Chen’s Formulas of the Fine-structure Constant

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract
This paper gives two series of formulas of the fine-structure constant α which are reasonable, precise, smart and elegant. It also demonstrates there are two values of α, i.e., \( \alpha_1 = 1/137.035999037435 \) and \( \alpha_2 = 1/137.035999111818 \), which are consistent with but much more accurate than those experiment measured values. The formulas consist of 2π-e formulas and some factors related to nucleon numbers of nuclides. A brief explanation of the fine-structure constant shows \( 1/\alpha \approx 137.036 \) is the equal ratio factor between 112 and 168 (more precisely 168 -1/3). Based on these, all 119th to 170th ideal extended elements were predicted, the speed of light in atomic units was mathematically calculated by \( c_{\text{au}} = 1/(\alpha_1 \alpha_2)^{1/2} = 137.035999074627 \), Schrödinger equation of hydrogen atom was simplified and correlated with \( \alpha_1/\alpha_2 \), classical electron radius was calculated to be 2.81794032658(43) fm and proton charge radius was hypothetically calculated to be 0.83302720299(13) fm. In the end, it was found that the approximate rational numbers of 2π marvelously related to nuclides, a mathematic shell model of nuclides was established and a picture of elements and ideal extended elements was depicted.

Keywords: formulas; the fine-structure constant; the ideal extended elements; the speed of light; Schrödinger equation of hydrogen atom; the proton charge radius; 2π.

1. Introduction
The fine-structure constant (Sommerfelt constant) is a critical dimensionless constant in physics, it is a century mystery of physics, it has been one of the biggest enigmas in physics since it was introduced by Arnold Sommerfeld in 1916. Its definition, some interpretations and the latest measured values are as follows:\(^1,^2:\)

\[
\alpha = \frac{\lambda_e}{2\pi a_0}, \quad \alpha = \frac{2\pi a}{\lambda_e}, \quad a_0 = \frac{1}{\alpha}, \quad \alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c}, \quad \frac{c}{v_e} = \frac{1}{\alpha}
\]

in atomic units, the speed of light \( c_{\text{au}} = \frac{1}{\alpha} \)

the 2014 CODADA recomended value: \( \alpha = 1/137.035999139(31) \)
the 2018 CODADA recomended value: \( \alpha = 1/137.035999084(21) \)
Science 13 April 2018 reported value: \( \alpha = 1/137.035999046(27) \)
The ratio of Bohr radius of hydrogen atom $a_0$ to the classical electron radius $r_e$ is $1/\alpha^2$. The ratio of the speed of light $c$ to the line velocity of ground state electron in hydrogen atom $v_e$ is $1/\alpha$, this means in atomic units $c=1/\alpha$ and $E=mc^2=m/\alpha^2$ or $\alpha^2=m/E$. In quantum electrodynamics it substantially characterizes the strength of electromagnetic interaction between elementary charged particles such as electron and proton, so it is the coupling constant of electric charges. It is one of the 25 fundamental constants (could not be calculated theoretically, could only be determined by experiments) in Standard Model of physics and should be the most important one. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. However, to our knowledge, up to now (except this work), no one knows how it comes from, no one could give reasonable explanations to it or formulas of it since it was introduced.

In 2016 Paul Davis gave the following comment: “Physicists have long wondered where this number, $1/137.035999$, comes from. Is there a deep reason why $\alpha$ has to be precisely this number for the world to function as it does? There is a long history of attempts to derive $\alpha$ from physical theory or to concoct a mathematical formula that has this value. For a brief time in the 1920s, when it looked as if $\alpha$ might be exactly $1/137$, astronomer Arthur Eddington searched for a theory that would throw up the numbers naturally, but his ideas ultimately led nowhere. Then in 1969 a young Swiss mathematician, Armand Wyler, pointed out that $(9/16\pi^3)(\pi/5!)^{1/4}$ comes close to $1/137.036$, which matched the value of $\alpha$ to the precision known at the time. However, his formula was not accompanied by any credible theory and was regarded as little more than a numerical curiosity. Several other attempts at $\alpha$ numerology have been made since, none of which have gained traction in the physics community.”

As for the fascination of the fine-structure constant, in the middle of 1980s, Richard Feynman stated: “It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to $\pi$ or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the hand of God wrote that number, and we don't know how He pushed his pencil.”

This paper shows how God pushed his pencil to write the fine-structure constant and how God used it to coordinate elements.
2. \(2\pi-e\) formula(s)

\(2\pi-e\) formula, its related formulas and their preliminary applications were deduced independently by us from April to December of 2013.

Fig. 1. Diagram of \(y=1/x\).

Fig. 2. Diagram of \(y=\log(x)\).

Euler-Mascheroni constant \(\gamma\): \[\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots = \ln \infty + \gamma\]

As for \(y=1/x\) (Fig. 1), \(\gamma = 0.577215\cdots = 0.5 + 0.077215\cdots = \sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{2} + \gamma_i\)

\[\gamma_i = \sum_{n=1}^{N-1} \frac{1}{n} \left( \int_1^n \frac{1}{x} \, dx \right) - \frac{1}{2}, \quad \text{Generally, } \gamma = \lim_{N \to \infty} \left( \sum_{n=1}^{N-1} \frac{1}{n} \right) - \frac{1}{2}, \quad s \in \mathbb{N}\]

As for \(y=\log(x)\) (Fig. 2), \[\delta_{\log} = \int_1^{e^{x+1}} \frac{1}{x} \, dx = \frac{1}{2} \ln(n+1) - \frac{1}{2} \ln n = (x \ln x) \bigg|_{x=1}^{x=n+1} - \frac{1}{2} \ln(n+1) - \frac{1}{2} \ln n\]

\[\gamma_e = \gamma_e \cdot \frac{n!}{n} \exp(-\gamma_k \cdot e) = \frac{n!}{n} \exp(-\gamma_k \cdot e)\]

\[\ln(N!) = \sum_{n=1}^{N} \ln n = \int_1^{N} \frac{n+1}{n} \, dx - \sum_{n=1}^{N} \frac{1}{n} \exp(-\gamma_k \cdot e) = \int_1^{N} \frac{n+1}{n} \, dx - \sum_{n=1}^{N} \frac{1}{n} \exp(-\gamma_k \cdot e)\]

\[\ln(N!) = (N+1) \ln(e) - \frac{1}{2} \ln(N+1) = \frac{(N+1) \ln(e)}{e^{2\pi}} \approx \sqrt{2\pi N \left( \frac{N}{e} \right)^{N+1}}\]

\[\ln(N!) = (N+1)^{N+1} \left( \frac{N}{e} \right)^{N+1}\]

\(\gamma = 0.0810614668, \quad e^\gamma = 1.0844375\)

\[2\pi - e \text{ formula(s): } 2\pi = \left( \frac{e}{e^x} \right)^2 = e^2 - e^2 \left( \frac{1}{2} \right)^{\gamma_k} - e^2 \left( \frac{3}{2} \right)^{\gamma_k} - e^2 \left( \frac{5}{2} \right)^{\gamma_k} - \cdots\]

\[\gamma_k = 0.0810614668, \quad e^\gamma = 1.0844375\]
2π-e formula is an expanding form of Stirling formula. To our knowledge, it was first deduced by us. If it was new, it could be named Chen’s 2π-e formula.

3. Some Formulas Related to 2π-e Formula

The following formulas which correlate each other and has similar form could be called Chen’s natural group formulas, and the form is called natural group.

\[ 1 = 4\gamma_1 + \frac{4\gamma_2}{1(1+1)} + \frac{4\gamma_3}{2(2+1)} + \frac{4\gamma_4}{3(3+1)} + \cdots \]

\[ = \left[|B| \frac{\pi}{2} + \sum_{n=1}^{N} \left|B_{2n}\right| \left(\frac{\pi}{2}\right)^{2n} \right] \left(\frac{2}{(2n)!}\right) = \sum_{n=1}^{N} \left|B_{2n}\right| \left(\frac{\pi}{2}\right)^{2n} = -\left|B\right| \frac{3\pi}{2} + \sum_{n=1}^{N} \left|B_{2n}\right| \left(\frac{3\pi}{2}\right)^{2n} \left(\frac{2}{(2n)!}\right) \]

\[ N \sim \frac{3}{2} \left|B\right| + \sum_{n=1}^{N} \left|B_{2n}\right| \left(\frac{2\pi}{2}\right) \left(\frac{2}{(2n)!}\right) \]

\[ e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \]

\[ 2\pi = \frac{\left(\frac{\pi}{e}\right)^{2}}{\left(\frac{\pi}{e}\right)^{2}} + \frac{e^{2}}{\left(\frac{\pi}{e}\right)^{2}} + \frac{e^{3}}{\left(\frac{\pi}{e}\right)^{2}} + \cdots \]

B, B_{2n}: the Bernoulli numbers such as \(-\frac{1}{2}, \frac{1}{6}, \frac{1}{30}, \frac{1}{42}, \frac{1}{30}, \cdots\)

\[ \gamma_{c} = \lim_{N \to \infty} \left\{ \frac{2}{N} \sum_{n=1}^{N} \log(n) - \frac{\log(N+1)}{2} \right\} = 0.0810614668 \]

\[ \gamma_{s} = \lim_{N \to \infty} \left\{ \frac{1}{N} \sum_{n=1}^{N} 1 - \frac{1}{2s} \right\}, \quad s \in \mathbb{N} \]

\[ \gamma_{1} = 0.077215, \quad \gamma_{2} = 0.144934, \quad \gamma_{3} = 0.248999, \quad \gamma_{4} = 0.36122, \quad \gamma_{6} = 0.433349, \quad \cdots \]

\[ \gamma_{c}, \gamma_{1}, \gamma_{2}, \gamma_{3}, \cdots \] are called Chen's natural group constants (analog to Bernoulli numbers).

The following are some other formulas related to 2π-e Formula.

\[ \sqrt{2\pi} = e^{\gamma_{c}}, \quad e = \sqrt{2\pi e^{\gamma_{c}}} = \sqrt{2\pi (1 + \sum_{n=1}^{N} \frac{\gamma_{c}^{n}}{n!})} \]

\[ \gamma_{c} = \sum_{n=1}^{N} \left(\ln(1 + \frac{1}{n}) - 1\right) = \sum_{n=1}^{N} \frac{(2^{n} - 1)}{2(2n+1)} \left|B_{2n}\right| \pi^{2n} - 2(2n)! = \frac{1}{4} \sum_{s=1}^{n} \gamma_{s} s(s+1) \]

\[ \gamma_{s} = \sum_{n=1}^{N} \left(\ln(1 + \frac{1}{n}) - 1\right) - \int_{1}^{N} \left(\ln(x) - 1\right) \ln(1 + \frac{1}{x}) \, dx \]

\[ \gamma_{c} = \frac{1}{2} \lim_{N \to \infty} \left\{ \frac{1}{N} \sum_{n=1}^{N} \left(\frac{2^{n} - 1}{n} \left|B_{2n}\right| \pi^{2n} \right) - \ln N \right\} \]

\[ \frac{\pi}{2} = \frac{e}{e^{\gamma_{c}}}, \quad e = \sqrt{\frac{\pi}{2}} = \sqrt{\frac{\pi}{2} \left(1 + \sum_{n=1}^{N} \frac{\gamma_{c}^{n}}{n!}\right)}; \quad \frac{\pi}{2} = \frac{e^{\gamma_{c}}}{e^{\gamma_{c}}} \quad \gamma_{c} = \ln \left(\frac{\pi}{2} + 2 \gamma_{c}\right) \]

\[ \gamma_{c} = \gamma_{c} - \ln 2 = 1 - \frac{\gamma_{c}}{2} - \gamma_{c} - \ln 2, \quad \gamma_{c} = 1 + \sum_{s=1}^{n} \frac{\gamma_{s}}{s(s+1)} - \ln 2 \]

\[ \gamma_{c} = 0.0810614668, \quad \gamma_{c} = 0.7742086474, \quad \gamma_{c} = 0.0628164798 \]

\[ \frac{\pi}{2} = \sum_{n=1}^{N} \left|B_{2n}\right| \pi^{2n} = \sum_{n=1}^{N} \left|B_{2n}\right| \left(\frac{\pi}{2}\right)^{2n} \frac{1}{(2n)!}; \quad \sum_{n=1}^{N} \zeta(2n) - 1 = \frac{3}{4}, \quad \zeta(2n) = \sum_{k=1}^{N} \frac{1}{k^{2n}} \]

\[ \sum_{n=1}^{N} \frac{1}{2n} \left|B_{2n}\right| \left(\frac{2\pi}{2}\right)^{2n} = \sum_{n=1}^{N} \left|B_{2n}\right| \left(\frac{2\pi}{2}\right)^{2n} \frac{1}{(2n)!} \]

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4. Some Applications of 2π-e Formula and its Related Formulas

(1). 2π-e formula is basically an algebraic expanding of Stirling formula, but it is more meaningful, it exhibits the relationship between 2π and e. In 2π-e formula, γc is a real constant with geometric definition like Euler-Mascheroni constant γ. With 2π-e formula and its related formulas, 2π can be calculated from e and vice versa. So it is the real 2π-e relationship formula.

