{e^2}{\left(\frac{e^2}{2}\right)^{\frac{1}{11}}} = 1/137.035999111818$

Emerging Factors: 19 38 57 71 114 145 187 208 209 210 et al.

Supplement 16: Some formulas of $\alpha_{2}$

$$\alpha_{2.45-Walls} = \frac{2 \cdot 5 \cdot 9 \cdot (2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 3234 \cdot 4 \cdot (8 \cdot 101 + 1))}{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 3235 \cdot 2 \cdot 3 \cdot 4 \cdot 9 \cdot 11 + 1} = 1/137.035999111818$$

$$\alpha_{2.45-GL} = \frac{4 \cdot 5 \cdot 9 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + 1}{2 \cdot 2 \cdot (24 \cdot 43 - 1) + 1} = 1/137.035999111818$$
\[
\alpha_{2-173} = \frac{1}{112 - \frac{3.53 \cdot (20 - 27 + 1)}{25 \cdot 10^3}}
\]

\[
\alpha_{2-173-Wallis} = \frac{1}{5.7 \cdot 19}
\]

\[
\alpha_{2-173-GL} = \frac{1}{112 - \frac{1}{4.7 \cdot 23 - 59.89}}
\]

\[
\alpha_{49} = \frac{49 \cdot e^2 \left( \frac{2}{(\frac{1}{2})^3} \right) \left( \frac{e^2}{(\frac{1}{2})^3} \right) \left( \frac{e^2}{(\frac{1}{2})^3} \right) \left( \frac{e^2}{(\frac{1}{2})^3} \right)}{13.29 \cdot 112 - \frac{1}{7 \cdot (6.11^2 + 1) + \frac{11.19}{3 \cdot 97}}}
\]

\[
\alpha_{49-Wallis} = \frac{1}{13.29 \cdot 112 - \frac{1}{10 \cdot (30 - 17 + 1) + \frac{2 \cdot 7.11}{300}}}
\]

\[
\alpha_{49-GL} = \frac{8.49 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 36 \cdot 11 + 1})}{13.29 \cdot 112 - \frac{1}{11^2 \cdot 47 - \frac{36}{125}}}
\]
Supplement 17: Some formulas of the speed of light

\[ c_{\text{em}} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-81}\alpha_{2-49}}} = \frac{9}{4.7} \sqrt{\frac{29}{2} \left(\frac{2\pi}{9}ight)^{15.1007} - \frac{1}{2.23} + \frac{1}{7 \cdot (2.11 \cdot 19 + 1) - \frac{13}{7.11}}} \]

\[ = 137.035999074627 \]

\[ c_{\text{em}} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_{1-81}\alpha_{2-49}}} = \frac{9}{4.7} \sqrt{\frac{29}{2} \left(\frac{\pi}{2}\right)^{7.1331} - \frac{1}{4.13} + \frac{1}{9.7 \cdot 11.13 + 17 \cdot 18}} \]

\[ = 137.035999074627 \]

Supplement 18: Other formulas of \( \alpha_2 \)

\[ \alpha_{2-42} = \frac{2 \cdot 3 \cdot 7 \cdot e^2 \cdot e^2 \cdot e^2}{\left(\frac{2}{3}\right)^3 \left(\frac{2}{3}\right)^5 \cdots} \frac{42 \cdot 11}{20 \cdot 23 + 1}^{13.71} \]

\[ = \frac{1}{137.035999111818} \]

\[ \alpha_{2-42 - Wallis} = \frac{8 \cdot 3 \cdot 7 \cdot (2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdots)}{3 \cdot 3 \cdot 5 \cdot 7 \cdot 7} \frac{1384}{1385} \frac{2 \cdot 9 \cdot 7 \cdot 11}{2 \cdot 4 \cdot 173 + 1} \]

\[ = 1/137.035999111818 \]

\[ \alpha_{2-42 - Wallis} = \frac{16 \cdot 3 \cdot 7 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \cdots +)}{17 \cdot 19} \frac{1}{112 - \frac{1}{2 \cdot 23 \cdot 113 - 1} + \frac{10}{59}} \]

\[ = 1/137.035999111818 \]
\[
\alpha_{2,61} = \frac{61 \cdot e^2 - e^2 - e^2 \cdots}{\left( \frac{1}{3} \right)^3 \left( \frac{1}{2} \right)^5} \left( 9 \cdot \frac{173}{4 \cdot (4 \cdot 97 + 1)} \right)^{11(6-47+1)}
\]

\[
\alpha_{2,61} = \frac{1}{1.37035999111818}
\]

\[
\alpha_{2,61-\text{Wallius}} = \frac{7 \cdot 67}{112 - \frac{1}{2 \cdot 6 \cdot 13 \cdot (8 \cdot 7 \cdot 11 \cdot 13 + 1)}
\]

\[
\alpha_{2,61-\text{Wallius}} = \frac{1}{1.37035999111818}
\]

\[
\alpha_{2,61-\text{GL}} = \frac{7 \cdot 67}{112 - \frac{1}{4 \cdot 3 \cdot 5 \cdot (2 \cdot 7 \cdot 173 + 1) - 37}{62}}
\]

\[
\alpha_{2,77} = \frac{16 \cdot 37}{112 - \left( \frac{5}{4} \cdot 9 \cdot 7 \cdot 13 \cdot 17 - 1 \right) + \frac{5}{16}}
\]

\[
\alpha_{2,77-\text{Wallius}} = \frac{4 \cdot 37}{112 - \frac{1}{2 \cdot 15 \cdot 17 \cdot 29 \cdot 31 - \frac{5}{6}}
\]

\[
\alpha_{2,77-\text{GL}} = \frac{7 \cdot 11 \cdot e^2 - \frac{e^2}{(\frac{1}{3})^3 \left( \frac{1}{2} \right)^5} \cdots}{\left( \frac{11}{19} \right)^{420}}
\]

\[
\alpha_{2,77-\text{GL}} = \frac{1}{1.37035999111818}
\]

\[
\alpha_{2,77-\text{GL}} = \frac{7 \cdot 11 \cdot 2 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdots}{3 \cdot 3 \cdot 5 \cdot 7 \cdot 7} \left( \frac{3 \cdot 5 \cdot 7 \cdot 7}{3 \cdot 5 \cdot 7 \cdot 7} \right) + 1
\]

\[
\alpha_{2,77-\text{GL}} = \frac{1}{1.37035999111818}
\]

\[
\alpha_{2,77-\text{GL}} = \frac{7 \cdot 11 \cdot 2 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdots}{3 \cdot 3 \cdot 5 \cdot 7 \cdot 7} \left( \frac{3 \cdot 5 \cdot 7 \cdot 7}{3 \cdot 5 \cdot 7 \cdot 7} \right) + 1
\]

\[
\alpha_{2,77-\text{GL}} = \frac{1}{1.37035999111818}
\]
\[
\alpha_{2-93} = \frac{93 \cdot e^2}{1} \left( \frac{3}{2} \right) \left( \frac{3}{2} \right)^5 \cdots \left( \frac{12 \cdot 227}{7 \cdot (4 \cdot 97 + 1)} \right)^{13(10-42-1)} \frac{1}{5 \cdot 11-13} = 1/137.035999111818
\]

\[
\alpha_{2-93-\text{Wallis}} = \frac{93 \cdot e^2}{1} \left( \frac{3}{2} \right) \left( \frac{3}{2} \right)^5 \cdots \left( \frac{12 \cdot 227}{7 \cdot (4 \cdot 97 + 1)} \right)^{13(10-42-1)} \frac{1}{5 \cdot 11-13} \frac{1}{10-12 \cdot (8 \cdot 9 \cdot 7 \cdot 13 + 1) + \frac{7}{12}} = 1/137.035999111818
\]

\[
\alpha_{2-93-\text{GL}} = \frac{93 \cdot e^2}{1} \left( \frac{3}{2} \right) \left( \frac{3}{2} \right)^5 \cdots \left( \frac{12 \cdot 227}{7 \cdot (4 \cdot 97 + 1)} \right)^{13(10-42-1)} \frac{1}{5 \cdot 11-13} \frac{1}{60 \cdot (30 \cdot 11-23 + 1) + \frac{14}{33}} = 1/137.035999111818
\]

\[
\alpha_{2-93-\text{Wallis}} = \frac{93 \cdot e^2}{1} \left( \frac{3}{2} \right) \left( \frac{3}{2} \right)^5 \cdots \left( \frac{12 \cdot 227}{7 \cdot (4 \cdot 97 + 1)} \right)^{13(10-42-1)} \frac{1}{5 \cdot 11-13} \frac{1}{3 \cdot 4 \cdot (4 \cdot 7 \cdot 13^2 + 1) + \frac{3}{4}} = 1/137.035999111818
\]

Note: 8 \cdot 137 + 1 = 1816 - 1

\[
\alpha_{2-93-\text{GL}} = \frac{93 \cdot e^2}{1} \left( \frac{3}{2} \right) \left( \frac{3}{2} \right)^5 \cdots \left( \frac{12 \cdot 227}{7 \cdot (4 \cdot 97 + 1)} \right)^{13(10-42-1)} \frac{1}{5 \cdot 11-13} \frac{1}{4 \cdot 19 \cdot 19 + 1} = 1/137.035999111818
\]
\[
\alpha_{\text{2-141-Gu}} = \frac{2 \cdot 3 \cdot 47 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{2 \cdot 4 \cdot (10 \cdot 11 \cdot 12 + 1) + 1})}{16 \cdot 17 - 1} - \frac{1}{112} - \frac{4 \cdot 109 \cdot (2 \cdot 3 \cdot 47 - 1)}{25 \cdot 10^{11}}
\]

\[
\alpha_{\text{2-189}} = \frac{27 \cdot 7 \cdot e^{2} \cdot e^{2} \cdot \left(\frac{2}{1}\right)^{3} \cdot 2}{(2 \cdot 1)^{3} \cdot 2} \cdot \left(\frac{12 \cdot (36 - 23 - 1) \text{ or } (14 - 59 + 1)}{189 \cdot 223}\right) \cdot 112 - \frac{2 \cdot 3 \cdot 7 \cdot 113}{25 \cdot 10^{11}}
\]

\[
\alpha_{\text{2-189-Wallis}} = \frac{4 \cdot 27 \cdot 7 \cdot (2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 3 \cdot 3 \cdot 5 \cdot 7 \cdot 5 \cdot 29770 \cdot (36 \cdot (36 - 23 - 1) \text{ or } (14 - 59 + 1))}{19771} - \frac{2 \cdot 5 \cdot 13 \cdot (12 \cdot 19 + 1) + 1}{112} - \frac{\delta_{2}}{25 \cdot 10^{11}}
\]

\[
\alpha_{\text{2-189-Gu}} = \frac{12 \cdot 11^{2} + 1}{112} - \frac{16 \cdot (6 \cdot 7 \cdot 11^{2} - 1) \text{ or } 3 \cdot 7 \cdot 19 \cdot 163}{25 \cdot 10^{11}}
\]

\[
\alpha_{\text{2-205}} = \frac{5 \cdot 41 \cdot e^{2} \cdot e^{2} \cdot \left(\frac{1}{2}\right)^{3} \cdot 3 \cdot 7 \cdot 9 \cdot 17 \cdot 10 \cdot 2 \cdot 105 \cdot 1}{2 \cdot 8 \cdot 2 \cdot 3 \cdot 103 + 1} - \frac{1}{112} - \frac{2 \cdot (2 \cdot 4 \cdot 11 \cdot (16 \cdot 17 + 1) - 1)}{25 \cdot 10^{11}}
\]

\[
\alpha_{\text{2-205-Wallis}} = \frac{2 \cdot 4 \cdot 4 \cdot 7 \cdot 6 \cdot 8 \cdot 3 \cdot 3 \cdot 5 \cdot 7 \cdot 7 \cdot 37208}{37209 \cdot 2 \cdot 4 \cdot (6 \cdot 25 \cdot 31 + 1) + 1} - \frac{1}{2 \cdot 197} - \frac{4 \cdot 3 \cdot 5 \cdot 37^{2} + 1}{2 \cdot 10^{12}}
\]

\[
\alpha_{\text{2-205-Gu}} = \frac{5 \cdot 41 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{2 \cdot 8 \cdot 9 \cdot 7 + 1})}{197} - \frac{1}{112} - \frac{3 \cdot 41 \cdot (16 \cdot 7 \cdot 11 + 1) + 1}{2 \cdot 10^{12}}
\]

\[
\alpha_{\text{2-139-Pm}} = \frac{5 \cdot 41 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{2 \cdot 8 \cdot 9 \cdot 7 + 1})}{197} - \frac{1}{112} - \frac{3 \cdot 41 \cdot (16 \cdot 7 \cdot 11 + 1) + 1}{2 \cdot 10^{12}}
\]

\[
\alpha_{\text{2-205-Pm}} = \frac{5 \cdot 41 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{2 \cdot 8 \cdot 9 \cdot 7 + 1})}{197} - \frac{1}{112} - \frac{3 \cdot 41 \cdot (16 \cdot 7 \cdot 11 + 1) + 1}{2 \cdot 10^{12}}
\]