\[
2\pi = \left( \frac{e^{e^{x}}}{e^{x}} \right)^{2} = e^{2} \prod_{n=1}^{\infty} \frac{e^{x}}{(1 + \frac{1}{n})^{2n+1}} = e^{2} \left( \frac{\gamma_{c}}{1} \right)^{2} \left( \frac{\gamma_{c}}{2} \right)^{3} \left( \frac{\gamma_{c}}{3} \right)^{5} \left( \frac{\gamma_{c}}{4} \right)^{7} \ldots
\]

\[
e = \sqrt{2\pi e^{x}} = \sqrt{2\pi \left( 1 + \sum_{n=1}^{\infty} \frac{\gamma_{c}^{n}}{n!} \right)}, \quad \gamma_{c} = \sum_{n=1}^{\infty} \frac{(2^{2n-1} - 1)B_{2n}}{2(2n+1)!}
\]

(2). 2π-e formula demonstrates 2π is a natural constant rather than π. π/2 is somewhat fundamental but not as complete as 2π. π is neither fundamental nor complete. In 2001 mathematician Bob Palais said “π is wrong”\(^5\). 2π-e formula and the Taylor expansion of e have similar form (natural group form), this should give a conclusive proof that 2π is a real natural constant and π is not.

\[
2\pi = \left( \frac{e^{e^{x}}}{e^{x}} \right)^{2} = e^{2} \left( \frac{\gamma_{c}}{1} \right)^{2} \left( \frac{\gamma_{c}}{2} \right)^{3} \left( \frac{\gamma_{c}}{3} \right)^{5} \left( \frac{\gamma_{c}}{4} \right)^{7} \ldots \Rightarrow 2\pi \text{ or } \sqrt{2\pi} \text{ is a natural constant}
\]

\[
\frac{\pi}{2} = \left( \frac{e^{e^{x}}}{e^{x}} \right)^{2} = \left( \frac{\gamma_{c}^{2}}{e^{x}} \right)^{2} \Rightarrow \frac{\pi}{2} \text{ or } \sqrt{\frac{\pi}{2}} \text{ is almost a natural constant}
\]

\[
\pi = \left( \frac{e^{e^{x}}}{e^{x}} \right)^{2} = \left( \frac{\gamma_{c}^{2}}{e^{x}} \right)^{2} \Rightarrow \pi \text{ or } \sqrt{\pi} \text{ is not a natural constant}
\]

Table 1 lists some points of view of Piist who support π is a natural constant, Tauist who support 2π is a natural constant and this work which supports the later.

Table 1. Comparison of points of view of Piist, Tauist and this work.

<table>
<thead>
<tr>
<th></th>
<th>Piist</th>
<th>Tauist</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference of a circle</td>
<td>πd</td>
<td>2πR</td>
<td>2πR</td>
</tr>
<tr>
<td>Area of a circle</td>
<td>πR(^2)</td>
<td>(1/2)(2πR)R</td>
<td></td>
</tr>
<tr>
<td>Volume of sphere</td>
<td>(4/3) πR(^3)</td>
<td>(2/3)(2π)R(^3)</td>
<td>(2πR(^2)/3)2R</td>
</tr>
<tr>
<td>Volume of n-dimension sphere</td>
<td>(\frac{\pi^{n/2}}{\Gamma(n/2+1)}R^{n})</td>
<td>(\frac{(2\pi)^{n/2}}{\Gamma(n/2+1)}R^{n})</td>
<td>(\frac{2\pi R^{2}}{n} V_{n-2})</td>
</tr>
<tr>
<td>Euler’s identity</td>
<td>(e^{i\pi} + 1 = 0)</td>
<td>(e^{2\pi i} = 1)</td>
<td>(e^{2\pi i} = 1)</td>
</tr>
<tr>
<td>Gauss integral</td>
<td>(\int_{-\infty}^{\infty} e^{-x^{2}} dx = \sqrt{\pi})</td>
<td>(\int_{-\infty}^{\infty} e^{-x^{2}} dx = \frac{e}{\sqrt{\pi}})</td>
<td>(\int_{-\infty}^{\infty} e^{-x^{2}} dx = \frac{e}{\sqrt{\pi}})</td>
</tr>
</tbody>
</table>

(3). As 2π is a square number, the frequent appearing of its square root in some
important equations such as Gaussian distribution (normal distribution) and Maxwell–Boltzmann distribution becomes reasonable and understandable. And the distributions can be transformed as follows.

Standard Normal Distribution: \( f(x, 0, 1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = e^{-\frac{x^2+2(1-\gamma)}{2}} \)

Maxwell–Boltzmann Distribution: \( f(v) = \frac{2}{\sqrt{2\pi}} v^2 \left( \frac{m^2}{kT} \right) e^{-\frac{m^2}{2kT}} = 2\left( \frac{m^2}{kT} \right)^{\frac{3}{2}} v^2 e^{-\frac{m^2}{2kT}} \)

(4). Euler’s identity (Euler’s equation) \( e^{i\pi} + 1 = 0 \) is called God formula and the most beautiful formula in mathematics. However, as \( 2\pi \) is the real natural constant and \( \pi \) is not, \( e^{2\pi} = 1 \) should be more beautiful.

(5). \( \gamma = \ln(2\pi) + \gamma_{CB} \) may help to prove \( \gamma \) is an irrational number or even a transcendental number.

(6). The natural group formulas help us to establish “Chen’s Periodic Table of Elements and Natural Group Theory”\(^\text{6} \) (2014-2017).

(7). The mathematic expression of chirality is \( \pm 2\pi \). This concept is helpful for us to establish “Chirality and Poetry Model of Atomic Nuclei”\(^\text{7} \) (2017/12-2018/3).

(8). Based on the above theories, Chen’s theory of the fine-structure constant was deduced (2018/4-6)\(^\text{8} \) and has been revised, modified and improved (2018/7-2020/1).

5. Original Inspiration for Formulas of the Fine-structure Constant

1. According to \( \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{\lambda_e}{2\pi a_0} = \frac{2\pi e\epsilon}{\lambda_e} \approx \frac{1}{137.036} \), the formulas of \( \alpha \) should relate to \( 2\pi \).

2. \( \frac{137.036}{2\pi} = \frac{137.036}{6.28318} \approx 21.81 \). \( 137.036 = 21.81 \times 2\pi \)

3. According to \( 2\pi \)-e formula: \( 2\pi = (\frac{e}{e^2})^2 = e^2 \left( \frac{2}{1} \right)^2 \left( \frac{3}{2} \right)^2 \left( \frac{4}{3} \right)^2 \cdots \)

\( 2\pi \) is a square number, suppose \( 21.81 = x^2 \), \( x = 4.670 \approx 14 / 3 \)

so: \( \frac{1}{\alpha} \approx (\frac{14}{3})^2 2\pi \) or \( \alpha \approx (\frac{3}{14})^2 \frac{1}{2\pi} \) (Discover: about 2 am on 2018/4/12)

4. Apply with \( 2\pi \)-e formula (in the afternoon of 2018/4/12, a meeting in the morning)

\( \alpha = (\frac{3}{14})^2 \left( \frac{1}{(2\pi)_{112}} \right) = (\frac{3}{14})^2 \left( \frac{1}{2\pi} \right)^2 \left( \frac{e^2}{\gamma_2} \right) \left( \frac{e^2}{\gamma_3} \right) \cdots = 137.035781520, \) closest to the real value.

\( \left( \frac{1}{2\pi} \right)^2 \left( \frac{e^2}{\gamma_2} \right) \left( \frac{e^2}{\gamma_3} \right) \cdots = \frac{113}{112}^{225} \)

As 112 is one of the most important stable numbers and the 112th element \( ^{205}_{112} \text{Cn} \) is the natural end of elements according to our Chen’s Chirality and Poetry Model of Atomic Nuclei\(^6 \).

So: Eureka! Subsequently transformed to: \( \alpha = \frac{6^2}{7(2\pi)_{112}} = 137.035781520, \)

Finally modified to: \( \alpha = \frac{6^2}{7(2\pi)_{112}} \left( \frac{1}{112} + \frac{1}{75} \right) = 137.035999037435 \)

6
6. Logical Deduction of Chen’s Formulas of the Fine-structure Constant

Physicist Richard Feynman noticed a hydrogen-like atom with Z protons and only one electron, according to Bohr model, the line velocity of the nth rank electron $v_{el/n}$ satisfies:

$$v_{el/n} = \frac{Z e^2}{n^2 \pi \alpha c} \approx \frac{Z e^2}{n^2 c}, \quad \text{as} \quad v_{el/n} \leq c, \quad \alpha = \frac{v_{el/n}}{c} \approx \frac{1}{Z_{\text{max-ideal}}} = \frac{1}{Fy} = \frac{1}{137}$$

The 137th hydrogen-like element Fy (Feynmanium) is an ideal (imaginative) element, in reality, the above formula should be modified to: $\alpha = f(Z_{\text{real}}) = \frac{1}{Z_{\text{max-real}}}$

According to Chen’s Chirality and Poteory Model of Atomic Nuclei,

$$Z_{\text{max-real}} = 112 = 2 \cdot 56, \quad \text{so} \quad \alpha = f(Z_{\text{real}}) = f(Z_{\text{real}}) = \frac{1}{112}$$

Compared to $\alpha = \frac{\lambda_c}{2\pi a_0}$, the formula should have a $2\pi$ factor:

$$\alpha = f(Z_{\text{real}}) = \frac{1}{Z_{\text{max-real}}} = \frac{n}{m(2\pi)} Z_{\text{max-real}} = \frac{6^2}{7(2\pi)112} = 1/136.8$$

Apply with $2\pi$-e formula: $2\pi e^2 \frac{e^2}{2} \frac{e^2}{3} \frac{e^2}{4} \ldots$

the formula is transformed to:

$$\alpha = \frac{n}{m(2\pi)} = \frac{6^2}{7(2\pi)112} = \frac{6^2}{7 \cdot \frac{1}{112} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{2} \cdot \frac{6}{3} \cdot \frac{7}{4} \cdot \frac{113}{112}} = 1/137.035782$$

Above deduction on 2018/4/12, only $(2\pi)_{112}$ gives the closest value to $\alpha$, this coincidence of one part per infinity proves the formula itself is correct.

Added an calibration factor ($\delta = 1/75^2$) on 2018/4/20, the accurate formula is:

$$\alpha = \frac{\lambda_c}{2\pi a_0} = \frac{6^2}{7 \cdot \frac{1}{112} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{2} \cdot \frac{6}{3} \cdot \frac{7}{4} \cdot \frac{113}{112}} = 1/137.035999037435$$


By the same procedure but compared to $\alpha = \frac{2\pi r_c}{\lambda_c}$, the other formula is:

$$\alpha = \frac{2\pi r_c}{\lambda_c} = \frac{m(2\pi)}{\lambda_c} \frac{1}{n Z_{\text{max-real}}} = \frac{13 \cdot \frac{e^2}{2} \frac{e^2}{3} \frac{e^2}{4} \ldots \frac{e^2}{279}}{10^7} \frac{\frac{112}{278}}{3 \cdot 29 \cdot 64} = 1/137.035999111818$$

Discover: 2018/4/24; Revise: 2018/9/18-20 (280 → 278, $- \frac{1}{39^2} + \frac{1}{780^2} \rightarrow - \frac{1}{3 \cdot 29 \cdot 64}$)

Another amazing coincidence is $6^2$ and $10^2$ are square numbers in accordance with $2\pi = (\frac{e}{e^2})^2$

This also demonstrates that $\alpha$ has two values with two kinds of formulas.

As $f(Z_{\text{real}}) = \frac{n}{m(2\pi)}$ or $f(Z_{\text{real}}) = \frac{m(2\pi)}{n}$, $m, n, k, \delta$ should relate to nucleon numbers of nuclides.
7. The Two Most Important Formulas

The above two formulas for $\alpha_1$ and $\alpha_2$ were our first gained formulas and are the most important formulas among their serial formulas which will be given followed in this paper. Calculation to give the values of $\alpha_1$ and $\alpha_2$ is shown in Fig. 3 and Table 2.

Fig. 3. Calculation diagram of $\alpha_1$ and $\alpha_2$ (2018/4-6).

Table 2. Calculation of $\alpha_1$ and $\alpha_2$ (2018/4-6).

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<thead>
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<th>$(2\pi)_k$</th>
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<th>$k$</th>
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In these two formulas (deduced from the modification of $Z_{\text{max}}$), there are some factors which are essentially related to nucleon numbers of some nuclides especially some important stable numbers (stipulated by Chen’s Chirality and Poetry Model of Atomic Nuclei) such as 28, 42, 56, 83, 84, 112, 126, 166, 167, 168 \textit{et al.} And these numbers correlate with each others. This kind of relationship is shown in the follows.

A brief illustration of the relationships between the fine-structure constant and nuclides:

\[ \alpha = \frac{6^2}{7 \cdot (2 \pi)^{1/2}} \frac{1}{112 + \frac{1}{75^2}} = \frac{6^2}{112 + \frac{1}{75^2}} = \frac{1}{1/137.035999037435} \]

Above nuclides indicate that 136 – 138, which can be called the fine-structure constant numbers, definitely relate to 112 and 166 – 168 (double of 56 and 83 – 84, the most stable numbers in nuclides).

\[ \alpha = \frac{6^2}{7 \cdot (2 \pi)^{1/2}} \frac{1}{112 + \frac{1}{75^2}} \]

Relations to nuclides (7 \cdot (2 \pi)^{1/2}) \approx 44; \text{ nuclu} \text{e}

\[ \frac{a}{b} \]

The value of the front part of each above formula is almost equal to 1/(3/2)\textsuperscript{1/2} (because 112 is the element natural proton end and 168 is the element neutral neutron end as shown in \textsubscript{112}Cn\textsubscript{168+5}), so the formulas can be transformed to the follows.

\[ \alpha = \frac{6^2}{7 \cdot (2 \pi)^{1/2}} \frac{1}{112 + \frac{1}{75^2}} \]

2019 / 4 / 25 Relations to nuclides:

\[ \frac{a}{b} \]

2019 / 4 / 25 Relations to nuclides:

\[ \frac{a}{b} \]
8. The Integrated Fine-structure Constant

Multiplication of \( \alpha_1 \) and \( \alpha_2 \) should almost divide out the \( 2\pi \) factors and give \( 3/2 \) and \( 112 \times 112 \) factors, this means \( \alpha_1 \alpha_2 \) is almost equal to \( 112 \times 168 \), so we define \( \alpha_c = (\alpha_1 \alpha_2)^{1/2} \) as the integrated fine-structure constant or Chen’s fine-structure constant.

\[
\frac{1}{\alpha_c^2} = \frac{1}{\alpha_1 \alpha_2} = \frac{2\pi \alpha_1}{\alpha_2} \frac{\lambda_1}{2\pi r_1} = \frac{\alpha_2}{r_2} = \left( \frac{c}{V} \right)^2
\]

\[= 112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{6 \cdot 29 \cdot 53 \cdot 59 \cdot 79 / 47}) \quad 2018 / 6 / 8-9, \ 9 / 18-19, \ 2019 / 4 / 19 \]

\[= 136(138 + \frac{1}{2} - \frac{1}{10 \cdot 29} + \frac{1}{12 \cdot 53 \cdot (6 \cdot 53 - 1) - 27 / 47}) \quad 2019 / 4 / 17-19 \]

\[= 137(137 + \frac{1}{13} - \frac{1}{7 \cdot 29} + \frac{1}{32 \cdot 33 \cdot 89 + 16 / 49}) \quad 2019 / 4 / 17-19 \]

\[= 112 \cdot 167.668437878408 = 18778.85042381 \]

\[
\alpha_c = \alpha_1 \alpha_2 = \frac{6^2}{7 \cdot (2\pi)^{1/12}} \left[ 1 + \frac{1}{112 + \frac{1}{75^2} \frac{13 \cdot (2\pi)^{1/278}}{112 - \frac{1}{3 \cdot 29 \cdot 64}} \right]
\]

\[= 13 \cdot 3^2 \left[ \frac{2 \cdot 3 \cdot 19}{113 \cdot \frac{9 \cdot 31}{2 \cdot 139} \cdot \frac{557}{112} \cdot \frac{1}{3 \cdot 29 \cdot 64} \right] \]

\[= 1 = 18778.85042381 \quad 2019 / 12 / 14 \]

9. A Brief Explanation of the Fine-structure Constant

According to Chen’s Chirality and Poetry Model of Atomic Nuclei, the ratio of neutron number \( N \) to proton number \( Z \) in nuclides increases from 1/1 to 3/2 (eventually slightly above 3/2) along with the increasing of atomic number, for example, from \( ^{14}\text{Si}_{14}, \ ^{26}\text{Fe}_{30}, \ ^{29}\text{Cu}_{34}, \ ^{56}\text{Ba}_{82}, \ ^{84}\text{Po}_{125} \) to \( ^{112}\text{Cn}_{168+5}^* \). In this process, \( (3/2)^{1/2} \) will act as a transition foothold. As for nuclide \( ^{112}\text{Cn}_{168+5}^* \) with \( Z=112, \ N=168+5 \) and \( 168/112=3/2, \ 137 \) is just right their \( (3/2)^{1/2} \) times intermediate stage. This should be why 137 exists and what’s the real meaning of 137.
\[
\frac{112}{1/\alpha_1} \approx 168 - 1/3 \quad \text{or} \quad \frac{112}{1/\alpha_2} \approx 168 - 1/3
\]

\[
137.036^2 \approx 112 \cdot (168 - 1/3)
\]

\[
112 \cdot \left( \frac{3}{2} - \frac{1}{336 + 1} \right)^2 \approx 137.036, \quad 137.036 \cdot \left( \frac{3}{2} - \frac{1}{336 + 1} \right)^2 \approx 168 - 1/3
\]

\[
\frac{112}{1/\alpha_1} \approx 168 \quad \text{or} \quad \frac{112}{1/\alpha_2} \approx 137
\]

\[
137^2 \approx 112 \cdot 168, \quad 112 \left( \frac{3}{2} \right)^2 \approx 137, \quad 137 \left( \frac{3}{2} \right)^2 \approx 168
\]

The relationships between the fine-structure constant and elements are mainly reflected by correlation of nucleon numbers of 56, 68-69, 82, 82-84, 112, 136-138, and 166-168 which are derived from the three key numbers 112, 137 and 168, and by correlation of nucleon numbers of the other factors in Chen’s formulas of the fine-structure constant. The former type is illustrated as follows.

Several clusters of ideal extended elements (ie) Fy and Ch are hence predicted.

10. Comparison to Experiment Determined Values

The above two calculated values of the fine-structure constant, i.e.,
\[\alpha_1 = 1/137.035999037435 \quad \text{and} \quad \alpha_2 = 1/137.035999111818\] are consistent with those experiment measured values\(^2\), but much more accurate with several more digits.

The above theoretical analysis and formulas also demonstrate there are two different values of the fine-structure constant, i.e. \(\alpha_1\) and \(\alpha_2\). Accordingly, we have found that up to now the experiment determinations of \(\alpha\) have almost proved this because the \(\alpha\) ranges measured by two different but accurate methods couldn’t overlap each other\(^2\). It seems that the time comes to a critical point to prove there are two values of the fine-structure constant theoretically and experimentally.

11. Theoretical Calculation of the Speed of Light

In atomic units, the line velocity of the ground state electron in hydrogen atom can be assigned as the natural unit of speed \((v_e/\alpha_u=1)\), then the speed of light becomes the reciprocal of the fine-structure constant, i.e., \(c_{au}=1/\alpha=137.035999\). However, we have demonstrated that there are two values of \(\alpha\), but the speed of light shouldn’t have two values, so by referring to Maxwell’s formula of calculating the speed of
electromagnetic wave or light, it should be reasonable to suppose the speed of light to be the integrated fine-structure constant, i.e., \( c_{au}=1/\alpha_{c}=1/(\alpha_{1}\alpha_{2})^{1/2}=137.035999074627 \).

It means we’ve theoretically/mathematically calculated the speed of light, the formula is intrinsically consistent with Maxwell’s formula, and the value is much accurate.

\[
\text{In atomic units } (e=m=e=1 \text{ and } \hbar=1) \text{, } v_{caw} = \alpha c_{aw} = \frac{e^2}{4\pi \hbar} = 1, \text{ so } c_{aw} = \frac{1}{\alpha}.
\]

There are two \( \alpha \) (\( \alpha_{1} \) and \( \alpha_{2} \)), but there shouldn’t be two \( c \) or \( c_{aw} \),

so it should be: \( c_{aw} = \frac{1}{\alpha_{1}\alpha_{2}} \) (\( \alpha_{1}\alpha_{2} \) atomic units)

Compared to Maxwell Formula \( c = \frac{1}{\sqrt{\mu_{0}\varepsilon_{0}}} \), \( c_{aw} = \frac{1}{\alpha_{1}\alpha_{2}} \) should be reasonable.

\[
c_{aw} = \frac{1}{\mu_{0aw}}, \quad \mu_{0aw} = \alpha_{1}\alpha_{2}, \quad \mu_{aw} = 4\pi\alpha_{1}\alpha_{2} \quad (2019/11/30)
\]

So the theoretical formula of the speed of light in atomic units is as follows:

\[
c_{aw} = \frac{1}{\alpha_{1}\alpha_{2}} = \frac{1}{\sqrt{7\cdot(2\pi)_{112}^{0.02} \cdot 112 + \frac{1}{75}}} = \frac{1}{\sqrt{13\cdot(2\pi)_{2186}}}
\]

\[
= \frac{5}{3} \sqrt{\frac{7}{13\cdot(2\pi)_{2186}}(112^2 - \frac{1}{30^2} + \frac{1}{60^2} - \frac{1}{120^2} - \frac{1}{15^5} - \frac{1}{15\cdot15})}
\]

\[
= \sqrt{\frac{5}{3} \cdot \frac{7}{11\cdot11\cdot11}} \cdot \frac{2\cdot7^2 \cdot 3^2 \cdot 7 \cdot 13}{10^8}
\]

\[
= \frac{3}{2} \cdot \frac{1}{3\cdot112+1} \cdot \frac{1}{7\cdot19\cdot29\cdot37-\frac{25}{44}} \cdot (112^2 - \frac{1}{30^2} + \frac{1}{60^2} - \frac{1}{120^2} - \frac{1}{15^5} - \frac{1}{15\cdot15})
\]

\[
= \frac{3}{2} \cdot \frac{1}{3\cdot112+1} \cdot \frac{1}{12^2\cdot13 \cdot 100 - \frac{1}{125\cdot100}} \cdot \frac{1}{14\cdot53\cdot193 - \frac{33}{2\cdot47}} \cdot 112
\]

\[
= \sqrt{137.03599907435\times137.03599911818} = 137.035999074627
\]

Note: \( 112/278 \approx 27/67, \ 12389/28186 \approx 11/25, \ 34450/28186 \approx 11/9 \approx 66/29 \)

Discover: 2019/12/16; Revise and Supplement: 2020/1/5–8, 2/24

12. The Special 29 and 75 Factors

In the above formulas some factors especially 29 and 75 appear several times. This feature should be analyzed and explained. Accompanying N/Z ratio from 1/1 to slightly above 3/2 along with the increasing of atomic number, \( _{29}\text{Cu}_{34,36} \) is the critical point of N/Z ratio approaching \((3/2)^{1/2}\) and \( _{75}\text{Re}_{110,112} \) is the critical point of N/Z ratio approaching 3/2 (Table 3, Fig. 4 and Fig. 5), so 29 and 75 are important factors and hence frequently appear in the formulas.

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Z: atomic number, \( N \): average neutron number or neutron number of the most stable isotope.

1. \( N/Z \) from 1/1 (\( ^6\)C) to slightly above 3/2 (such as \( ^{112}\)Cn which is the natural end of elements demonstrated by Chen’s Chirality and Poetry Model of Atomic Nuclei\(^7\)).
2. For \( ^{29}\)Cu, \( N/Z \) ratio 1.19 is near to \( (3/2)^{1/2} = 1.22 \), slightly less is because of stability effect.
3. For \( ^{29}\)Re, \( N/Z \) ratio 1.48 is near to \( 3/2 = 1.50 \), slightly less is because of stability effect.
4. From \( ^6\)C to \( ^{112}\)Cn, the middle of \( N/Z \) 1.5 range is at \( (76.5-5)/(112-5) = 0.668 \approx 2/3 \) position.

Fig. 4 and Fig. 5 shows that stability effect of nucleon number 64 makes the neutron numbers of \( ^{29}\)Cu’s isotopes are relatively less (34 and 36) than normal so that its \( N/Z \) ratio is a little less than \( (3/2)^{1/2} \) which is otherwise it should be. Also the
stability effect of nucleon numbers 110 and 112 make the neutron numbers of $^{75}\text{Re}$'s nuclides are relatively less (110 and 112) than normal so that its N/Z ratio is a little less than 3/2 which otherwise it should be.

**Fig. 4. Complete Graph of N/Z Ratios of Elements** (2019/4/23-24).

![Complete Graph of N/Z Ratios of Elements]

3 Li: 4/3
29 Cu: 35.5/29 = $(3/2)^{1/2}$
31 Ga: 38.75/31 = 5/4
64 Gd: 90.5/64 = $(2)^{1/2}$
75 Re: 112.5/75 = 3/2
3, 29, 31, 64, 75 and 112 are factors in $\alpha_1$ and $\alpha_2$.
Calculation is the ideal situation.

**Fig. 5. Partially Amplified Graph of N/Z Ratios of Elements** (2019/4/24).

![Partially Amplified Graph of N/Z Ratios of Elements]
The general trend of N/Z ratio of elements is from 1/1 (6^6C₆) to slightly above 3/2 (112^173Cn) definitely. However, the increasing process is not smooth, the N/Z ratio rising fluctuates consecutively. According to Chen’s Chirality and Poetry Model of Atomic Nuclei⁷, there are some stable numbers (magic numbers) which can bring about this kind of fluctuation (Table 4 and Fig. 6).