\[
\alpha_{\text{2-205-Pm}} = \frac{5 \cdot 41 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{2 \cdot 8 \cdot 9 \cdot 7 + 1})}{197} - \frac{1}{112} - \frac{3 \cdot 41 \cdot (16 \cdot 7 \cdot 11 + 1) + 1}{2 \cdot 10^{12}}
\]

\[
\alpha_{\text{2-205-Pm}} = \frac{5 \cdot 41 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{2 \cdot 8 \cdot 9 \cdot 7 + 1})}{197} - \frac{1}{112} - \frac{3 \cdot 41 \cdot (16 \cdot 7 \cdot 11 + 1) + 1}{2 \cdot 10^{12}}
\]
\[\alpha_{2-221} = \frac{13 \cdot 17 \cdot e^2 \cdot e^2 \cdot e^2 \ldots (6 \cdot 11 \cdot (2 \cdot 7 \cdot 17 + 1 \cdot 7 \cdot 17 + 1))^{(2/7)} \cdot (2 \cdot 8 \cdot 17 - 29 \cdot 1)}{17 \cdot 100 - 1} = \frac{1}{137.035999111818} \]

\[\alpha_{2-221-Wallis} = \frac{1}{17 \cdot 100 - 1} = \frac{1}{137.035999111818} \]

\[\alpha_{2-221-GL} = \frac{1}{17 \cdot 100 - 1} = \frac{1}{137.035999111818} \]

\[\alpha_{2-234} = \frac{8 \cdot 13 \cdot 17 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{3 \cdot 5 \cdot 7 \cdot 9 - 1})^{(2/3 \cdot 5 \cdot 7 \cdot 9 - 1)}}{2 \cdot 7 \cdot 25 \cdot 107 + 1} \]

\[\alpha_{2-234-Wallis} = \frac{1}{7 \cdot 25 \cdot 107 + 1} = \frac{1}{137.035999111818} \]

\[\alpha_{2-234-GL} = \frac{8 \cdot 9 \cdot 13 \cdot (2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \ldots 11556 \cdot 2 \cdot (20 \cdot 17^2 - 1))}{5 \cdot 11 \cdot (13 \cdot 100 - 1) + 14 \cdot 19} = \frac{1}{137.035999111818} \]

\[\alpha_{2-235} = \frac{3 \cdot 7 \cdot 9 \cdot e^2 \cdot e^2 \cdot e^2 \ldots (2 \cdot 0 \cdot (24 \cdot 43 - 1))^{(2/13 \cdot 23 \cdot 43 - 1)}}{2 \cdot 10 \cdot 17 \cdot 13 + 1} = \frac{1}{137.035999111818} \]

\[\alpha_{2-257-Wallis} = \frac{6 \cdot 7 \cdot 9 \cdot (2 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 10 \cdot 13 + 1) \cdot 18 \cdot (8 \cdot (12 \cdot 41 - 1) + 1)}{10 \cdot 7 \cdot 13 + 1} = \frac{1}{137.035999111818} \]
\[
\alpha_{237-Ge} = \frac{12 \cdot 79 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{19 - 1})}{10 \cdot 7 \cdot 13 + 1 + 48 \cdot 19 - 1} = \frac{1}{137.035999111818}
\]

Supplement 19: Relationships between Ramanujan–Sato series for \(1/\pi\) and nuclides

Ramanujan Series (1914): \(\frac{\pi}{16} = \frac{\sqrt{8}}{9801} \left( 1 - \frac{1}{2^4} \cdot \frac{99}{163} + \frac{3 \cdot 13}{13 \cdot 143} - \frac{4 \cdot 37}{4 \cdot 143} + \cdots \right)\)

Ramanujan-Sato-Chen Series: \(\frac{\pi}{16} = \frac{\sqrt{8}}{9801} \left( 1 - \frac{1}{2^4} \cdot \frac{99}{163} + \frac{3 \cdot 13}{13 \cdot 143} - \frac{4 \cdot 37}{4 \cdot 143} + \cdots \right)\)

Supplement 20: Rewriting of Einstein’s \(E=mc^2\)

Einstein Formula: \(E = mc^2\)  Maxwell Formula: \(c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\)

In atomic units, \(c_{au} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi}}\) or \(c_{au}^2 = \frac{1}{\alpha_1 \alpha_2}\)

So: \(E_{au} = m_{au} c_{au}^2 = \frac{m_{au}}{\alpha_1 \alpha_2}\), or \(E_{au} = \frac{m_{au}}{\alpha_1 \alpha_2}\), or \(m_{au} = E_{au} \alpha_1 \alpha_2\)

Supplement 21: Other formulas of the fine-structure constant

\[
\alpha_{1-3-Wallis} = \frac{1}{103 \cdot (24.37 - 1) + \frac{4}{5}} = \frac{1}{137.035999037435}
\]
Supplement 22: The fine-structure constant 13 billion years ago

In a recent paper\(^\text{12}\), Wilczynska and Webb \textit{et al.} reported measurements of the fine-structure constant in the location of the universe 13 billion light years away, the results indicated that there would be deviation from the terrestrial value, i.e.,

\[
\frac{\Delta \alpha}{\alpha} = \frac{(\alpha_z - \alpha_0)}{\alpha_0} = (2.18 \pm 7.27) \times 10^{-5}
\]

in this direction or location at the age of the universe of 0.8 billion years old.
According to our theories about the fine-structure constant, the formulas of it are related to nuclides, and the universe should have different nuclide contents and distribution in different ages or even in different directions of the universe which should result in a little different values of the fine-structure constant, so this deviation could be explained to be reasonable. We here try to give some different hypothetical formulas and values of the fine-structure constant in reference to Webb’s results as follows.

\[ \Delta \alpha = (\alpha_z - \alpha_0) / \alpha_0 = (-2.18 \pm 7.27) \times 10^{-5} \]

\[ \alpha_z \approx (1 - 2.18 \times 10^{-5}) \times \frac{1}{137.035999} = 1/137.03899 \]

\[ \alpha_{z-2-g} = \frac{7 \cdot e^2}{\left(\frac{2}{3}\right)^2 \left(\frac{2}{3}\right)^3} \frac{1}{49} \]

\[ \alpha_{z-2-7, Wallis} = \frac{6 \cdot 9}{112} = 1/137.04295 \]

\[ \alpha_{z-2-7, \text{nl}} = \frac{14 \cdot 2 \cdot 4 \cdot 6 \cdot 8 \cdot 144}{145 \cdot 2 \cdot 2 \cdot 36 + 1} \]

\[ \alpha_{z-2-7, \text{nl}} = \frac{28 \cdot (1 - \frac{1}{3} \frac{1}{5} \frac{1}{7} - \cdots + \frac{1}{2 \cdot 4 \cdot 23 + 1})}{27} = 1/137.03976 \]

\[ \alpha_{z-2-7, \text{nl}} = \frac{1}{112} = 1/137.04084 \]