Table 4. Effect of Stable Numbers on N/Z ratio’s fluctuation (2019/4/22).

<table>
<thead>
<tr>
<th>Element</th>
<th>Z</th>
<th>N(Average)</th>
<th>Z(3/2)^1/2</th>
<th>N-Z(3/2)^1/2</th>
<th>Stable Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>19</td>
<td>20.13</td>
<td>23.27</td>
<td>-3.17</td>
<td>20</td>
</tr>
<tr>
<td>Ca</td>
<td>20</td>
<td>20.12</td>
<td>24.49</td>
<td>-4.41</td>
<td>20+20</td>
</tr>
<tr>
<td>Sc</td>
<td>21</td>
<td>24</td>
<td>25.72</td>
<td>-1.74</td>
<td></td>
</tr>
<tr>
<td>Ti</td>
<td>22</td>
<td>25.92</td>
<td>26.94</td>
<td>-1.07</td>
<td>22+26=48</td>
</tr>
<tr>
<td>V</td>
<td>23</td>
<td>28.00</td>
<td>28.17</td>
<td>-0.23</td>
<td>28</td>
</tr>
<tr>
<td>Cr</td>
<td>24</td>
<td>28.06</td>
<td>29.39</td>
<td>-1.39</td>
<td>28</td>
</tr>
<tr>
<td>Mn</td>
<td>25</td>
<td>30</td>
<td>30.62</td>
<td>-0.68</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>26</td>
<td>29.91</td>
<td>31.84</td>
<td>-1.99</td>
<td>26+30=56</td>
</tr>
<tr>
<td>Co</td>
<td>27</td>
<td>32.00</td>
<td>33.07</td>
<td>-1.14</td>
<td></td>
</tr>
<tr>
<td>Ni</td>
<td>28</td>
<td>30.76</td>
<td>34.29</td>
<td>-3.60</td>
<td>28+30=58, 28+32=60</td>
</tr>
<tr>
<td>Cu</td>
<td>29</td>
<td>34.62</td>
<td>35.52</td>
<td>-0.97</td>
<td>64</td>
</tr>
<tr>
<td>Zn</td>
<td>30</td>
<td>35.45</td>
<td>36.74</td>
<td>-1.36</td>
<td>30+34=64, 30+36=66</td>
</tr>
<tr>
<td>Ga</td>
<td>31</td>
<td>38.80</td>
<td>37.97</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>Ge</td>
<td>32</td>
<td>40.71</td>
<td>39.19</td>
<td>1.44</td>
<td>32+40=72</td>
</tr>
<tr>
<td>As</td>
<td>33</td>
<td>42.00</td>
<td>40.42</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>Se</td>
<td>34</td>
<td>45.05</td>
<td>41.64</td>
<td>3.30</td>
<td>34+46=80</td>
</tr>
<tr>
<td>Br</td>
<td>35</td>
<td>44.98</td>
<td>42.87</td>
<td>2.03</td>
<td></td>
</tr>
<tr>
<td>Kr</td>
<td>36</td>
<td>47.89</td>
<td>44.09</td>
<td>3.71</td>
<td>36+48=84</td>
</tr>
</tbody>
</table>

Fig. 6. Effect of Stable Numbers on N/Z ratio’s fluctuation (2019/4/22-23)

If there were no stability effect, the N/Z ratio of Cu should be at the red spot.
13. \( \alpha_1/\alpha_2 \) in Schrödinger Equation of Hydrogen Atom

Stationary Schrödinger Equation \(-\frac{\hbar^2}{2m} \nabla^2 \psi + U \psi = E \psi\), applied to hydrogen atom:

\[
\nabla^2 \psi + \frac{2m}{\hbar^2} \left( E + \frac{e^2}{4\pi \varepsilon_0 r} \right) \psi = 0, \quad E = -\frac{m_e^2}{2n^2(4\pi \varepsilon_0)^2 \hbar^2}, \quad \text{do substitution and simplification:}
\]

\[
\frac{2m}{\hbar^2} \left( \frac{m_e^2}{2n^2(4\pi \varepsilon_0)^2 \hbar^2} - \frac{e^2}{4\pi \varepsilon_0 r} \right) \psi = \nabla^2 \psi, \quad \left[ \frac{1}{n^2} \left( \frac{m_e^2}{4\pi \varepsilon_0 \hbar^2} \right)^2 - \frac{2}{r} \frac{m_e^2}{4\pi \varepsilon_0 \hbar^2} \right] \psi = \nabla^2 \psi,
\]

As \( \sqrt{\alpha_1 \alpha_2} = \frac{v}{c} = \frac{\hbar}{m_e c}, \lambda_e = \frac{h}{m_e c} \) and \( \alpha_i = \frac{\lambda_e}{2\pi a_0} \):

\[
\frac{1}{n^2} \left( \frac{\sqrt{\alpha_1 \alpha_2}}{2\pi} \right)^2 = \frac{2}{\alpha_i \sqrt{\alpha_1 \alpha_2}} \psi = \nabla^2 \psi,
\]

\[
\frac{1}{n^2} \left( \frac{\sqrt{\alpha_1 \alpha_2}}{2\pi} \right)^2 = \frac{2}{\alpha_i \sqrt{\alpha_1 \alpha_2}} \psi = \nabla^2 \psi
\]

As \( \alpha_i / \alpha_2 \approx 1 \), simplified to: \( \left[ \frac{1}{n^2} \alpha_i^2 - \frac{2}{\alpha_i} \right] \psi = \nabla^2 \psi \) (factor 2 seems not beautiful)

In atomic units \( (au: e = m_e = \hbar = 1 \text{ and } \varepsilon_0 = \frac{1}{4\pi}) \),

\[
a_{0,uu} = 4\pi \varepsilon_0 \hbar^2 = 1, \quad v_{0,uu} = \frac{e^2}{4\pi \varepsilon_0 \hbar} = 1, \quad c_{0,uu} = \frac{v_{e,uu}}{\alpha_c} = 1 = \frac{1}{\sqrt{\alpha_1 \alpha_2}},
\]

\[
\left[ \frac{1}{n^2} \alpha_i / \alpha_2 \right] - \frac{2}{r_{uu} \sqrt{\alpha_1 / \alpha_2}} \psi = \nabla_{uu} \psi, \quad \text{or} \quad \left( \frac{c_{0,uu}}{\alpha_i^2} - \frac{2c_{0,uu}}{\alpha_i^2} \right) \psi = \nabla_{uu} \psi
\]

the above equation could be called Schrodinger-Chen equation of hydrogen atom, the later form of the equation shows factor 2 is still reasonable and beautiful.

As \( \alpha_i / \alpha_2 \approx 1 \), simplified to: \( \left[ \frac{1}{n^2} - \frac{2}{r_{uu}} \right] \psi = \nabla_{uu} \psi \)

Discover: 2018/4-6; Revise: 2019/12/13 (add \( au \) form)

\[
\alpha_i / \alpha_2 = 137.035999111818, 137.035999037435 = 1.0000000005428 = 1 + \frac{23 \cdot 59}{25 \cdot 10^{11}} = (1 + \frac{23 \cdot 59}{50 \cdot 10^{11}})^2
\]

\[
\sqrt{\alpha_i / \alpha_2} = 1 + \frac{23 \cdot 59}{50 \cdot 10^{11}} = 1.0000000002714
\]

Relations to nuclides:

\[
_{\text{11}}^{23}Na, _{\text{25}}^{50}V, _{\text{52}}^{55}Mn, _{\text{90}}^{99}Ru, _{\text{56}}^{105}Pd, _{\text{81}}^{137}Ba
\]

\[
_{\text{69}}^{118}Sn, _{\text{82}}^{141}Pr, _{\text{100}}^{169}Tm
\]

\[
_{\text{100}}^{185,187}Re, _{\text{112}}^{169}Ra
\]

2019/8/28 – 29

Solution of Schrödinger equation of hydrogen atom gives some quantum numbers such as \( n, l \) and \( m_l \) which determine the electron shell structure and the chemical
properties of atoms. That means Schrödinger equation of hydrogen atom is the base of chemical periodicity of elements. On the other hand, from above analysis, we have already demonstrated the formulas of the fine-structure constant $\alpha$ are derived from Chen’s Chirality and Poetry Model of Atomic Nuclei and hence mainly connected to the stability of atomic nuclei. So, a question is whether and how $\alpha$ is connected to Schrödinger Equation of hydrogen atom. This question should reveal the connection of the theory of electron shell of atoms and the theory of nuclei of elements. The above deduction provides the answer. The fine-structure constant $\alpha$ relates to Schrödinger Equation of hydrogen atom in $\alpha_1/\alpha_2$ way which is subtle and negligible but could show the equation is really reasonable and beautiful.

14. The Two Kinds of General Formulas of the Fine-structure Constant

Based on the above two formulas of $\alpha_1$ and $\alpha_2$, it should be reasonable to assume there are two kinds of serial formulas of $\alpha_1$ and $\alpha_2$ which are listed in follows. Among these formulas, the above two first discovered formulas are the most fundamental and important. Some formulas both with a big $m$ and an extra large $k$ should be more important referring to the trend of the approximate values of $\alpha$.

Approximate formulas:

$$\alpha_{1-\text{approx}} = \frac{n}{m \cdot (2\pi)_k} \frac{1}{112} = \frac{n}{m \cdot \frac{e^2}{(2)^3 \left(\frac{3}{2}\right)^5 \left(\frac{k+1}{k}\right)^{2k+1}}} \approx 1/137.036$$

$$\alpha_{2-\text{approx}} = \frac{m \cdot (2\pi)_k}{n} \frac{1}{112} = \frac{m \cdot \frac{e^2}{(2)^3 \left(\frac{3}{2}\right)^5 \left(\frac{k+1}{k}\right)^{2k+1}}}{n} \approx 1/137.036$$

Accurate Formulas:

$$\alpha_{1-\text{accurate}} = \frac{n}{m \cdot (2\pi)_k} \frac{1}{112 + \delta_1} = \frac{n}{m \cdot \frac{e^2}{(2)^3 \left(\frac{3}{2}\right)^5 \left(\frac{k+1}{k}\right)^{2k+1}}} \frac{1}{112} = 1/137.035999037435$$

$$\alpha_{2-\text{accurate}} = \frac{m \cdot (2\pi)_k}{n} \frac{1}{112 - \delta_2} = \frac{m \cdot \frac{e^2}{(2)^3 \left(\frac{3}{2}\right)^5 \left(\frac{k+1}{k}\right)^{2k+1}}}{n} \frac{1}{112 - \delta_2} = 1/137.035999111818$$

Discover: 2019/6/27; Revise: 2019/7/2-3
### Table 5. Parameters and Results of Approximate Formulas of $\alpha_1$ (2019/7/2).

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>k</th>
<th>$\alpha_{1-m'}$</th>
<th>m</th>
<th>n</th>
<th>k</th>
<th>$\alpha_{1-m'}$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>1</td>
<td>122.265854937</td>
<td>24</td>
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<td>2</td>
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<td>98</td>
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<td>137.035999031</td>
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</tbody>
</table>

### Fig. 7. Results of Approximate Formulas of $\alpha_1$ (2019/7/2).

1. $\alpha_{1,7'}$ is the first most accurate and reasonable formula, so assume $\alpha_1=\alpha_{1,7'}$.
2. The $m$ and $k$ are bigger, the more accurate the $\alpha_{1-m'}$ is. So there should be a big $m=M$ make $\alpha_{1,M'}$ is much close to the real $\alpha_1$. Here $M$ is assumed to be 133 or 170.
\[ \alpha_{-1} = \frac{6}{1} \cdot e^{2} \left( \frac{2}{1} \right) 112 + \frac{17}{2} \cdot e \left( \frac{2}{3} \right) 112 + \frac{1}{2} \cdot e \left( \frac{2}{5} \right) \frac{1}{5} = \frac{1}{137.035999037434} \]

\[ \alpha_{-2} = \frac{2}{1} \cdot e^{2} \left( \frac{2}{3} \right) 112 + \frac{1}{2} \cdot e \left( \frac{2}{3} \right) 112 + \frac{1}{2} \cdot e \left( \frac{2}{5} \right) \frac{1}{7} = \frac{1}{137.035999037435} \]

\[ \alpha_{-3} = \frac{3}{1} \cdot e^{2} \left( \frac{2}{3} \right) 112 + \frac{1}{2} \cdot e \left( \frac{2}{3} \right) 112 + \frac{1}{2} \cdot e \left( \frac{2}{5} \right) \frac{1}{7} = \frac{1}{137.035999037435} \]

\[ \alpha_{-4} = \frac{21}{1} \cdot e^{2} \left( \frac{2}{3} \right) 112 + \frac{1}{2} \cdot e \left( \frac{2}{3} \right) 112 + \frac{1}{2} \cdot e \left( \frac{2}{5} \right) \frac{1}{7} = \frac{1}{137.035999037435} \]

\[ \alpha_{-5} = \frac{5}{1} \cdot e^{2} \left( \frac{2}{3} \right) 112 + \frac{1}{2} \cdot e \left( \frac{2}{3} \right) 112 + \frac{1}{2} \cdot e \left( \frac{2}{5} \right) \frac{1}{7} = \frac{1}{137.035999037435} \]
\[ \alpha_{1-9} = 12 + \frac{1}{2} \cdot \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \ldots \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \] \[ \begin{array}{c}
\alpha_{1-11} = 11 \cdot \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \ldots \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \\
\alpha_{1-13} = 13 \cdot \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \ldots \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \\
\alpha_{1-15} = 16 \cdot \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \ldots \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \\
\alpha_{1-17} = 17 \cdot \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \ldots \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \\
\alpha_{1-19} = 19 \cdot \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \ldots \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \\
\alpha_{1-20} = 2 \cdot \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \ldots \left( \frac{1}{2} \right)^{0.2} \left( \frac{1}{2} \right)^{0.2} \\
\end{array} \]
\[
\alpha_{1,22} = \frac{2 \cdot e^2}{\left( \frac{3}{2} \right)^{5/2}} \frac{e^2}{27 \cdot 29 \cdot 2 \cdot 17 \cdot 23} \cdot 112 + \frac{1}{2 \cdot [2 \cdot 3 \cdot 17 \cdot (10 \cdot 19 + 1)] + 1} \cdot \frac{29}{49} = 1/137.035999037435
\]

\[
\alpha_{1,23} = \frac{23 \cdot e^2}{\left( \frac{3}{2} \right)^{5/2}} \frac{e^2}{23 \cdot 22 \cdot 17} \cdot 112 + \frac{1}{35} \cdot \frac{4 \cdot 13 - 43 - 2 \cdot 29}{16 - 17 - 1} = 1/137.035999037435
\]

\[
\alpha_{1,25} = \frac{5^2 \cdot e^2}{\left( \frac{3}{2} \right)^{5/2}} \frac{e^2}{34 \cdot 32 \cdot 30 \cdot 28 \cdot 26} \cdot 132 + \frac{1}{11-19} \cdot \frac{1}{13^3 (16-17) + \frac{1}{25}} = 1/137.035999037435
\]

\[
\alpha_{1,27} = \frac{27 \cdot e^2}{\left( \frac{3}{2} \right)^{5/2}} \frac{e^2}{67 \cdot 66 \cdot 710} \cdot 112 + \frac{1}{11-47 + \frac{18}{23} + \frac{1}{23 \cdot 6 \cdot 23-137}} = 1/137.035999037434
\]

\[
\alpha_{1,29} = \frac{29 \cdot e^2}{\left( \frac{3}{2} \right)^{5/2}} \frac{e^2}{14 \cdot 23 \cdot 643} \cdot 112 + \frac{1}{6 \cdot 8 \cdot (12 \cdot 26 - 1) + \frac{11}{18}} = 1/137.035999037434
\]

\[
\alpha_{1,31} = \frac{31 \cdot e^2}{\left( \frac{3}{2} \right)^{5/2}} \frac{e^2}{33 \cdot 32} \cdot 112 + \frac{1}{12 \cdot 11-17 \cdot 4.49 + \frac{1}{4.51}} = 1/137.035999037434
\]

\[
\alpha_{1,32} = \frac{2 \cdot 4^2 \cdot e^2}{\left( \frac{3}{2} \right)^{5/2}} \frac{e^2}{41 \cdot 40} \cdot 112 + \frac{1}{25 \cdot 29 - \frac{5 \cdot 83}{19} - \frac{23}{19 \cdot 23}} = 1/137.035999037435
\]
\[
\alpha_{1,3,3} = 170 e^{-2 \frac{V}{1}} - \frac{2}{3} \frac{e^{-2 \frac{V}{1}}}{3} - \ldots - \frac{e^{-2 \frac{V}{1}}}{2} + \frac{1}{2} = \frac{1}{137.035999037436}
\]

\[
\alpha_{1,3,4} = 34 \cdot e^{-2 \frac{V}{1}} - \frac{2}{3} \frac{e^{-2 \frac{V}{1}}}{3} - \ldots - \frac{e^{-2 \frac{V}{1}}}{2} + \frac{1}{2} = \frac{1}{137.035999037436}
\]

\[
\alpha_{1,3,5} = 100 \cdot e^{-2 \frac{V}{1}} - \frac{2}{3} \frac{e^{-2 \frac{V}{1}}}{3} - \ldots - \frac{e^{-2 \frac{V}{1}}}{2} + \frac{1}{2} = \frac{1}{137.035999037436}
\]

\[
\alpha_{1,3,6} = 5 \cdot 3 \cdot \frac{e^{-2 \frac{V}{1}}}{3} - \ldots - \frac{e^{-2 \frac{V}{1}}}{2} + \frac{1}{2} = \frac{1}{137.035999037436}
\]

\[
\alpha_{1,4,3} = 43 \cdot e^{-2 \frac{V}{1}} - \frac{2}{3} \frac{e^{-2 \frac{V}{1}}}{3} - \ldots - \frac{e^{-2 \frac{V}{1}}}{2} + \frac{1}{2} = \frac{1}{137.035999037436}
\]

\[
\alpha_{1,4,4} = 100 \cdot e^{-2 \frac{V}{1}} - \frac{2}{3} \frac{e^{-2 \frac{V}{1}}}{3} - \ldots - \frac{e^{-2 \frac{V}{1}}}{2} + \frac{1}{2} = \frac{1}{137.035999037436}
\]

\[
\alpha_{1,5,9} = 59 \cdot e^{-2 \frac{V}{1}} - \frac{2}{3} \frac{e^{-2 \frac{V}{1}}}{3} - \ldots - \frac{e^{-2 \frac{V}{1}}}{2} + \frac{1}{2} = \frac{1}{137.035999037436}
\]
\[
\alpha_{-96} = \left( \frac{1}{2} \right)^3 \left( \frac{3}{2} \right)^5 \frac{e^2}{1} \left( \frac{16 \cdot 3 \cdot 11 - 1}{27 \cdot 5 \cdot 43 + 1} \right)^{7947} 112 + \frac{163 \cdot (8 \cdot 21 \cdot 37 + 1)}{50 \cdot 10^{11}} \\
= 1/137.035999037435
\]

\[
\alpha_{-103} = \left( \frac{1}{2} \right)^3 \left( \frac{3}{2} \right)^5 \frac{e^2}{1} \left( \frac{3 \cdot 19 - 23}{7 \cdot 11 - 17 + 1} \right)^{1262} 112 + \frac{6 \cdot (12 - (6 \cdot 7 + 1) + 1)}{3 \cdot 4} \\
= 1/137.035999037435
\]

\[
\alpha_{-133} = \left(\frac{1}{2}\right)^3 \left(\frac{3}{2}\right)^5 \frac{e^2}{1} \left(\frac{59 \cdot 210}{13 \cdot (17 \cdot 56 + 1)}\right)^{71(12.29-1)} 112 + \frac{7 \cdot 13 \cdot 23}{50 \cdot 10^{11}} \\
= 1/137.035999037435
\]

\[
\alpha_{-140} = \left(\frac{1}{2}\right)^3 \left(\frac{3}{2}\right)^5 \frac{e^2}{1} \left(\frac{4 \cdot 13 \cdot 37}{3 \cdot (64 \cdot 10 + 1)}\right)^{5487} 112 + \frac{4 \cdot 9 \cdot (2 \cdot 32 \cdot 29 + 47) + 29}{54} \\
= 1/137.035999037435
\]

\[
\alpha_{-155} = \left(\frac{1}{2}\right)^3 \left(\frac{3}{2}\right)^5 \frac{e^2}{1} \left(\frac{19 \cdot 210 - 1}{3 \cdot (2 \cdot 13 \cdot 17 + 1)}\right)^{7977} 112 + \frac{5 \cdot 17 \cdot 31 \cdot (2 \cdot 13 \cdot 17 + 1) - 15}{43} \\
= 1/137.035999037435
\]

\[
\alpha_{-170} = \left(\frac{1}{2}\right)^3 \left(\frac{3}{2}\right)^5 \frac{e^2}{1} \left(\frac{47(12 \cdot 61 + 1)}{2 \cdot 25 \cdot 13 \cdot 53}\right)^{717493} 112 + \frac{43 \cdot 97}{8 \cdot 10^7} \\
= 1/137.035999037435
\]
Table 6. Parameters and Results of Approximate Formulas of $\alpha_2$ (2019/7/3).

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>k</th>
<th>$\alpha_{2-m'}$</th>
<th>m</th>
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Fig. 8. Results of Approximate Formulas of $\alpha_2$ (2019/7/3).

3. $\alpha_{2,13}$ is the first most accurate and reasonable formula, so assume $\alpha_2=\alpha_{2,13}$.
4. The m and k are bigger, the more accurate the $\alpha_{2-m'}$ is. So there should be a big m=M make $\alpha_{2,M'}$ is much close to the real $\alpha_2$. Here M is assumed to be 29 or 253.
\[ \alpha_{2-1} = \frac{e^2}{1} + \frac{e^2}{2} + \frac{e^2}{3} + \frac{e^2}{4} = 1/1.37035999111816 \]

\[ \alpha_{2-4} = \frac{2^2 \cdot e^2}{31} + \frac{2^2 \cdot e^2}{32} + \frac{2^2 \cdot e^2}{33} + \frac{2^2 \cdot e^2}{34} = 1/1.37035999111818 \]

\[ \alpha_{2-5} = \frac{5 \cdot e^2}{39} + \frac{5 \cdot e^2}{40} + \frac{5 \cdot e^2}{41} + \frac{5 \cdot e^2}{42} = 1/1.37035999111818 \]

\[ \alpha_{2-6} = \frac{6 \cdot e^2}{47} + \frac{6 \cdot e^2}{48} + \frac{6 \cdot e^2}{49} + \frac{6 \cdot e^2}{50} = 1/1.37035999111818 \]

\[ \alpha_{2-7} = \frac{7 \cdot e^2}{63} + \frac{7 \cdot e^2}{64} + \frac{7 \cdot e^2}{65} + \frac{7 \cdot e^2}{66} = 1/1.37035999111818 \]

\[ \alpha_{2-9} = \frac{3 \cdot e^2}{70} + \frac{3 \cdot e^2}{71} + \frac{3 \cdot e^2}{72} + \frac{3 \cdot e^2}{73} = 1/1.37035999111818 \]

\[ \alpha_{2-11} = \frac{2^2 \cdot e^2}{11^2} + \frac{2^2 \cdot e^2}{12^2} + \frac{2^2 \cdot e^2}{13^2} + \frac{2^2 \cdot e^2}{14^2} = 1/1.37035999111818 \]
\[
\alpha_{2\to 9} = \frac{19 + \epsilon^2}{\left(\frac{a}{b}\right)^2 + \frac{c^2}{d}} + \frac{1}{112} - \frac{1}{44} + \frac{1}{16} \cdot (4 \cdot 37 + 1) - \frac{23}{6 \cdot 47 + 1} = 1/\lambda_{35999111818} \\
\]

\[
\alpha_{2\to 23} = \frac{23 - \epsilon^2}{\left(\frac{a}{b}\right)^2 + \frac{c^2}{d}} \equiv \frac{1}{112} - \frac{1}{16} \cdot (4 \cdot 23 - 1) + \frac{9}{32 \cdot 10} = 1/\lambda_{35999111818} \\
\]

\[
\alpha_{2\to 24} = \frac{2 \cdot 6 \cdot \epsilon^2}{\left(\frac{a}{b}\right)^2 + \frac{c^2}{d}} \equiv \frac{1}{112} - \frac{1}{257} + \frac{1}{10 \cdot (12 - 13 \cdot 83 + 1)} + \frac{23}{81} = 1/\lambda_{35999111818} \\
\]

\[
\alpha_{2\to 25} = \frac{5 \cdot \epsilon^2}{\left(\frac{a}{b}\right)^2 + \frac{c^2}{d}} \equiv \frac{1}{112} - \frac{1}{193} + \frac{1}{8 \cdot 43} - \frac{1}{18} \cdot 23 \cdot (32 - 27 - 1) - \frac{3}{7} = 1/\lambda_{35999111818} \\
\]

\[
\alpha_{2\to 27} = \frac{3 \cdot 3 \cdot \epsilon^2}{\left(\frac{a}{b}\right)^2 + \frac{c^2}{d}} \equiv \frac{1}{112} - \frac{1}{10 \cdot 41} + \frac{1}{2 \cdot 27 \cdot 43 \cdot (3 \cdot 64 - 1)} - \frac{19}{26} = 1/\lambda_{35999111818} \\
\]

\[
\alpha_{2\to 29} = \frac{29 - \epsilon^2}{\left(\frac{a}{b}\right)^2 + \frac{c^2}{d}} \equiv \frac{1}{112} - \frac{1}{29 \cdot 59 \cdot (12 \cdot 19 + 1) + \frac{19}{29}} = 1/\lambda_{35999111818} \\
\]
\[ \alpha_{2,31} = \frac{31 \cdot e^2}{\left( \frac{3}{2} \right) \cdot \left( \frac{3}{2} \right) \cdot \ldots \cdot \left( \frac{59}{58} \right)} = \frac{1}{137.035999111819} \]