**Supplement 23:** The most possible elements to be synthesized after the 118th element

Up to now human beings have already discovered and synthesized 118 elements which fully fill 7 periods in Periodic Table of Elements. In this paper, we predicted the 119-170th elements, and called the 113-170th elements were ideal extended elements which implied some of them could be synthesized and many of them shouldn’t. Some questions at present are whether people could synthesize new elements to open the 8th period in Periodic Table of Elements, what would be the next element to be synthesized after the 118th element and so on. As 126, 128 and 137 are special numbers according to our theories, we here predict the following ideal extended elements could be relatively easier to be synthesized in future.

\[
\begin{align*}
124 \text{Ch}^{16} & \quad 126 \text{Ch}^{188} & \quad 128 \text{Ch}^{198} & \quad 126 \text{Ch}^{191-193} & \quad 128 \text{Ch}^{198} \\
310 \text{Ch}^{198} & \quad 312 \text{Ch}^{197} & \quad 314 \text{Ch}^{195} & \quad 318-320 & \quad 326 \text{Ch}^{198}
\end{align*}
\]

Among them 126Ch\text{188} should be the easiest to be synthesized. So, we predict one of these ideal extended elements (most likely 126Ch\text{188}) would open the 8th period in Periodic Table of Elements.
Supplement 24: Relationships between Chudnovsky algorithm for $1/\pi$ and nuclides

Chudnovsky algorithm (1987): \[
\frac{1}{\pi} = 12\sum_{k=0}^{\infty} \frac{(-1)^k(13591409 + 545140134k)}{(6k)!} \frac{640320^{13+3k}}{(k!)^3}
\]

Chudnovsky-Chen algorithm:

\[
2\pi = \frac{1}{6\sum_{k=0}^{\infty} \frac{(-1)^k(6k)!(-1 + 2 \cdot 3 \cdot 5 \cdot 7 \cdot 61 \cdot (4 \cdot 5 \cdot 53 + 1) + 2 \cdot 9 \cdot 7 \cdot 11 \cdot 19 \cdot 127 \cdot 163 \cdot k)}{(3k!)^3}640320^{13+3k}}
\]

\[
= \frac{1}{6\sum_{k=0}^{\infty} \frac{(-1)^k(6k)!(-1 + 2 \cdot 3 \cdot 5 \cdot 7 \cdot 61 \cdot (2 \cdot 9 \cdot 59 - 1) + 2 \cdot 9 \cdot 7 \cdot 11 \cdot 19 \cdot 127 \cdot 163 \cdot k)}{(3k!)^3}640320^{13+3k}}
\]

According to our theories, the formulas of the fine-structure constant are related to nuclides. As some formulas and values of $2\pi$ are incorporated in formulas of the fine-structure constant, they would be related to nuclides directly. In Supplement 19, 24 and 28, we gave two examples of Ramanujan-Sato series for $1/\pi$ which are related to nuclides. In 2012, G. Almkvist and J. Guillera\textsuperscript{13} stated that they used the theory of Calabi-Yau differential equations to obtain all the parameters of Ramanujan-Sato-like series for $1/\pi^2$ along with a conjecture to find new examples of series of non-hypergeometric type. So, we here guess that Calabi-Yau differential equations would be somewhat related to the formulas of the fine-structure constant and nuclides.
Supplement 25: Other formulas of $\alpha_2$

$$\alpha_{2,157} = \frac{157 \cdot e^2 \cdot e^2}{\left(\frac{2}{1}\right)^3 \cdot \frac{3}{5} \cdot \frac{2}{5}} \cdots \frac{e^2}{2^{817 \cdot 23}} \cdot \frac{8 \cdot 5 \cdot 163}{17 \cdot 71} \cdot \frac{112 - 1}{8 \cdot 27 \cdot 7 \cdot 13 \cdot 127}$$

$$= 1/137.035999111818$$

Supplement 26: Other formulas of $\alpha_1$

$$\alpha_{1,73} = \frac{3 \cdot 125}{\left(\frac{7}{1}\right)^2 \cdot \left(\frac{3}{2}\right)^2 \cdot \left(\frac{500 - 1}{83}\right)^{997}} \cdot \frac{1}{112 + 2 \cdot \frac{7 \cdot (2100 - 1) + 30}{67}}$$

$$= 1/137.035999037435$$
Supplement 27: The analogies between graph of the approximate formulas of the fine-structure constant and graph of the stability of nuclides

The stability of nuclides (Fig. 14) determines the abundance of elements in the universe (Fig. 15). For example, $^{56}\text{Fe}$ is the most stable nuclide, so the abundance of Fe in the universe is relatively high (called Fe peak) because massive stars create it; $^4\text{He}$ is a much stable nuclide, so He is the second rich element in the universe because the Big Bang and stars create it.
We noticed that there are morphological analogies between graph of approximate formulas of the fine-structure constant (Fig. 7 and Fig. 8) and graph of the stability of nuclides (Fig. 14). So we suppose that there would be corresponding relationships between the formulas of the fine-structure constant and elements (Table 9).

The corresponding relationships described in Table 9 are approximate rather than accurate. Among them, \( \alpha_{1\text{-}7} \) to \( ^2\text{He} \) and \( \alpha_{2\text{-}13} \) to \( ^2\text{He} \) should be the most important. \( \alpha_{1\text{-}22} \) to \( ^6\text{C} \), \( \alpha_{2\text{-}29} \) to \( ^6\text{C} \), \( \alpha_{1\text{-}170} \) to \( ^{26}\text{Fe} \) and \( \alpha_{1\text{-}253} \) to \( ^{26}\text{Fe} \) are also special. Essentially, these formulas of \( \alpha \) and the corresponding elements (\(^2\text{He} \) and \(^{26}\text{Fe} \)) are special respectively.
Table 9. The corresponding relationships between the formulas of $\alpha$ and elements

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>n</th>
<th>k</th>
<th>Elements</th>
<th>$\alpha_2$</th>
<th>n</th>
<th>k</th>
<th>Elements</th>
</tr>
</thead>
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<tr>
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<td>112</td>
<td>$\alpha He$</td>
<td>$\alpha_{2.13}$</td>
<td>100</td>
<td>278</td>
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<td>782</td>
<td>$\alpha C$</td>
<td>$\alpha_{2.29}$</td>
<td>223</td>
<td>655</td>
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<td>$\alpha {^{22}Ti}$</td>
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<td>3988</td>
<td>$\alpha {^{24}Cr}$</td>
<td>$\alpha_{2.205}$</td>
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<td>2092</td>
<td>$\alpha {^{28}Ni}$</td>
<td>$\alpha_{2.237}$</td>
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<td>20619</td>
<td>$\alpha {^{24}Cr}$</td>
</tr>
<tr>
<td>$\alpha_{1.207}$</td>
<td>1063</td>
<td></td>
<td></td>
<td>$\alpha_{2.253}$</td>
<td>1945</td>
<td>28186</td>
<td>$\alpha {^{28}Fe}$</td>
</tr>
</tbody>
</table>

Supplement 28: Relationships between Ramanujan series for $1/\pi$ (1914) and nuclides

$$1 = \frac{1}{4} \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!}{n!^4 \cdot (2n)!} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!}{n!^4 \cdot (2n)!} = \frac{(-1)^n(4n)!}{n!^4} \left( \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!}{n!^4 \cdot (2n)!} \right)$$

$$2\pi = \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!}{n!^4} \left( \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!}{n!^4 \cdot (2n)!} \right)$$

$$1 = \frac{1}{3528} \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!}{n!^4 \cdot (2n)!} = \frac{1}{42 \cdot 84} \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!}{n!^4 \cdot (2n)!}$$

$$2\pi = \sum_{n=0}^{\infty} \frac{(-1)^n(4n)!}{n!^4 \cdot (2n)!}$$

Elements: $\alpha_{1.7}$, $\alpha_{1.22}$, $\alpha_{1.29}$, $\alpha_{1.36}$, $\alpha_{1.43}$, $\alpha_{1.50}$, $\alpha_{1.59}$, $\alpha_{1.73}$, $\alpha_{1.81}$, $\alpha_{1.96}$, $\alpha_{1.103}$, $\alpha_{1.133}$, $\alpha_{1.140}$, $\alpha_{1.155}$, $\alpha_{1.170}$, $\alpha_{1.199}$, $\alpha_{1.207}$
\[
\frac{1}{\pi} = \frac{1}{72} \sum_{n=0}^{\infty} (-1)^n (4n)! (260n + 23) = \frac{1}{8.9} \sum_{n=0}^{\infty} (-1)^n (4n)! (n! 4^{4n} \cdot (2 - 9)^{2n})
\]

\[
2\pi = \frac{(4 \cdot 3)^{2}}{\sum_{n=0}^{\infty} (-1)^n (4n)! (1 - 8 \cdot 3 + 4 \cdot 5 \cdot 13 \cdot n)}
\]

Supplement 29: Other formulas of \(\alpha_1\) with Nilakantha formula for \(\pi\)

Nilakantha (1444-1544) formula for \(\pi\):

\[
\pi = 3 + \frac{4}{2 \cdot 3 \cdot 4} - \frac{4}{4 \cdot 5 \cdot 6} + \frac{4}{6 \cdot 7 \cdot 8} - \frac{4}{8 \cdot 9 \cdot 10} + \cdots
\]

Nilakantha-Chen formula for \(2\pi\):

\[
2\pi = 6 + \frac{1}{1(1 + 1/2)(1 + 1)} - \frac{1}{2(2 + 1/2)(2 + 1)} + \frac{1}{3(3 + 1/2)(3 + 1)} - \frac{1}{4(4 + 1/2)(4 + 1)} + \cdots
\]

\[
= 6 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n(n + 1/2)(n + 1)}
\]

\[
(2\pi)_{\text{NC-}k} = 6 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n(n + 1/2)(n + 1)}
\]

\[
\alpha_{1-7-\text{NC}} = \frac{36}{7 \cdot (2\pi)_{\text{NC-}7}} = \frac{1}{1/137.035999037435}
\]

\[
\alpha_{1-22-\text{NC}} = \frac{113}{2 \cdot (2\pi)_{\text{NC-}22}} = \frac{1}{1/137.035999037435}
\]

\[
\alpha_{1-29-\text{NC}} = \frac{149}{29 \cdot (2\pi)_{\text{NC-}29}} = \frac{1}{1/137.035999037435}
\]
\[\alpha_{1-36-NC} = \frac{5 \cdot 37}{4 \cdot 9 \cdot (2\pi)_{NC-5}} = \frac{1}{1/137.035999037435} = 1/137.035999037435\]

\[C = 4 \cdot 13 \cdot (16 \cdot 5 \cdot 17 \cdot 37 + 1) \approx 7 \cdot 11 \cdot 17 \cdot (2 \cdot 27 \cdot 37 + 1) \approx 2 \cdot 193 \cdot (4 \cdot 3 \cdot 5 \cdot 113 - 1)\]

\[\alpha_{1-81-NC} = \frac{4 \cdot 2 \cdot 26}{9 \cdot (2\pi)_{NC-9}} = \frac{1}{1/137.035999037435} = 1/137.035999037435\]

\[\alpha_{1-96-NC} = \frac{17 \cdot 29}{32 \cdot (2\pi)_{NC-15}} = \frac{1}{1/137.035999037435} = 1/137.035999037435\]

\[\alpha_{1-103-NC} = \frac{23}{2 \cdot 103 \cdot (2\pi)_{NC-15}} = \frac{1}{1/137.035999037435} = 1/137.035999037435\]

\[\alpha_{1-133-NC} = \frac{4 \cdot 9 \cdot 19 \cdot 1}{7 \cdot 19 \cdot (2\pi)_{NC-19}} = \frac{1}{1/137.035999037435} = 1/137.035999037435\]

\[\alpha_{1-140-NC} = \frac{4 \cdot 9 \cdot 20 \cdot 1}{7 \cdot 20 \cdot (2\pi)_{NC-9}} = \frac{1}{1/137.035999037435} = 1/137.035999037435\]

\[\alpha_{1-155-NC} = \frac{5 \cdot 31 \cdot (2\pi)_{NC-13}}{2 \cdot 3 \cdot 16 \cdot 43 - \frac{32 \cdot 47 \cdot (8 \cdot 7 \cdot 17 + 1)}{1012}} = 1/137.035999037435\]

\[\alpha_{1-170-NC} = \frac{9 \cdot 97}{170 \cdot (2\pi)_{NC-25}} = \frac{1}{1/137.035999037435} = 1/137.035999037435\]
Supplement 30: Other formulas of $\alpha_2$ with Nilakantha series for $\pi$