\[ \alpha_{2,32} = \frac{2 \cdot 4^2 \cdot e^2}{\left( \frac{1}{2} \right) \cdot \left( \frac{3}{2} \right) \cdot \ldots \cdot \left( \frac{42}{41} \right)} = \frac{1}{137.035999111819} \]

\[ \alpha_{2,33} = \frac{33 \cdot e^2}{\left( \frac{1}{2} \right) \cdot \left( \frac{3}{2} \right) \cdot \ldots \cdot \left( \frac{139}{138} \right)} = \frac{1}{137.035999111819} \]

\[ \alpha_{2,36} = \frac{6^2 \cdot e^2}{\left( \frac{1}{2} \right) \cdot \left( \frac{3}{2} \right) \cdot \ldots \cdot \left( \frac{191}{190} \right)} = \frac{1}{137.035999111819} \]

\[ \alpha_{2,37} = \frac{37 \cdot e^2}{\left( \frac{1}{2} \right) \cdot \left( \frac{3}{2} \right) \cdot \ldots \cdot \left( \frac{4 \cdot 3}{5 \cdot 17} \right)} = \frac{1}{137.035999111819} \]
\[
\alpha_{-125} = \frac{5 \cdot 5^2 \cdot e^2 \left(\frac{2}{1}\right)^3 \left(\frac{3}{2}\right)^5 \cdots e^2}{31^2} \bigg[ \frac{4294}{4293} \bigg]^{8857} \bigg[ \frac{1}{112} \bigg]^{1/2} - \frac{1}{2159481}
\]

\[
\alpha_{-253} = \frac{5 \cdot 5^2 \cdot e^2 \left(\frac{2}{1}\right)^3 \left(\frac{3}{2}\right)^5 \cdots e^2}{31^2} \bigg[ \frac{253}{28187} \bigg]^{5673} \bigg[ \frac{1}{1945} \bigg]^{1/2} - \frac{1}{10411} \times 8 \times 10^{11}
\]

\[
\alpha_{-269} = \frac{5 \cdot 5^2 \cdot e^2 \left(\frac{2}{1}\right)^3 \left(\frac{3}{2}\right)^5 \cdots e^2}{31^2} \bigg[ \frac{269}{41655} \bigg]^{83019} \bigg[ \frac{1}{2068} \bigg]^{1/2} - \frac{1}{112} \times 5.317 \times 10^{-9}
\]

In above formulas, there are many amazing coincidences. As 136=8×17 and 138=6×23, 17 and 23 both appear in \( \alpha_{1-1} \), \( \alpha_{1-17} \), \( \alpha_{1-22} \), \( \alpha_{1-23} \), \( \alpha_{1-25} \), \( \alpha_{1-59} \), \( \alpha_{1-103} \), \( \alpha_{1-133} \), \( \alpha_{2-17} \) and \( \alpha_{2-23} \), 17 frequently appears in \( \alpha_1 \) and 23 frequently appears in \( \alpha_2 \). 157 and 257 in \( \alpha_{1-50} \) should relate to \( \text{Fm}^{*} \), 173 in \( \alpha_{1-16} \) should relate to \( \text{Cn}^{*} \), and so on. As the factors in formulas of \( \alpha \) are reasonably assumed to relate to nuclides, some ideal extended elements such as \( \text{Fm}^{*}, \text{Cn}^{*} \) and \( \text{Cn}_{257}^{*} \) are predicted.

15. Radius of Electron and Proton

The classical electron radius \( r_e \) has been calculated very accurately. However, the proton charge radius \( r_p \) hasn't yet been determined precisely. Recent two experiments...
measured $r_p$ and had given the best results up to now which was $r_p = 0.833(19) \text{ fm}$ and $r_p = 0.831(19) \text{ fm}$, and hence CODADA revised its recommended data of $r_p$ to $0.8414(19)$ fm. Here we give our calculation results of $r_e$ and $r_p$. And it seems there is $\alpha_p$ similar to $\alpha$. $\alpha_p$ could be called “the second fine-structure constant”.

Ratio of Bohr radius of hydrogen atom to classical electron radius:

$$\frac{a_0}{r_e} = \frac{1}{\alpha} = \frac{1}{\alpha_a} = \frac{112 \times (168 - \frac{1}{3} + \frac{1}{2^2 \cdot 3 \cdot 47} - \frac{1}{2 \cdot 3 \cdot 29 \cdot 53 \cdot 59 - 79 / 47}) = 18788.865042381$$

$$r_e = \alpha^2 a_0 = \alpha_a a_0 = 5.29177210903(80) \times 10^{-13} \frac{m}{18788.865042381} = 2.81794032658(43) \text{ fm}$$

Comparable to CODATA recomended value $r_e = 2.8179403262(13) \text{ fm}$ but more precise.

Ratio of Bohr radius of hydrogen atom to the proton charge radius should have the similar form, and is assumed to have the following hypothetical formulas:

$$\frac{a_0}{r_p} = \frac{1}{\alpha_p} = \frac{1}{\alpha_{p_2}} \approx \frac{1}{\alpha_{p_2}} \approx 252.04, \text{ $\alpha_p$ could be called the second fine-structure constant.}$$


$$\alpha_{p_2} = \frac{1}{5 \cdot 13 + 30 \cdot (28 \cdot (2 \cdot 100 - 1) + 1) + \frac{8}{45}} = 252.040872632515^2$$

$$r_p = \alpha^2 a_0 = \alpha_{p_2} a_0 = 5.29177210903(80) \times 10^{-13} \frac{m}{63524.60147736} = 0.83302720999(13) \text{ fm}$$

$$\alpha_{p_2} \approx \alpha_{p_1} = \alpha_{p_2} \approx 252.04. \text{ $\alpha_p$ could be called the second fine-structure constant.}$$

2020/1/2.
\[ \alpha_{p}\ = \frac{22 \cdot (2\pi)_{164}}{5 \cdot 31} = \frac{1}{25.040872632512} \ 
\text{2020/1/3} \]

\[ \alpha_{p}\ = \frac{21 \cdot (2\pi)_{126}}{2 \cdot 37} = \frac{1}{25.040872632512} \ 
\text{2020/1/3} \]

16. Direct Relationships of 2\(\pi\) with Nuclides

In Chen’s formulas of the fine-structure constant, there are \(2\pi\)-e formulas, in
which \(k\) gets certain numbers and relate to nucleon numbers of some nuclides. So in
the end of this paper we feel curious about whether \(2\pi\) directly relate to nuclides.

\[ 2\pi = 6.2831853 \cdots = \frac{4 \cdot 157}{100} \approx 6.28 \approx \frac{3 \cdot 7 \cdot 44 \cdot 68}{100^2} = 6.2832 \]

\[ 2\pi \approx 16 \cdot 3 \cdot 7 \cdot 11 \cdot 17 = \frac{48 \cdot 7 \cdot 11 \cdot 17}{100} \cdots = \frac{3 \cdot 112 \cdot 11 \cdot 17}{100^2} = \frac{2 \cdot 168 \cdot 11 \cdot 17}{100^3} = \frac{2 \cdot 3 \cdot 7 \cdot 11 \cdot 13 \cdot 16}{100^4} = \cdots = 6.2832 \]

\[ \text{2020/1/8-10} \]
The approximate rational numbers of \(2\pi\) (could be called \(2\pi\) formulas) relate to nuclides marvelously. This means \(2\pi\) (along with \(2\pi\)-e formula) plays important roles in atomic nuclei, and acts as a rational number rather than an irrational number in the world of atomic nuclei.

17. Correlations among \(\alpha\), \(2\pi\) and nuclides

Some Chen’s formulas of the fine-structure constant and \(2\pi\) formulas correlate with each others with the same factors and all together relate to the same nuclides. For example, \(\alpha_{1.50}\) and \(2\pi \approx 4 \times 157/100\) have the same 157 and 100 factors, \(\alpha_{1.50}\) and \(2\pi \approx 3 \times 7 \times 44 \times 68/100^2\) have the same 100, 7, 11 and 16 factors, and they relate to the same corresponding nuclides. They also have common factors with \(\alpha_{1.7}\) and \(\alpha_{2.13}\) which should relate to \(2\pi \approx 5 \times 7^2/3/13\) and \(2\pi \approx 13 \times 29/4/3/5\).

\[
\alpha_{1.9} = \frac{47}{3^2 \cdot e^2 \cdot \left(\frac{2}{1}\right)^3 \cdot \left(\frac{3}{2}\right)^{\frac{1}{5}} \cdot \left(\frac{10}{9}\right)^{19}} + \frac{1}{12} + \frac{1}{16} + \frac{1}{161}
\]

\[
\alpha_{1.50} = \frac{2 \cdot 257}{100} \cdot \frac{e^2}{\left(\frac{2}{1}\right)^3 \cdot \left(\frac{3}{2}\right)^{\frac{1}{3}} \cdot \left(\frac{14 \cdot 13}{181}\right)^{\frac{1}{3}}} + \frac{1}{29 - 61 + \frac{157}{161}}
\]

\[
2\pi \approx \frac{4 \cdot 157}{100} = \frac{157}{25} = 6.28
\]

\[
2\pi \approx \frac{3 \cdot 7 \cdot 44 \cdot 68}{100^2} = \frac{3 \cdot 7 \cdot 11 \cdot 17}{25^2} = 6.2832
\]

\[
\alpha_{1.5} = \frac{47}{3^2 \cdot e^2 \cdot \left(\frac{2}{1}\right)^3 \cdot \left(\frac{3}{2}\right)^{\frac{1}{5}} \cdot \left(\frac{10}{9}\right)^{19}} + \frac{1}{12} + \frac{1}{16} + \frac{1}{161}
\]

\[
\alpha_{2.13} = \frac{13 \cdot e^2}{\left(\frac{2}{1}\right)^3 \cdot \left(\frac{3}{2}\right)^{\frac{1}{3}} \cdot \left(\frac{79}{278}\right)^{\frac{1}{5}}} + \frac{1}{10^2} + \frac{1}{112 - \frac{1}{3 \cdot 29 - 64}}
\]

\[
2\pi \approx \frac{3 \cdot 7 \cdot 44 \cdot 68}{100^2} = \frac{3 \cdot 7 \cdot 11 \cdot 17}{25^2} = 6.2832
\]

\[
\alpha_{1.22}\) relates to \(2\pi \approx 2 \times 22/7, 2\pi \approx 17^2/7/23\) and \(2\pi \approx 2 \times 355/113\) as follows. And \(2\pi \approx 17^2/7/23\) also relates to \(\alpha_{1.1}, \alpha_{1.17}, \alpha_{1.22}, \alpha_{1.23}, \alpha_{1.25}, \alpha_{1.59}, \alpha_{1.103}, \alpha_{1.133}, \alpha_{2.17}\) and \(\alpha_{2.23}\), in which both 17 and 23 factors appear.

\[
\alpha_{1.22} = \frac{113}{22 \cdot e^2 \cdot \left(\frac{2}{1}\right)^3 \cdot \left(\frac{3}{2}\right)^{\frac{1}{5}} \cdot \left(\frac{17^2}{2 \cdot 17 \cdot 23}\right)^{\frac{2}{5}}} + \frac{1}{112 + \frac{1}{2 \cdot 23 \cdot (10 \cdot 19 + 1) + \frac{1}{49}}}
\]

\[
2\pi \approx \frac{2 \cdot 22}{7} = 6.2857\ldots, 2\pi \approx \frac{17^2}{7/23} = 6.2826\ldots, 2\pi \approx \frac{2 \cdot 355}{113} = \frac{4 \cdot 5 \cdot 71}{2 \cdot 113} = 6.2831858\ldots
\]
\[ \alpha_{1,13} \text{ and } \alpha_{1,43} \text{ relate to } 2\pi \approx 3 \times 67/32, \ 2\pi \approx 5 \times 7^2/39, \ 2\pi \approx 17^2/46 \text{ and others as follows.} \]

\[
\alpha_{1,13} = \frac{67}{13 \cdot e^2 - e^2 \cdot \left( \frac{2}{1} \right)^3 \cdot \left( \frac{3}{2} \right)^5 \cdot \left( e^2 \right)^{31} + \frac{1}{112 + \frac{1}{240 + \frac{1}{627 + \frac{1}{13}}}} + \frac{1}{24 \cdot 31 + 401}}
\]

\[
\alpha_{1,43} = \frac{13 \cdot 17}{43 \cdot e^2 - e^2 \cdot \left( \frac{2}{1} \right)^3 \cdot \left( \frac{3}{2} \right)^5 \cdot \left( e^2 \right)^{31} + \frac{1}{112 + \frac{1}{240 + \frac{1}{627 + \frac{1}{13}}}} + \frac{1}{24 \cdot 31 + 401}}
\]

\[2\pi \approx 3 \cdot 67/32, \ 2\pi \approx 5 \cdot 7^2/39, \ 2\pi \approx 17^2/46 \text{ and others as follows.} \]

\[\alpha_{1,11}, \ \alpha_{1,36}, \ \alpha_{2,24}, \ \alpha_{2,23}, \ \alpha_{2,37} \text{ and } \alpha_{2,125} \text{ relate to } 2\pi \approx 9 \times 37/53, \ 2\pi \approx 15 \times 31/2/37 \text{ and } 2\pi \approx (30 \times 17-1)/81 \text{ as follows.} \]

\[
\alpha_{1,11} = \frac{57}{11 \cdot e^2 - e^2 \cdot \left( \frac{2}{1} \right)^3 \cdot \left( \frac{3}{2} \right)^5 \cdot \left( \frac{19}{18} \right)^{31} + \frac{1}{112 + \frac{1}{240 + \frac{1}{627 + \frac{1}{13}}}} + \frac{1}{24 \cdot 31 + 401}}
\]

\[
\alpha_{1,36} = \frac{37}{6 \cdot e^2 - e^2 \cdot \left( \frac{2}{1} \right)^3 \cdot \left( \frac{3}{2} \right)^5 \cdot \left( \frac{2 \cdot 81}{7 \cdot 23} \right)^{31} + \frac{1}{112 + \frac{1}{240 + \frac{1}{627 + \frac{1}{13}}}} + \frac{1}{24 \cdot 31 + 401}}
\]

\[2\pi \approx 9 \times 37/53, \ 2\pi \approx 15 \times 31/2/37 \text{ and } 2\pi \approx (30 \times 17-1)/81 \text{ as follows.} \]
18. Chen’s Mathematic Shell Model of Nuclides

In overall, there are multi-correlations among $\alpha$, $2\pi$ and nuclides. It seems there should be a mathematical shell model of nuclides, in which the core is $2\pi$ formulas and the middle layer is $2\pi$-e formulas and the outer layer is Chen’s formulas of the fine-structure constant (Fig. 9). The nucleon numbers, stability and abundance of nuclides are regulated by these formulas, especially by their integer factors.