$\alpha_{1-199-\text{NC}} = \frac{2 \cdot 7 \cdot 73}{199 \cdot (2\pi)_{\text{NC}-9}} \frac{1}{112 + \frac{1}{29 \cdot (4 \cdot 73 - 1)}} = \frac{1}{137.035999037435}$

$\alpha_{1-207-\text{NC}} = \frac{2 \cdot 9 \cdot 59 + 1}{9 \cdot 23 \cdot (2\pi)_{\text{NC}-46}} \frac{1}{112 + \frac{1}{8 \cdot 3 \cdot (8 \cdot 3 \cdot 7 + 1)}} = \frac{1}{137.035999037435}$

$\alpha_{2-13-\text{NC}} = \frac{13 \cdot (2\pi)_{\text{NC}-5}}{100} \frac{1}{112 - \frac{1}{4 \cdot 9} + \frac{1}{2 \cdot 9 \cdot 89 - 1} - \frac{1}{3 \cdot 173 \cdot (16 \cdot 17 \cdot 23 + 1)}} = \frac{1}{137.035999111818}$

$\alpha_{2-29-\text{NC}} = \frac{29 \cdot (2\pi)_{\text{NC}-5}}{223} \frac{1}{112 - \frac{1}{3 \cdot 29} + \frac{1}{8 \cdot 3 \cdot 5 \cdot 19^2 + \frac{8}{15}}} = \frac{1}{137.035999111818}$

$\alpha_{2-36-\text{NC}} = \frac{36 \cdot (2\pi)_{\text{NC}-5}}{12 \cdot 23 + 1} \frac{1}{112 - \frac{1}{17} + \frac{1}{2 \cdot 11 \cdot 79} - \frac{1}{25 \cdot 17 \cdot (2 \cdot 25 \cdot 11 \cdot 17 - 1)}} = \frac{1}{137.035999111818}$

$\alpha_{2-45-\text{NC}} = \frac{45 \cdot (2\pi)_{\text{NC}-5}}{2 \cdot 173} \frac{1}{112 - \frac{1}{4 \cdot 11} + \frac{1}{47 \cdot (10 \cdot 23 - 1) - \frac{10}{199}}} = \frac{1}{137.035999111818}$

$\alpha_{2-61-\text{NC}} = \frac{61 \cdot (2\pi)_{\text{NC}-9}}{7 \cdot 67} \frac{1}{112 - \frac{1}{2 \cdot 157 - 1} + \frac{1}{B}} = \frac{1}{137.035999111818}$

$B = 16 \cdot 7 \cdot 13 \cdot 73 + \frac{23}{27} = 157 \cdot (4 \cdot 13^2 + 1) - \frac{4}{27} = 157 \cdot (6 \cdot 113 - 1) - \frac{4}{27}$
\[
\alpha_{2-77} = \frac{77 \cdot (2\pi)_{\text{NC}}}{16 \cdot 37} - \frac{1}{112} - \frac{1}{9 \cdot (8 \cdot 3 \cdot 5 \cdot 11 + 1) + \frac{3.37}{5.29}} = \frac{1}{137.035999111818}
\]
\[
\alpha_{2-93} = \frac{3.31 \cdot (2\pi)_{\text{NC}}}{5 \cdot 11.3} - \frac{1}{112 - \frac{2 \cdot 7.41}{1 + 4.5 \cdot 107.227}} = \frac{1}{137.035999111818}
\]
\[
\alpha_{2-109} = \frac{109 \cdot (2\pi)_{\text{NC}}}{121 - \frac{1}{112} - \frac{1}{3 \cdot 100} + \frac{25 \cdot 10^{11}}{1}} = \frac{1}{137.035999111818}
\]
\[
\alpha_{2-125} = \frac{125 \cdot (2\pi)_{\text{NC}}}{311} \frac{1}{4 \cdot (16 \cdot 17 - 1)} = \frac{1}{137.035999111818}
\]
\[
\alpha_{2-141} = \frac{3 \cdot 47 \cdot (2\pi)_{\text{NC}}}{112} \frac{1}{4 \cdot (16 \cdot 17 - 1)} = \frac{1}{137.035999111818}
\]
\[
\alpha_{2-157} = \frac{157 \cdot (2\pi)_{\text{NC}}}{171.71} \frac{1}{112} - \frac{1}{6 \cdot (2 \cdot 27.17 + 1) - 1} + \frac{1}{2 \cdot 7 \cdot 17 \cdot 111111111111111}
\]
\[
\alpha_{2-173} = \frac{173 \cdot (2\pi)_{\text{NC}}}{2 \cdot 5 \cdot 17} \frac{1}{112} - \frac{1}{6 \cdot (2 \cdot 27.17 + 1) - 1} + \frac{1}{2 \cdot 7 \cdot 17 \cdot 111111111111111}
\]
\[
\alpha_{2-189} = \frac{2 \cdot 7 \cdot (2\pi)_{\text{NC}}}{112} \frac{1}{4 \cdot 3 \cdot 111111111111111} = \frac{1}{137.035999111818}
\]
\[
\alpha_{2-205} = \frac{5 \cdot 41 \cdot (2\pi)_{\text{NC}}}{112} \frac{1}{8 \cdot 197} = \frac{1}{137.035999111818}
\]
\[
\alpha_{2-205} = \frac{5 \cdot 41 \cdot (2\pi)_{\text{NC}}}{112} \frac{1}{8 \cdot 197} = \frac{1}{137.035999111818}
\]
\[
\alpha_{2-205} = \frac{5 \cdot 41 \cdot (2\pi)_{\text{NC}}}{112} \frac{1}{8 \cdot 197} = \frac{1}{137.035999111818}
\]
\[ \alpha_{2-221-NC} = \frac{13 \cdot 17 \cdot (2\pi)_{N_{C-19}}}{1700 - 1} - \frac{1}{112} - \frac{3 \cdot 23 \cdot (2 \cdot 9 \cdot 11 + 1)}{5 \cdot 41} = 1/137.035999111818 \]

\[ \alpha_{2-237-NC} = \frac{3 \cdot 79 \cdot (2\pi)_{N_{C-21}}}{2 \cdot (2 \cdot 5 \cdot 7 \cdot 13 - 1)} - \frac{1}{112} - \frac{8 \cdot 25 \cdot 71}{11 \cdot 19} + \frac{49}{53} = 1/137.035999111818 \]

\[ \alpha_{2-253-NC} = \frac{11 \cdot 23 \cdot (2\pi)_{N_{C-23}}}{5 \cdot (4 \cdot 97 + 1)} - \frac{1}{112} - \frac{3 \cdot 5 \cdot 11^2 \cdot 29}{3 + 29} = 1/137.035999111818 \]

\[ \alpha_{2-269-NC} = \frac{(4 \cdot 67 + 1) \cdot (2\pi)_{N_{C-27}}}{4 \cdot 11 \cdot 47} - \frac{1}{112} - \frac{1}{11 \cdot 13 \cdot 163} - \frac{83}{13 \cdot 17} = 1/137.035999111818 \]

**Supplement 31:** Other formulas of \( \alpha \) with Beeler formula for \( \pi/2 \)

Beeler formula for \( \pi/2: \)

\[ \frac{\pi}{2} = 1 + \sum_{n=1}^{\infty} \frac{n!}{(2n+1)!!} = 1 + \frac{1}{3} + \frac{1 \cdot 2}{3 \cdot 5} + \frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7} + \cdots \]

\[ (\frac{\pi}{2})_{Beeler-k} = 1 + \sum_{n=0}^{k} \frac{n!}{(2n+1)!!} = 1 + \frac{1}{3} + \frac{1 \cdot 2}{3 \cdot 5} + \frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7} + \cdots + \frac{k!}{(2k+1)!!} \]

\[ \alpha_{1-207-Beeler} = \frac{2 \cdot 9 \cdot 59 + 1}{4 \cdot 9 \cdot 23 \cdot (\frac{\pi}{2})_{Beeler-17}} - \frac{1}{112} + \frac{2 \cdot 9 \cdot 59 + 1}{2 \cdot 9 \cdot 23 \cdot (\frac{\pi}{2})_{Beeler-17}} = 137.035999037435 \]
Supplement 32: Other formulas of $\alpha_2$

\[
\alpha_{2-522} = \frac{(18 \cdot 223 - 1) \cdot e^2 \cdot e^2 \cdot e^2 \cdot \ldots \cdot e^2}{(2^2 \cdot 3^5 \cdot 5 \cdot 83)^{163}} \cdot \frac{1}{18.29} \cdot \frac{1}{112 - \delta_2}
\]

\[
= 1/137.035999111818
\]

\[
\delta_2 = \frac{27 \cdot 13 \cdot 47}{25 \cdot 10^{11}} \quad \text{or} \quad \frac{3 \cdot 53 \cdot 83}{2 \cdot 10^{12}} \quad \text{or} \quad \frac{2 \cdot 73 \cdot 113}{25 \cdot 10^{11}}
\]

\[
\alpha_{2-522-\text{Wallis}} = \frac{1}{18.29} \cdot \frac{1}{112 - \frac{4 \cdot (12 \cdot (8 \cdot 9 \cdot 7 - 1) + 1)}{25 \cdot 10^{11}}}
\]

\[
= 1/137.035999111818
\]

\[
\alpha_{2-522-\text{GL}} = \frac{(18 \cdot 223 - 1) \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots + \frac{1}{2 \cdot 2 \cdot 4 \cdot 9 \cdot (14 \cdot 47 + 1) + 1}\right)}{18.29} \cdot \frac{1}{112 - \frac{37 \cdot (2 \cdot 3 \cdot 47 - 1)}{25 \cdot 10^{11}}}
\]

\[
= 1/137.035999111818
\]

\[
\alpha_{2-522-\text{NC}} = \frac{(18 \cdot 223 - 1) \cdot (2\pi)_{\text{NC}-25}}{18.29} \cdot \frac{1}{112 - \frac{5 \cdot 7}{3 \cdot 31}} \cdot \frac{1}{9 \cdot (2 \cdot 7 \cdot 173 + 1)} - \frac{2 \cdot 19}{101}
\]

\[
= 1/137.035999111818
\]
\[ \alpha_{2,\text{-}27,09} = \frac{3 \cdot (4 \cdot 3 \cdot 13^2 - 1) \cdot e^2 \cdot \frac{2}{1} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdots \cdot \frac{2 \cdot 7 \cdot 9 \cdot 89}{23 \cdot (10 \cdot 157 + 1)}^{33/17 + 109}}{7 \cdot 113} = \frac{1}{112 - \frac{7 \cdot (4 \cdot 100 + 1)}{4 \cdot 10^2}} \]

\[ \alpha_{2,\text{-}27,091 - \text{Wallis}} = \frac{3 \cdot (4 \cdot 3 \cdot 13^2 - 1) \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots - \frac{1}{2 \cdot 128 - 49 + 11 + 1})}{7 \cdot 113} = \frac{1}{112 - \frac{17 \cdot 23}{8 \cdot 10^2}} \]

\[ \alpha_{2,\text{-}27,091 - \text{GL}} = \frac{3 \cdot (4 \cdot 3 \cdot 13^2 - 1) \cdot (2 \pi)^{\text{NC,25}}}{7 \cdot 113} = \frac{1}{112 - \frac{2 \cdot 5 \cdot 49 \cdot 193}{3 \cdot 7 \cdot 2 \cdot 11}} = \frac{1}{112 - \frac{2 \cdot 9 \cdot 103}{5 \cdot 10^2}} \]

\[ \alpha_{2,\text{-}29,092 - \text{Wallis}} = \frac{5 \cdot 23 \cdot 73 \cdot e^2 \cdot \frac{2}{1} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdots \cdot \frac{4 \cdot 23 \cdot (2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 - 1)}{3^2 \cdot 11^2 \cdot 43} \cdot e^2}{4 \cdot 3 \cdot 7 \cdot 13} = \frac{1}{112 - \frac{2 \cdot 29 \cdot 103}{5 \cdot 10^2}} \]

\[ \alpha_{2,\text{-}29,092 - \text{GL}} = \frac{5 \cdot 23 \cdot 73 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots - \frac{1}{2 \cdot 23 \cdot 73})}{4 \cdot 3 \cdot 7 \cdot 13} = \frac{1}{112 - \frac{2 \cdot 7 \cdot (2 \cdot 125 + 1)}{25 \cdot 10^{11}}} \]

\[ \alpha_{2,092,902 - \text{Wallis}} = \frac{5 \cdot 23 \cdot 73 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots - \frac{1}{2 \cdot 23 \cdot 73})}{4 \cdot 3 \cdot 7 \cdot 13} = \frac{1}{112 - \frac{2 \cdot 7 \cdot (2 \cdot 125 + 1)}{25 \cdot 10^{11}}} \]

\[ \alpha_{2,092,902 - \text{GL}} = \frac{5 \cdot 23 \cdot 73 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots - \frac{1}{2 \cdot 23 \cdot 73})}{4 \cdot 3 \cdot 7 \cdot 13} = \frac{1}{112 - \frac{8 \cdot 107 + 1}{25 \cdot 10^{11}} \text{ or } \frac{6 \cdot (6 \cdot 47 - 1)}{5 \cdot 10^2}} = \frac{1}{112 - \frac{8 \cdot 107 + 1}{25 \cdot 10^{11}}} \]
Regarding the fine-structure constant, Richard Feynman said: “is it related to \( \pi \) or perhaps to the base of natural logarithms?\(^4\) Our answer is that it relates to \( 2\pi-e \), \( 2\pi \), \( \pi/2 \) and \( \pi/4 \) formulas. He also deduced that the maximum element should be the 137\(^{th} \) element Fynmanium (Fy) based on the analyses of the electron line velocity of his ideal hydrogen-like atoms. Our answer is that the natural end of the elements is the 112\(^{th} \) element Copernicium (Cn\(^* \)), but the elements could have some ideal extensions, and above all, the fine-structure constant does relate to elements.