![Chen’s Mathematic Shell Model of Nuclides]

Dr. Gang Chen (2020/1/12-13)

Fig. 9

19. Ideal Extended Elements

In the deduction of Chen’s formulas of the fine-structure constant, it was reasonably assumed the factors in them related to nucleon numbers of nuclides, and it seems this assumption is quite correct. So by somewhat correlation and decoding methodology, all 119th to 170th ideal extended elements were predicted (Table 7). In addition, nuclides can even relate to naked $2\pi$’s approximate rational numbers ($2\pi$ formulas). Some typical examples of correlations of ideal extended elements with formulas of $\alpha$ and $2\pi$ are listed as follows.

**Example 1:** Correlations of 100, 121,125,126,157, 257, 169, et al.

\[
\alpha_{1-9} = \frac{47}{3^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdots e^2} \cdot \frac{1}{112 + \frac{10}{9}} \cdot \frac{1}{1 - \frac{3\cdot17}{157}}
\]

\[
\alpha_{1-50} = \frac{2257}{100 \cdot e^2 \cdot e^2 \cdot e^2 \cdots e^2} \cdot \frac{1}{112 + \frac{14\cdot13}{181}} \cdot \frac{1}{29 - \frac{61}{157}} \cdot \frac{1}{16 - \frac{11}{1117}}
\]

\[
\alpha_{2-24} = \frac{5\cdot37}{2^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdots e^2} \cdot \frac{1}{63 \cdot 62} \cdot \frac{1}{112 + \frac{1}{257}} \cdot \frac{1}{10 - \frac{3\cdot83 - 1}{81}}
\]

\[
2\pi \approx \frac{4\cdot157}{100}, \quad 2\pi \approx \frac{16\cdot3\cdot7\cdot11\cdot17}{100}, \quad 2\pi \approx \frac{4\cdot11}{7}, \quad 2\pi \approx \frac{17^2}{2\cdot23}, \quad 2\pi \approx \frac{30\cdot31}{4\cdot37}, \quad 2\pi \approx \frac{4\cdot5\cdot71}{2\cdot113}
\]

\[
\begin{align*}
104 & Ru_{56} \\
\end{align*}
\]
Example 2: Correlations of 83, 126, 84, 125, 209, 112, 173, 285, 115 and 137

\[ \alpha_{16} = \frac{83}{4^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2} = \frac{1}{112 + \frac{1}{28} - \frac{1}{6 \cdot (18 \cdot 41 + 1) + \frac{1}{173}} - \frac{1}{2 \cdot (2 \cdot 75 - 1)}} \]

\[ \alpha_{21} = \frac{3 \cdot 43}{5^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2} = \frac{1}{112 + \frac{1}{319} - \frac{1}{131} - \frac{1}{11 \cdot 9} - \frac{1}{133} \cdot (2 \cdot 136 - 1) + \frac{1}{25}} \]

\[ \alpha_{32} = \frac{15 \cdot 11}{2 \cdot 2^2 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2} = \frac{1}{112 + \frac{1}{22} - \frac{1}{25 - 29} - \frac{1}{5 \cdot 83} - \frac{1}{19 \cdot 23}} \]

\[ \alpha_{20} = \frac{77}{10 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2} = \frac{1}{112 + \frac{1}{11 - 13} - \frac{1}{6 \cdot 37 \cdot (5 \cdot 210 - 1) + \frac{10}{11}}} \]

\[ \alpha_{10} = \frac{67}{2^2 \cdot 22 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2} = \frac{1}{112 + \frac{1}{137} - \frac{1}{2 \cdot 19 - 23 \cdot 59 - \frac{30}{100}}} \]

\[ \alpha_{1} = \frac{139}{17 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2} = \frac{1}{112 + \frac{1}{137} - \frac{1}{11 \cdot 47 + \frac{18}{23} + \frac{1}{6 \cdot 23} - \frac{137}{100}}} \]

\[ \alpha_{2} = \frac{139}{27 \cdot e^2 \cdot e^2 \cdot e^2 \cdot e^2} = \frac{1}{112 + \frac{1}{137} - \frac{1}{2 \cdot 47} + \frac{1}{2 \cdot 31 - (16 \cdot 17 - 1) + \frac{137}{23}}} \]

\[ 2\pi = \frac{16 \cdot 3 \cdot 7 \cdot 11 \cdot 17}{100} \approx \frac{411}{7} \approx \frac{17^2}{23} \approx \frac{17^2}{23} \approx \frac{817}{1176} \approx 0.623 \]

\[ \text{Table 7. Correlations of Ideal Extended Elements (IEE) with Formulas of } \alpha \text{ and } 2\pi. \]

<table>
<thead>
<tr>
<th>IEE</th>
<th>Page</th>
<th>(\alpha)</th>
<th>(2\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\text{H}</td>
<td>171</td>
<td>(\alpha_{16})</td>
<td>2\pi=4\times355/226</td>
</tr>
<tr>
<td>1\text{F}</td>
<td>75</td>
<td>(\alpha_{21})</td>
<td>2\pi=17/46</td>
</tr>
<tr>
<td>1\text{M}</td>
<td>73</td>
<td>(\alpha_{32})</td>
<td>2\pi=17/46</td>
</tr>
<tr>
<td>1\text{L}</td>
<td>77</td>
<td>(\alpha_{20})</td>
<td>2\pi=622/99</td>
</tr>
<tr>
<td>1\text{O}</td>
<td>76</td>
<td>(\alpha_{1})</td>
<td>2\pi=44/7</td>
</tr>
<tr>
<td>1\text{N}</td>
<td>182</td>
<td>(\alpha_{2})</td>
<td>2\pi=44/7 et al.</td>
</tr>
<tr>
<td>1\text{C}</td>
<td>183/185</td>
<td>(\alpha_{2})</td>
<td>2\pi=333/53=465/74 et al</td>
</tr>
</tbody>
</table>

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20. Chen's Picture of Elements and Ideal Extended Elements

<table>
<thead>
<tr>
<th>Periodic Table</th>
<th>Er</th>
<th>82 Pb</th>
<th>92 U</th>
<th>100 Fm</th>
<th>112 Cn</th>
<th>118 Fy</th>
<th>136 Ba</th>
<th>140-142 Ba</th>
<th>166 Hf</th>
<th>168 Hf</th>
<th>169 Hf</th>
<th>170 Hf</th>
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<tr>
<td>Group 4</td>
<td>1</td>
<td>14</td>
<td>26</td>
<td>44</td>
<td>56</td>
<td>69</td>
<td>84</td>
<td>Po</td>
<td>197</td>
<td>113</td>
<td>118</td>
<td>139</td>
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<td>Group 5</td>
<td>H</td>
<td>Si</td>
<td>Fe</td>
<td>Ru</td>
<td>Ba</td>
<td>Sm</td>
<td>At</td>
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<td>Group 6</td>
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</tbody>
</table>

Chen's Picture of Elements and Ideal Extended Elements

Dr. Gang Chen (2018/1-3; 2020/2-2, 5, 17, 19, 22-26)
The relationships between elements and ideal extended elements (the frontier of elements) and an overall picture of them were depicted as above.

21. Some Supplements

Supplement 1:

\[
2\pi \approx \frac{16 \cdot 3 \cdot 7 \cdot 11 \cdot 17}{100^2} = \frac{3 \cdot 7 \cdot 44 \cdot 68}{100^2} = \frac{3 \cdot 112 \cdot 11 \cdot 17}{100^2} = \frac{2 \cdot 168 \cdot 11 \cdot 17}{100^2} = \frac{2 \cdot 3 \cdot 7 \cdot 11 \cdot 136}{100^2} = \cdots = 6.2832
\]

Refer to Section 16; Supplements: \(32,33,34\) and some of the follows

Supplement 2:

\[
2\pi \approx \frac{9 \cdot 7 \cdot 12 \cdot (4 \cdot 13 \cdot 151 + 1)}{10^7} = \frac{63 \cdot 127 \cdot (52 \cdot 151 + 1)}{10 \cdot 100^3} = \frac{63 \cdot 127 \cdot (2 \cdot 3 \cdot 7 \cdot 11 \cdot 17 - 1)}{10 \cdot (8 \cdot 125)^2} = \cdots = 6.2831853
\]

Supplement 3:

Table 8. Relationships of factors in \(\alpha_{1-7}\) and \(\alpha_{2-13}\) with primordial nuclides (2020/2/16-17).

<table>
<thead>
<tr>
<th>Nuclides</th>
<th>(3)Li(_4)</th>
<th>(29)Cu(_{34})</th>
<th>(31)Ga(_{40})</th>
<th>(64)Gd(_{92})</th>
<th>(75)Re(_{112})</th>
</tr>
</thead>
<tbody>
<tr>
<td>PN</td>
<td>5</td>
<td>70</td>
<td>78</td>
<td>209</td>
<td>252</td>
</tr>
<tr>
<td>PN all</td>
<td>285</td>
<td>285</td>
<td>285</td>
<td>285</td>
<td>285</td>
</tr>
<tr>
<td>Ratios</td>
<td>1/57</td>
<td>14/57</td>
<td>26/95</td>
<td>11/15</td>
<td>84/95</td>
</tr>
</tbody>
</table>

1. \(3, 29, 31, 64, 75\) and 112 are factors in \(\alpha_1\) and \(\alpha_2\).
2. PN: primordial nuclides; PN all: usually regarded as 286.
3. Nucleon number 285 of \(112\)Cn would relate to PN all, or PN all should be 285 rather than 286.
4. \(235\)U\(_{141}\) should not be a primordial nuclide, its relative stability (but not much stable) should come from relative stable nucleon numbers 92=96-4 and 143=11 \(\times\) 13, so number of PN would become 285 from 286.
Supplement 4: Correlations of factors in $\alpha$ ($\alpha_{1-7}$, $\alpha_{2-13}$ and $\alpha_{1-50}$) and nuclides

Dr. Gang Chen, 2020 / 2 / 18 – 19

In this scheme there are several important clues based on factors in the formulas of $\alpha_{1-7}$, $\alpha_{2-13}$ and $\alpha_{1-50}$ such as 6 (36, 48, 138, 144, et al), 7 (56, 84, 112, 126, 166-168, 210, 252), 10 (30, 50, 70, 100, 120, 130, 170, 200, 210, 220, 250, 310, 330, 400, 420), 11 (44, 88, 121, 134, 176, 187, 209, 220, 330, 363), 13 (26, 143, 169, 221, 364), 29 (87, 145, 348), 25 (75, 100, 125, 200, 250, 400), 31 (93 124 186 310, 372), 29 (87, 145, 348), 25 (75, 100, 125, 200, 250, 400), 31 (93 124 186 310, 372), 61 (122, 244), 64 (136 et al), 137 (68, 69, 136,138), 139, 157 (314), 257, et al. And these clues correlate each others. These relationships are strong proofs that Chen’s formulas of the fine-structure constant are correct, otherwise so many coincidences couldn’t be explained.

In addition, numbers 7, 13 and 50 in $\alpha_{1-7}$, $\alpha_{2-13}$ and $\alpha_{1-50}$ may have the following relationships: $(13+7) (13-7) = 50+70 = 120$ and $50\text{Sn}^{70}$. And Sn is special, it has the most stable nuclides (up to 10) among which $50\text{Sn}^{70}$ has the most relative abundance.