So, based on the analyses of ideal and real natural maximum element, Chen’s Chirality and Poetry Model of Atomic Nuclei\(^7\) and \( 2\pi-e \) formula\(^6.7.8,\) we deduced two series of Chen’s formulas of the fine-structure constant which gave two values \( \alpha_1 = 1/137.035999037435 \) and \( \alpha_2 = 1/137.035999111818 \). The factors in the formulas are much coincident to nucleon numbers of some nuclides, this means the formulas should be correct (too many coincidences mean too few possibilities to be wrong, or too many coincidences imply science). And we indicate the reason of \( \alpha \approx 1/137.036 \) is that it’s almost the equal ratio factor between 112 and 168 (more precisely 168-1/3) which are the key stable numbers (magic numbers) in Chen’s Chirality and Poetry Model of Atomic Nuclei\(^7\).

With Chen’s formulas of the fine-structure constant, we predicted the nucleon numbers of all 119\(^{th} \) to 170\(^{th} \) ideal extended elements; we theoretically or mathematically calculated the speed of light in atomic units, i.e., \( \varepsilon_{\text{au}} = 1/\alpha = 1/(\alpha_1 \alpha_2)^{1/2} = 137.035999074627 \); we deduced a concise Schrödinger-Chen equation of hydrogen atom which included \( \alpha_1/\alpha_2 \) factor; in analogy to \( \alpha \) and its formulas, \( \alpha_p \) (the second fine-structure constant) and its formulas were hypothesized, and the proton charge radius \( r_p \) was supposed to be 0.833027203 fm; in the end we discovered that the approximate rational numbers of \( 2\pi \) marvelously and directly related to nuclides.
Based on these, a mathematic shell model of elements was established and a picture of elements and ideal extended elements was depicted.

In their relations to nuclides, \(2\pi\) formulas can only be certain approximate rational numbers and \(2\pi\)-e formulas in Chen’s formulas of the fine-structure constant can only take certain \(k\) values. So we also believe the two values of the fine-structure constant should be rational numbers with definite digits rather than irrational numbers with infinite digits, and actually the fine-structure constant is transformed to nucleon numbers of 136, 137 and 138 in the world of nuclides.

In a recent paper\textsuperscript{11}, physicist Nicolas Gisin commented that in 1920s there once was a debate between mathematicians David Hilbert and Luitzen Egbertus Jan Brouwer. Hilbert was promoting formalized mathematics, in which every real number with its infinite series of digits is a completed individual object. On the other side the Luitzen Egbertus Jan Brouwer was defending the view that each point on the line should be represented as a never-ending process that develops in time, a view known as intuitionistic mathematics. Hilbert and his supporters clearly won the debate. In consequence, formalized mathematics has been adopted as the language of physics. In the end of his paper, Nicolas Gisin said: “Physics can be as successful if built on intuitionistic mathematics, even if this breaks its marriage to determinism. Contrary to usual expectations, I bet that the next physical theory will not be even more abstract than quantum field theory, but might well be closer to human experience.”

In this paper we adopted mathematical language like intuitionistic mathematics, but we go ahead even more. The formulas of \(2\pi\), \(2\pi\)-e and the fine-structure constant consist of integer factors and relate to nucleon numbers of nuclides, and hence correlate with each others. So in this paper we may use super-intuitionistic mathematics or decoding methodology with features of multi-correlations of integer factors or rational numbers which relate to nucleon numbers of nuclides, and it seems it is the real language in the world of nuclides. As we know an atomic nucleus is a N-body system and chaos should be its real state, so it seems N-body chaos returns to integers. In overall, Leopold Kronecker’s famous saying “God made the integers, all else is the work of man” should be correct in the world of nuclides or even in other fields of the real world. It seems an irrational number can only be a rational number to play roles in the real world.
“God is a pure mathematician!” declared British astronomer Sir James Jeans (1877-1946). The physical Universe does seem to be organized around elegant mathematical relationships. The fine-structure constant may be the most important number in physics. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. And we have successfully given reasonable and precise formulas of it. In some sense, we explain the bridge between mathematics and physics, or we may realize the unification of mathematics and physics. It seems we prove the saying “God is a pure mathematician”. At least, it seems that good mathematics means good physics, and some pure mathematical numbers do have scientific meanings.

References
1. Wikipedia. The fine-structure constant.
6. G. Chen and T. Chen, Copyright Registration, Chen’s Periodic Table of Elements and Natural Group Theory, GuoZuoDengZi-2018-L-00472808.
Acknowledgements

The main author Dr. Gang Chen studied in the Department of Chemistry at Sichuan University from 1983 to 1987 (B. Sc.), in Institute of Chemistry of the Academy of Sciences of China from 1987-1990 (M. Sc. Under the supervision of Prof. Rongben Zhang), in the Hong Kong Polytechnic University from 1999 to 2004 (PhD and research assistant under the supervision of Prof. Albert Sun-Chi Chan) and in Kyoto University from 2004 to 2005 (postdoctoral research under the supervision of Prof. Tamio Hayashi).

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## Appendix I: Research History

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### Parameters and results of $\alpha_{2,m}$

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*Note:* The parameters and results are presented in a tabular format, with each parameter listed along with its corresponding date range. The dates range from 2018/4/12 to 2020/3/5, with some dates indicating a specific duration or event. The table includes various parameters with different subscripts, indicating different data points or categories. The last entry includes a note indicating a transition or reference to another source (280→278 et al.).
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### Direct relationships between 2π and nuclides

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### Chen’s Picture of Elements and Ideal Extended Elements

| 41 | 2018/1-3 | 2020/2/2-5 | 2020/2/12, 16, 17, 19, 22-24 |

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Note: Dates were recorded according to Beijing Time; ie means ideal extended elements; GL means Gregory-Leibniz formula.