Supplement 5: Other two formulas of the fine-structure constant

\[ \alpha_{1-9/11} = \frac{9}{11 + \frac{1}{84} - \frac{1}{117.53 + \frac{2}{191}}} + \frac{1}{122 + \frac{1}{75^7}} = 1/137.035999037435 \]

\[ \alpha_{2-20/25} = \frac{1}{20 + \frac{1}{2} - \frac{1}{14} + \frac{1}{251} - \frac{1}{8 \cdot (12.43 - 227 + 1) - \frac{8}{37}}} = 1/137.035999111818 \]
Supplement 6: Other formulas of the speed of light $c_{au}$

$$c_{au} = \frac{1}{\sqrt{\alpha_1 \cdot 0.1 \alpha_2 \cdot 20/25}}$$

\[
\begin{align*}
(11 + 1/84) = & \frac{1}{11 \cdot 17 \cdot 53 + 3 \cdot 157} (112 + 1/75) \cdot 25 \cdot (112 - 1/3 \cdot 29 \cdot 64) \\
= & 9 \cdot 20 + 1/2 - 1/14 + 1/251 - 1/8 \cdot (12 \cdot 43 \cdot 227 + 1) - 8/37
\end{align*}
\]

\[
\begin{align*}
(11 + 1/84) = & \frac{1}{11 \cdot 17 \cdot 53 + 3 \cdot 157} (112^2 - 7 \cdot 19) + 2^3 \cdot 3^2 \cdot 5^2 \cdot 29 \cdot 2^3 \cdot 3^2 \cdot 5^2 \cdot 29 \\
= & 5 \cdot 5 \cdot 3 \cdot 112 + 2 \cdot 191 \cdot 2 \cdot 173 \cdot 116 + 37 \cdot 177 \cdot 117 \cdot 177 \cdot 157 \cdot 243 \cdot 169 \cdot 257
\end{align*}
\]

\[
\begin{align*}
(11 + 1/84) = & \frac{1}{11 \cdot 17 \cdot 53 + 3 \cdot 157} (112^2 - 1/3 \cdot 3 \cdot 37 \\
= & 20 + 1/2 - 1/14 + 1/251 - 1/8 \cdot (12 \cdot 43 \cdot 227 + 1) - 8/37
\end{align*}
\]

$$= \sqrt{137.035990037435 \times 137.035999111818} = 137.035999074627$$

Note: $112 \times 5/3 \approx 187 = 11 \times 17$, $112 \times 25/3 \approx 5 \times 11 \times 17$ 2020 / 2 / 24

$$c_{au} = \frac{25 \cdot 112}{3} \sqrt{11 \cdot 12 \cdot 13 - \frac{1}{2 \cdot 17 \cdot 41 \cdot 163 + 47}} = 137.035999074628$$

Note: $112 \times 5/3 \approx 187 = 11 \times 17$, $112 \times 25/3 \approx 5 \times 11 \times 17$ 2020 / 2 / 24
Supplement 7: Comparison of formulas of 1, N, e, 2π, π/2, φ, α, αc, cau and αp/c

\[
1 = 4\gamma_e + \frac{4\gamma_i}{(1+1)} + \frac{4\gamma_2}{2(2+1)} + \frac{4\gamma_3}{3(3+1)} + \cdots
\]

\[
= \left|\beta\right| \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{|B_{2n}|(\pi/2)^{2n}}{(2n)!} = \sum_{n=1}^{\infty} \frac{|B_{2n}|\pi^{2n}}{(2n)!} = \left|\beta\right| \frac{3\pi}{2} + \sum_{n=1}^{\infty} \frac{|B_{2n}|(3\pi/2)^{2n}}{(2n)!}
\]

\[N \sim -\frac{3}{2} \left|\beta\right| + \sum_{n=1}^{\infty} \frac{|B_{2n}|(2\pi)^{2n}}{2(2n)!} \]

\[e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\]

\[2\pi = \left(\frac{e}{e^i}\right)^2 = e^2 \frac{e^2}{2} \frac{e^2}{3} \frac{e^2}{4} \cdots, \quad \frac{\pi}{2} = \left(\frac{e^i}{e^i}\right)^2 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\]

\[2\pi \approx \frac{4.157}{100} \approx \frac{9.737}{53} \approx \frac{4.571}{15^2 + 1} \approx \cdots, \quad \frac{\pi}{2} \approx \frac{157}{25} \approx \frac{9(9+1/4)}{53} \approx \frac{5.71}{15^2 + 1} \approx \cdots\]

\[\phi_1 = \sqrt{\frac{5}{2}} - \frac{1}{2} = 0.618 \cdots, \quad \phi_2 = -\frac{\sqrt{5} + 1}{2} = -1.618 \cdots\]

\[\sqrt{\frac{5}{2}} + 2 - \frac{\sqrt{5} + 1}{2} = \frac{e^{\frac{2\pi}{2}}} {1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \cdots}}} \quad \text{(Ramanujan Formula)}\]

\[\alpha_1 = \frac{6^2}{7 \cdot e^2 \frac{e^2}{2} \frac{e^2}{3} \cdots} = \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435\]

\[\alpha_2 = \frac{13 \cdot e^2 \frac{e^2}{2} \frac{e^2}{3} \cdots} {10^2} = \frac{1}{112 - \frac{1}{3 \cdot 29 \cdot 64}} = 1/137.0359991118181\]

\[\frac{1}{\alpha_1 \alpha_2} = 112 \times \frac{1}{168 - \frac{1}{3}} + \frac{1}{12 \cdot 47} + \frac{1}{6 \cdot 29 \cdot 53 \cdot 59 - 79/47} = 137.035999074627^2 = 18778.865042381\]

\[c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_c \alpha_1 \alpha_2}} = 5 \frac{7 \cdot (2\pi)^{112}}{3} \frac{13 \cdot (2\pi)^{278}}{112^2 \frac{1}{30^2 \cdot 5} + \frac{1}{60^2 \cdot 15} - \frac{1}{120^2 \cdot 15 \cdot 29}}\]

\[= \frac{25 \cdot 112}{3} \frac{1}{47} + \frac{1}{40 \cdot 89} - \frac{1}{16 \cdot 9 \cdot (2 \cdot 21 \cdot 31 \cdot 43 + 1) + \frac{3}{4}}\]

\[= \frac{25 \cdot 112}{3} \frac{1}{46} - \frac{1}{55 \cdot 100} + \frac{1}{9 \cdot 25 \cdot 13 \cdot (20 \cdot 293 + 1) - \frac{4}{23}} = \cdots = 137.035999074627\]

\[\frac{1}{\alpha_{p/c}} = \frac{1}{\alpha_p \alpha_{p/2}} = 252.040872632515^2 = 63524.60147736 \quad \text{(Supposed)}\]
The relations of the above formulas are sophisticated. In general, some formulas such as 1, N, e and 2π have similar form (called the natural group form), some formulas such as φ, α, αc and c_{au} can be divided into rational parts and irrational parts for each which may imply they have the same reasonability. In addition, 2π, π/2, φ, α, αc, c_{au} and α_{p/c} are all proportional constants, so they should have some similar or the same regularities.

Supplement 8: Comparison of pictures of elements and φ

With the hints of the above formulas, it is not strange that the gold section (φ≈0.618) appears in the elements, it should appears in some places with some forms.

Imagine a one-dimension creature lives in the line 0-1, he is familiar with 0-0.618 line space and can reach 0.618-1 line space, if he is enough smart, he may feel there should be an ideal extended line space from 0 to -1.618, but he couldn't reach all or can only get the margin of it. The same situation is suitable for us, we live in the space of elements, we mainly utilize the stable elements and can use some radioactive elements before the 112th element Cn, moreover, there should be a space for ideal extended elements from the 119th to the 170th, a few of which we can synthesize, many of which we can't, but this space should exist. This situation is also suitable for our lives in the earth, the solar system and the universe, or even in the mater, dark matter and dark energy, except that the proportion ratios should be different.
22. Discussion and Conclusion

Regarding the fine-structure constant, Richard Feynman said: “is it related to \( \pi \) or perhaps to the base of natural logarithms?” Our answer is that it relate to \( 2\pi \)-e formula. He also deduced that the maximum element should be the 137\textsuperscript{th} element Fynmanium (Fy) based on the analyses of the electron line velocity of his ideal hydrogen-like atoms. Our answer is that the natural end of the elements is the 112\textsuperscript{th} element Copernicium (Cn\textsuperscript{7}) , but the elements could have some ideal extensions, and above all, the fine-structure constant does relate to elements.

So, based on the analyses of ideal and real natural maximum element, Chen’s Chirality and Poetry Model of Atomic Nuclei\textsuperscript{7} and \( 2\pi \)-e formula\textsuperscript{6,7,8}, we deduced two series of Chen’s formulas of the fine-structure constant which gave two values \( \alpha_1=1/137.035999037435 \) and \( \alpha_2=1/137.035999111818 \). The factors in the formulas are much coincident to nucleon numbers of some nuclides, this means the formulas should be correct (too many coincidences mean too few possibilities to be wrong, or too many coincidences imply science). And we indicate the reason of \( \alpha \approx 1/137.036 \) is that it’s almost the equal ratio factor between 112 and 168 (more precisely 168-1/3) which are the key stable numbers (magic numbers) in Chen’s Chirality and Poetry Model of Atomic Nuclei\textsuperscript{7}.

With Chen’s formulas of the fine-structure constant, we predicted the nucleon numbers of all 119\textsuperscript{th} to 170\textsuperscript{th} ideal extended elements; we theoretically or mathematically calculated the speed of light in atomic units, i.e., \( c_{\text{nu}}=1/\alpha_c=1/(\alpha_1\alpha_2)^{1/2}=137.035999074627 \); we deduced a concise Schrödinger-Chen equation of hydrogen atom which included \( \alpha_1/\alpha_2 \) factor; in analogy to \( \alpha \) and its formulas, \( \alpha_p \) (the second fine-structure constant) and its formulas were hypothesized, and the proton charge radius \( r_p \) was supposed to be 0.833027203 fm; in the end we discovered that the approximate rational numbers of \( 2\pi \) marvelously and directly related to nuclides. Based on these, a mathematic shell model of elements was established and a picture of elements and ideal extended elements was depicted.

In their relations to nuclides, \( 2\pi \) formulas can only be certain approximate rational numbers and \( 2\pi \)-e formulas in Chen’s formulas of the fine-structure constant
can only take certain k values. So we also believe the two values of the fine-structure constant should be rational numbers with definite digits rather than irrational numbers with infinite digits, and actually the fine-structure constant is transformed to nucleon numbers of 136, 137 and 138 in the world of nuclides.

In a recent paper, physicist Nicolas Gisin commented that in 1920s there once was a debate between mathematicians David Hilbert and Luitzen Egbertus Jan Brouwer. Hilbert was promoting formalized mathematics, in which every real number with its infinite series of digits is a completed individual object. On the other side the Luitzen Egbertus Jan Brouwer was defending the view that each point on the line should be represented as a never-ending process that develops in time, a view known as intuitionistic mathematics. Hilbert and his supporters clearly won the debate. In consequence, formalized mathematics has been adopted as the language of physics. In the end of his paper, Nicolas Gisin said: “Physics can be as successful if built on intuitionistic mathematics, even if this breaks its marriage to determinism. Contrary to usual expectations, I bet that the next physical theory will not be even more abstract than quantum field theory, but might well be closer to human experience.”

In this paper we adopted mathematical language like intuitionistic mathematics, but we go ahead even more. The formulas of $2\pi$, $2\pi e$ and the fine-structure constant consist of integer factors and relate to nucleon numbers of nuclides, and hence correlate with each others. So in this paper we may use super-intuitionistic mathematics or decoding methodology with features of multi-correlations of integer factors or rational numbers which relate to nucleon numbers of nuclides, and it seems it is the real language in the world of nuclides. As we know an atomic nucleus is a N-body system and chaos should be its real state, so it seems N-body chaos returns to integers. In overall, Leopold Kronecker’s famous saying “God made the integers, all else is the work of man” should be correct in the world of nuclides or even in other fields of the real world. It seems an irrational number can only be a rational number to play roles in the real world.

“God is a pure mathematician!” declared British astronomer Sir James Jeans (1877-1946). The physical Universe does seem to be organised around elegant
mathematical relationships\textsuperscript{3}. The fine-structure constant may be the most important number in physics. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. And we have successfully given reasonable and precise formulas of it. In some sense, we explain the bridge between mathematics and physics, or we may realize the unification of mathematics and physics. It seems we prove the saying “God is a pure mathematician”. At least, it seems that good mathematics means good physics, and some pure mathematical numbers do have scientific meanings.

References
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Acknowledgements
The main author Dr. Gang Chen studied in the Department of Chemistry at Sichuan University from 1983 to 1987 (Bachelor’s degree), in Institute of Chemistry of the Academy of Sciences of China from 1987-1990 (Master’s degree under supervision of Prof. Rongben Zhang),
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Thank Ramanujan, the famous Indian mathematician. His story, the picture of his Goddess and the film about him gave me some inspirations in this work.

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Appendix I: Research History

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**Direct relationships of 2π with nuclides**

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**Some correlations of formulas of 2π and α**

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**Chen’s Mathematic Shell Model of Nuclides**

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### Table: Direct relationships of 2π with nuclides

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